

MODULE 4

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CENTROID AND MOMENT OF INERTIA OF ENGINEERING SECTION...

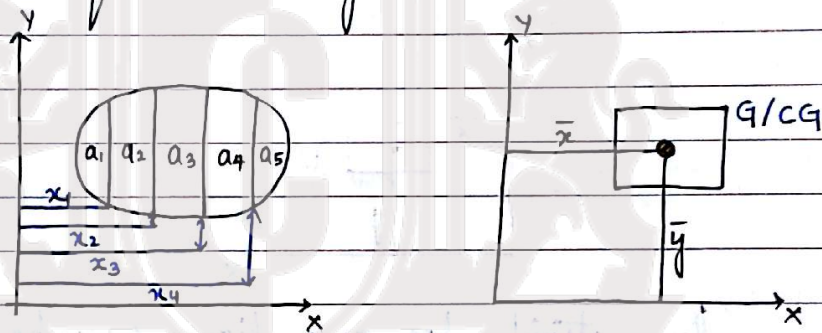
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- **CENTRE OF GRAVITY:** Centre of gravity of a body is a point through which the whole weight of the body acts at the centre. It is denoted by centre of gravity (CG) or G .
- **CENTROID:** The point at which the total area of the plane figures is assumed to be concentrated at the centre.

Centroid and centre of gravity are at same point.

- **Determination of centroid by the moment :-**



$$\bar{x} = \frac{\sum ax}{\sum a} \quad \bar{y} = \frac{\sum ay}{\sum a}$$

Let us consider a body of weight of w as shown in figure. The centre of gravity of whole figure is located at a distance $\bar{x}$ from y -axis and $\bar{y}$ from x -axis.

Let us divide the whole figure into no. of elemental strips of weight a_1, a_2, a_3, a_4 and whose centroids are located by the distance x_1, x_2, x_3, x_4 from y -axis and y_1, y_2, y_3, y_4 from x -axis.

Now by applying the theorem of moment we have

$$\sum ax = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4$$

$$\Sigma ay = a_1 y_1 + a_2 y_2 + a_3 y_3 + a_4 y_4$$

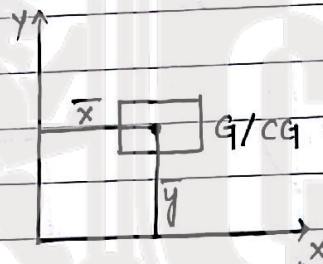
$$\Sigma a = a_1 + a_2 + a_3 + a_4$$

From y-axis, $\bar{x}$ is given by :-

$$\bar{x} = \frac{\Sigma ax}{\Sigma a}$$

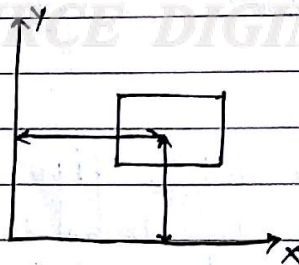
From x-axis, $\bar{y}$ is given by :-

$$\bar{y} = \frac{\Sigma ay}{\Sigma a}$$

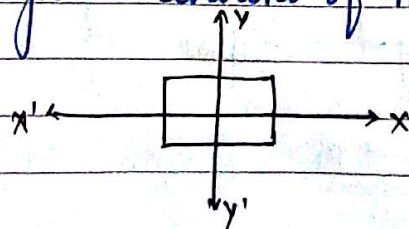


The centroid for any figure can be determined by calculating $\bar{x}$ and $\bar{y}$.

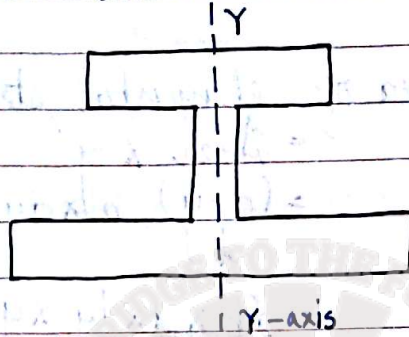
- AXIS REFERENCE: These are the axis with respect to which the centroid of the given figure is determined.



- CENTROIDAL AXIS: This is the axis which passes through a centroid of the given figure.



- SYMMETRICAL AXIS:- The axis which divides the whole figure into equal parts is known as symmetrical axis.

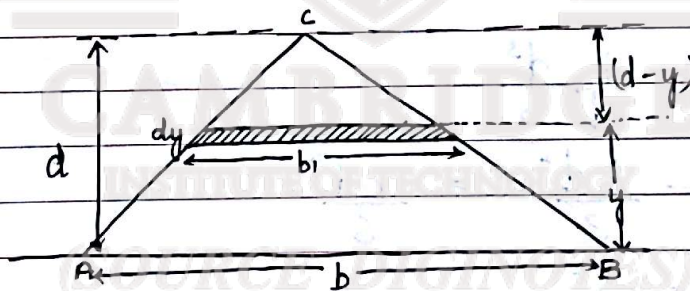


If the section is symmetrical about x-axis then $\bar{y} = 0$ and $\bar{x} = \frac{\sum ax}{\sum a}$

If the section is symmetrical about y-axis then $\bar{x} = 0$ and $\bar{y} = \frac{\sum ay}{\sum a}$

If the section is symmetrical about both axes then $\bar{x} = 0$ and $\bar{y} = 0$

- DERIVATION OF CENTROID OF TRIANGLE :



Consider a triangular lamina ABC of area $\frac{1}{2}bh$ as shown in figure:-

Now consider an elemental stripe of area of $(b_1 \times dy)$ at a distance of y from AB. Using property of similar Δ , $\frac{b_1}{b} = \frac{d-y}{d}$

$$\Rightarrow b_1 = \frac{(d-y)b}{d}$$

Area of elemental strip is given by $b \cdot x dy$
 $= \frac{(d-y)b}{d} dy$

Moment of Area of elemental strip about AB =

$$= \text{Area} \times y$$

$$= \frac{(d-y)b}{d} \times dy \times y$$

$$= \frac{b \times d \times dy \times y}{d} - \frac{y \times b \times dy \times y}{d}$$

$$= bxy \times dy - \frac{y^2 \times b \times dy}{d}$$

Now area of the whole Δ is

$$\int_0^d bxy \times dy - \int_0^d \frac{y^2 b}{d} dy$$

$$= b \left[\frac{y^2}{2} \right]_0^d - \frac{b}{d} \left[\frac{y^3}{3} \right]_0^d$$

$$= \frac{bd^2}{2} - \frac{b}{d} \frac{d^3}{3}$$

$$= \frac{bd^2}{2} - \frac{bd^2}{3} = \frac{bd^2}{6}$$

$$\therefore \Sigma ay = \frac{bd^2}{6}$$

$$\Sigma a = \frac{1}{2} \times b \times d$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{\frac{bd^2}{6} \times 2}{\frac{1}{2} \times bd} = \frac{d}{3} \Rightarrow \bar{y} = \frac{d}{3}$$

from the base $\bar{y} = d/3$

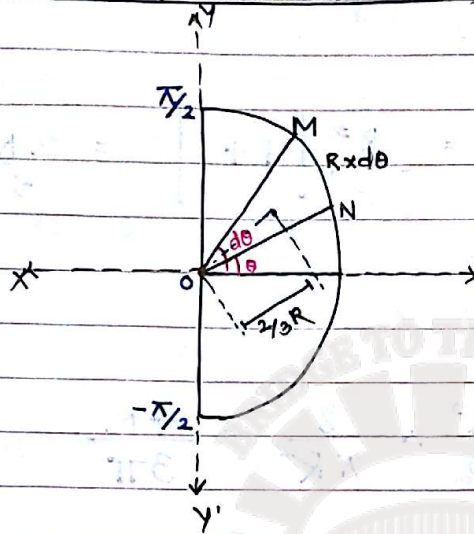
from the apex its $\bar{y} = 2d/3$ //

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• DERIVATION OF CENTROID OF SEMICIRCLE :-

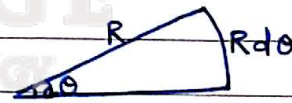


Consider a semicircular lamina of area $\frac{\pi R^2}{2}$ as shown in figure.

Now consider a triangular elemental strip OMN of area $\left(\frac{1}{2} \times R \times R d\theta\right)$ at an angle of θ from x-axis and whose centre of gravity is at a distance of $\frac{2}{3} R$ from the point O.

$$CG = \frac{2}{3} R$$

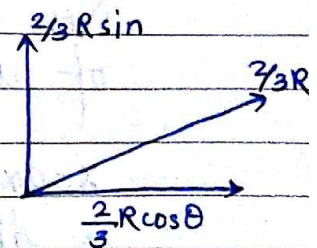
$$\Rightarrow \text{Area of OMN} = \frac{1}{2} \times R \times R d\theta$$



$$\Rightarrow CG = \frac{2}{3} R$$

The projection of centre of gravity on x-axis is given by $\frac{2}{3} R \cos \theta$.

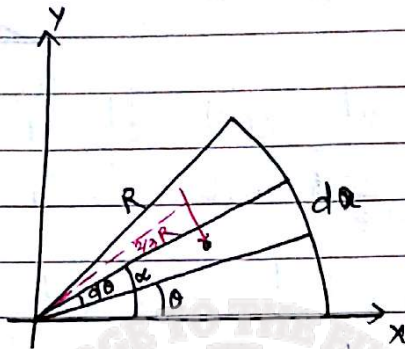
Moment of area of elemental strip about y-axis is given by :-



$$A \times \frac{2}{3} R \times \cos \theta$$

$$\Rightarrow \frac{1}{2} \times R^2 d\theta \times \frac{2}{3} R \cos \theta = \frac{R^3}{3} \cos \theta d\theta$$

• DERIVATION OF CENTROID OF SECTOR OF A CIRCLE:



Consider a sector of circular lamina as shown in figure.

Consider a Δ angular elementary strip ($\frac{1}{2} R^2 d\theta$) at an angle of θ from x-axis whose centre of gravity is $\frac{2}{3} R$ from point O & Projection is $\frac{2}{3} R \cos \theta$.

$$\text{Area of strip} = \frac{1}{2} \times R \times R \times d\theta$$

$$\text{Area of sector} = \int_0^\alpha \frac{1}{2} R^2 d\theta = \frac{1}{2} R^2 \alpha \left\{ \left[\frac{1}{2} R^2 \theta \right]_0^\alpha \right\}$$

$$\begin{aligned} \text{Moment of area of elementary strip} &= \frac{2}{3} R \cos \theta \times \frac{1}{2} R^2 d\theta \\ &= \frac{R^3 \cos \theta d\theta}{3} \end{aligned}$$

Sum of moment of all strips about y-axis

$$= \int_0^\alpha \frac{R^3 \cos \theta}{3} d\theta = \frac{R^3}{3} [\sin \theta]_0^\alpha$$

$$\Sigma ax = \frac{R^3}{3} \{ [\sin \alpha] - \sin 0 \} = \frac{R^3 \sin \alpha}{3}$$

$$\Sigma a = \frac{R^2 \alpha}{2}$$

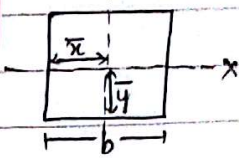
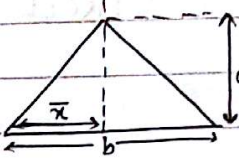
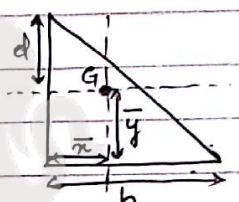
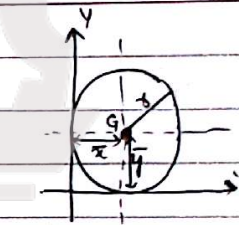
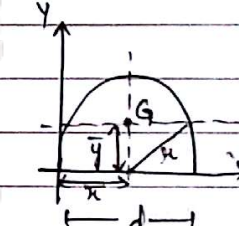
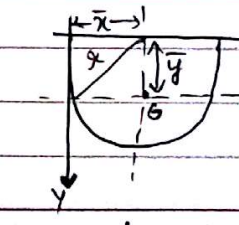
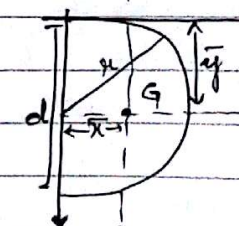
$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{R^3 \sin \alpha}{3} \times \frac{2}{R^2 \alpha} = \frac{2R \sin \alpha}{3\alpha}$$

FORMULAE

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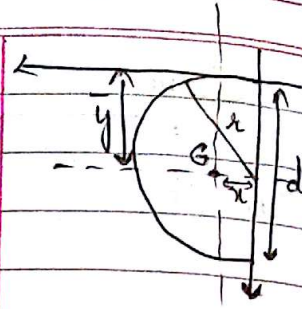
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Shape	Area	$\bar{x}$	$\bar{y}$	Figure
① RECTANGLE	$b \times d$	$\frac{b}{2}$	$\frac{d}{2}$	
② TRIANGLE	$\frac{1}{2} \times b \times d$	$\frac{b}{2}$	$\frac{d}{3}$	
③ RIGHT-ANGLED TRIANGLE	$\frac{1}{2} \times b \times d$	$\frac{b}{3}$	$\frac{d}{3}$	
④ CIRCLE	πr^2	$\bar{x} = r$	$\bar{y} = r$	
⑤ SEMI-CIRCLE	$\frac{\pi r^2}{2}$	$\frac{d}{2}$	$\frac{4r}{3\pi}$	
		$\frac{d}{2}$	$-\frac{4r}{3\pi}$	
		$\frac{4r}{3\pi}$	$\frac{d}{2}$	

$$-\frac{4r}{3\pi}$$

$$\frac{d}{2}$$

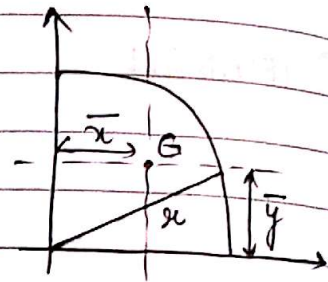


QUARTER
CIRCLE

$$\frac{\pi r^2}{4}$$

$$\frac{4r}{3\pi}$$

$$\frac{4r}{3\pi}$$



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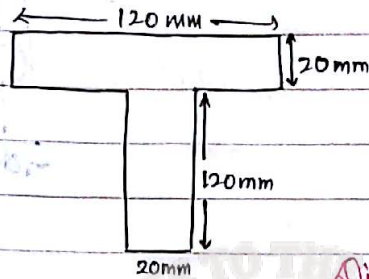
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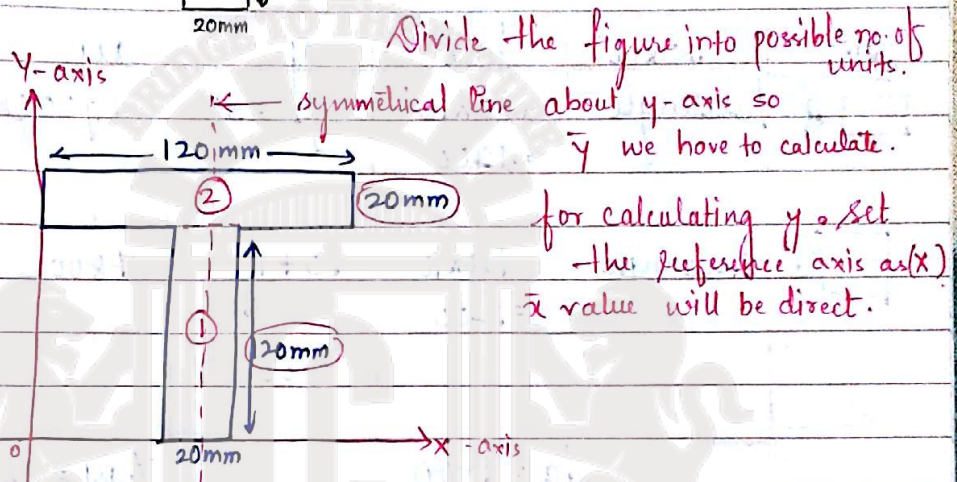
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Q.1- Find the centroid for the given figure shown :-



Ans -



Component	Area (mm ²)	y (mm)	ay (mm ³)
① Rectangle	120 × 20 = 2400 mm ²	$\frac{120}{2}$ = 60 mm	144000
② Rectangle	120 × 20 = 2400 mm ²	$\frac{120 + 20}{2}$ = 130 mm	312000

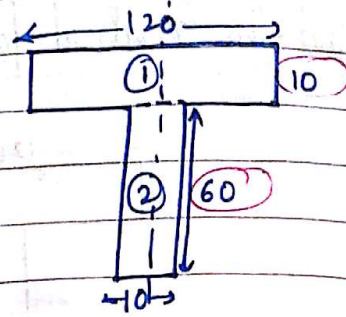
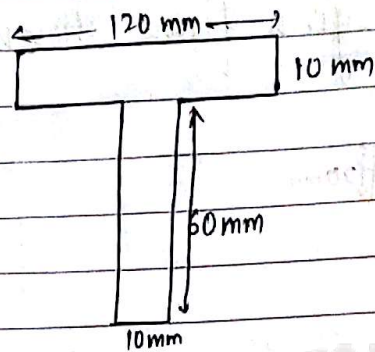
$$\Sigma a = 4800$$

$$\Sigma ay = 456000$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{456000}{4800} = 95 \text{ mm} //$$

$$\bar{x} = (\text{Reference is y-axis}) = \frac{120}{2} = 60 \text{ mm} //$$

Q.2-



Ans-

Component
RectangleArea
 60×10
 $= 600$ y
 $\frac{60}{2} = 30$ ay
 18000 

Rectangle

 120×10
 $= 1200$ $60 + \frac{10}{2}$
 $= 65$ 78000

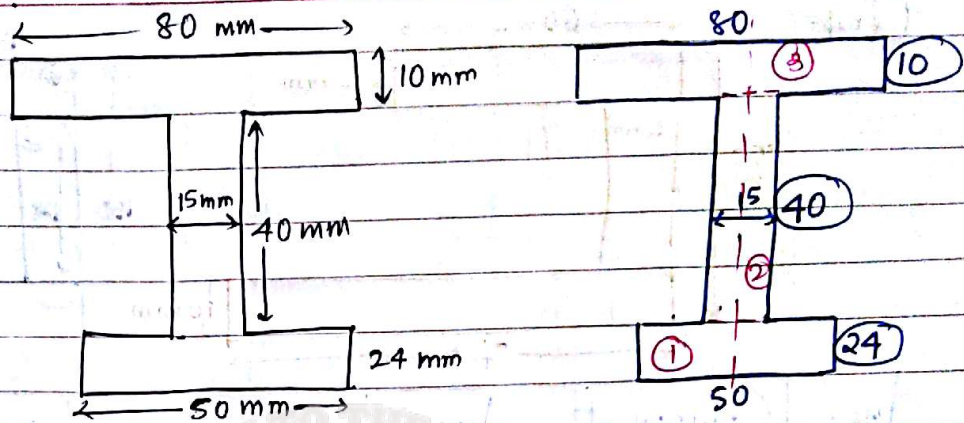
$$\Sigma a = 1800$$

$$\Sigma ay = 96000$$

$$\therefore \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{96000}{1800} = 53.33 \text{ mm.}$$

$$\bar{x} = \frac{120}{2} = 60 \text{ mm}$$

Q.3-



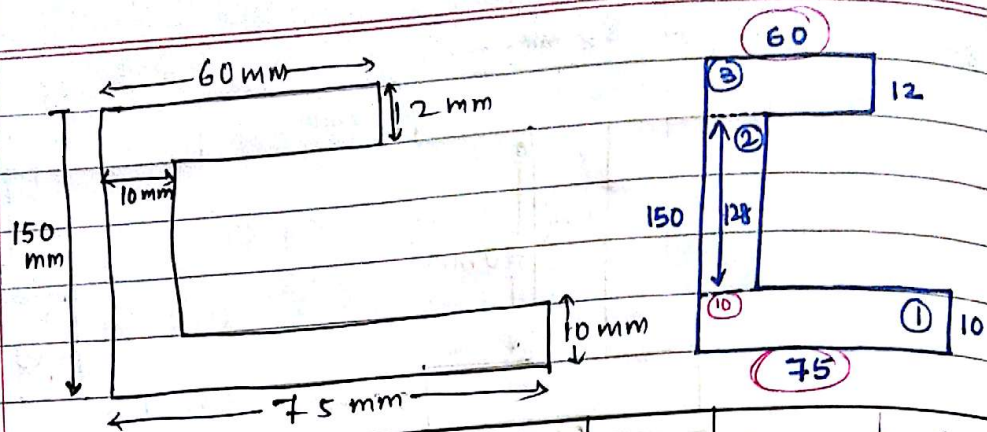
Ans -	Component	Area	y	ay
 ①	Rectangle	24×50 $= 1200$	$\frac{24}{2} = 12$	14400
 ②	Rectangle	15×40 $= 600$	$\frac{24 + 40}{2}$ $= 44$	26400
 ③	Rectangle	80×10 $= 800$	$\frac{24 + 40 + 10}{2}$ $= 69$	55200
		$\Sigma a = 2600$	$\Sigma ay = 96000$	

$\therefore$ Since the figure is symmetrical about y-axis :-

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{96000}{2600} = 36.923 \text{ mm}$$

$$\bar{x} = \frac{80}{2} = 40 \text{ mm}$$

Q.4-



Ans-
①

Component	Area	x	y	ax	ay
① Rectangle	75×10 $= 750$	$\frac{75}{2}$ $= 37.5$	$\frac{10}{2}$ $= 5$	28125	3750
② Rectangle	128×10 $= 1280$	$\frac{10}{2} = 5$	$\frac{10+128}{2}$ $= 74$	6400	94720
③ Rectangle	60×12 $= 720$	$\frac{60}{2} = 30$	$\frac{10+128}{2} + \frac{12}{2}$ $= 144$	21600	103680

$$\Sigma a = 2750$$

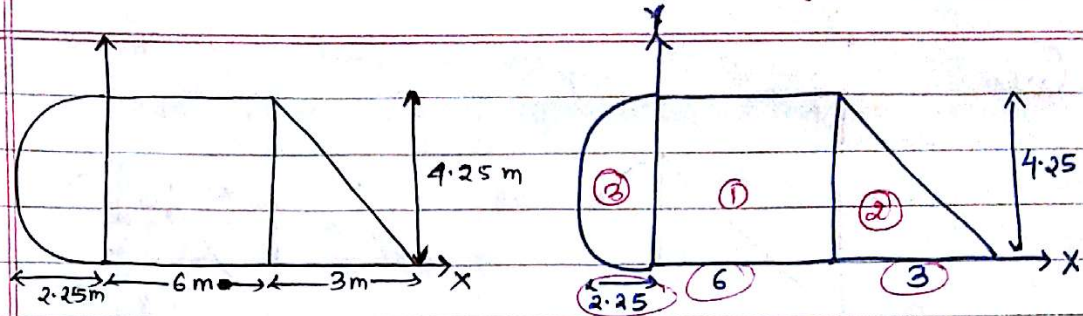
$$\Sigma ax = 56125$$

$$\Sigma ay = 202150$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{202150}{2750} = 73.5 \text{ mm}$$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{56125}{2750} = 20.409 \text{ mm}$$

Imp
Q. 15-



Ans-

Component	Area	x	y	ax	ay
① Rectangle	6×4.25 $= 25.5$	$\frac{6}{2} = 3$	$\frac{4.25}{2}$ $= 2.125$	76.5	54.1875
② Triangle	$\frac{1}{2} \times 3 \times 4.25$ $= 6.375$	$\frac{3}{3} = 1$	$\frac{4.25}{3}$ $= 1.416$	6.375	9.027
③ Semicircle	$\frac{\pi \times (2.25)^2}{2}$ $\frac{3.14 \times (2.25)^2}{2}$ $= 7.9481$	$\frac{-4 \times 2.25}{8 \times 3.14}$ $= -0.95$	$\frac{4.25}{2}$ $= 2.125$	-7.55	16.889

$$\Sigma a = 39.8231$$

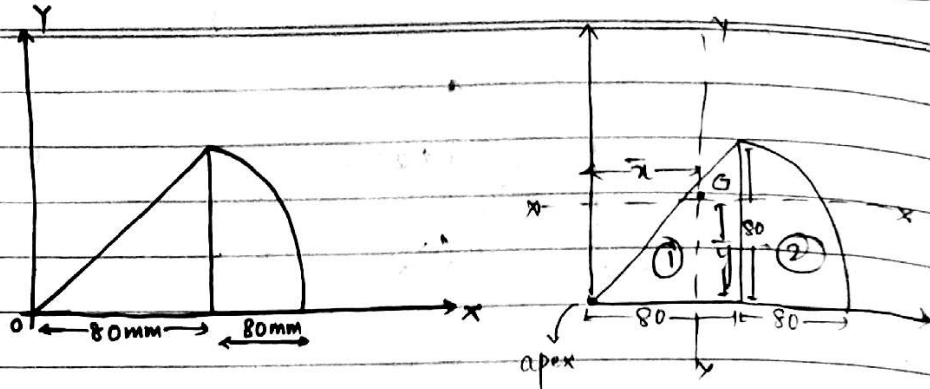
$$\Sigma ax = 75.325$$

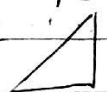
$$\Sigma ay = 80.1035$$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{75.325}{39.8231} = 1.8914 \text{ mm}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{80.1035}{39.8231} = 2.0114 \text{ mm}$$

Q.6-



Ans.	Component	Area (a)	$\bar{x}$	$\bar{y}$	$a\bar{x}$	$a\bar{y}$
	① Triangle	$\frac{1}{2} \times 80 \times 80$ $= 3200$	$\frac{2}{3} \times 80$ $= 53.33$	$\frac{80}{3}$ $= 26.66$	170656	85312
	② Quarter Circle	$\frac{3.14 \times (80)^2}{4}$ $= 5024$	$\frac{80 + 4 \times 80}{3 \times 3.14}$ $= 113.970$	$\frac{4 \times 80}{3 \times 3.14}$ $= 33.97$	542585.28	170666.60

$$\Sigma a = 8224$$

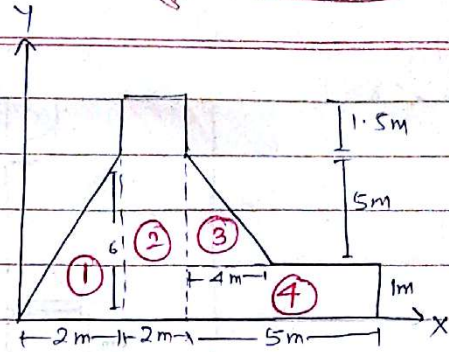
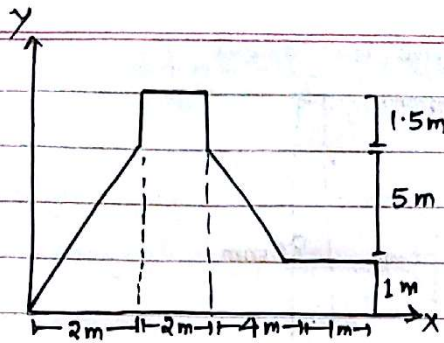
$$\Sigma a\bar{y} = 255978.66$$

$$\Sigma a\bar{x} = 743241.28$$

$$\therefore \bar{y} = \frac{\Sigma a\bar{y}}{\Sigma a} = \frac{255978.66}{8224} = 31.1258 \text{ mm}$$

$$\bar{x} = \frac{\Sigma a\bar{x}}{\Sigma a} = \frac{743241.28}{8224} = 90.374 \text{ mm}$$

Q.7-



Ans. Not symmetrical about any of the axis :-

Component	Area (a)	x	y	ax	ay
① Triangle	$\frac{1}{2} \times 2 \times 6$ = 6	$\frac{2}{3} \times 2$ = 1.33	$\frac{6}{3} = 2$ = 2	7.98	12
② Rectangle	7.5×2 = 15	$2 + \frac{2}{2}$ = 3	$\frac{7.5}{2}$ = 3.75	45	56.25
③ Triangle	$\frac{1}{2} \times 4 \times 5$ = 10	$\frac{2+2+4}{3}$ = 5.33	$1 + \frac{5}{3}$ = 2.66	53.3	26.6
④ Rectangle	5×1 = 5	$2+2+\frac{5}{2}$ = 6.5	$\frac{1}{2}$ = 0.5	32.5	2.5

$$\sum a = 36$$

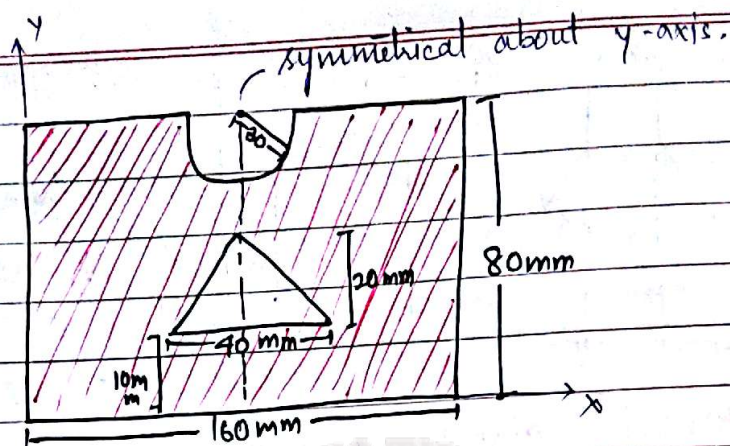
$$\sum ax = 138.78$$

$$\sum ay = 97.35$$

$$\therefore \bar{x} = \frac{\sum ax}{\sum a} = \frac{138.78}{36} = 3.855 \text{ m}$$

$$\bar{y} = \frac{\sum ay}{\sum a} = \frac{97.35}{36} = 2.704 \text{ m}$$

Q.8-



Determine the Centroid for the shaded area.

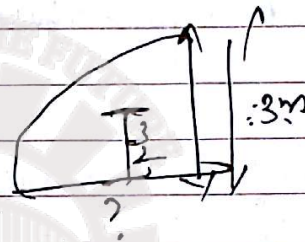
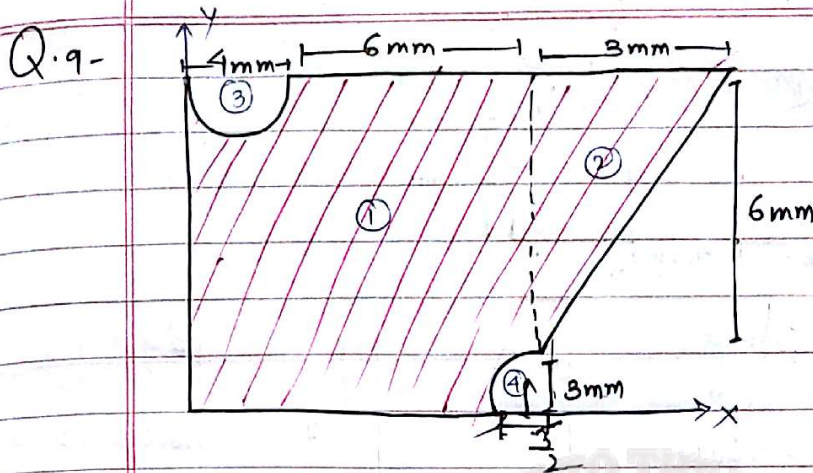
Ans- ①	Component	Area (mm ²)	y (mm)	Σay
	Rectangle.	160×80 $= 12800$	$\frac{80}{2} = 40$	512000
	Triangle.	$-\frac{1}{2} \times 40 \times 20$ $= -400$	$10 + \frac{20}{3}$ $= 16.66$	-6664
	Semi-circle.	$-\frac{3.14 \times (30)^2}{2}$ $= -1413$	$80 - \frac{4 \times 30}{3 \times 3.14}$ $= 67.261$	-95039.793

$$\therefore \Sigma a = 10987$$

$$\Sigma ay = 410296.207$$

$$\therefore \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{410296.207}{10987} = 37.34 \text{ mm}$$

$$\bar{x} = \frac{160}{2} = 80 \text{ mm}$$



Ans-	Component	Area	n	y	Σax	Σay
①	Rectangle	$10 \times 9 = 90$	$\frac{10}{2} = 5$	$\frac{9}{2} = 4.5$	450	405
②	Triangle	$\frac{1}{2} \times 3 \times 3 = 4.5$	$\frac{10 + 3}{3} = 11$	$\frac{3 + 2 \times 3}{3} = 7$	99	63
③	Semi-circle	$-\frac{3.14 \times (2)^2}{2} = -6.28$	$\frac{4}{2} = 2$	$\frac{4 \times 2}{3 \times 3.14} = 0.84$	-12.56	-5.2752
④	Quarter circle	$-\frac{3.14 \times (3)^2}{4} = -7.065$	$\frac{10 - 4 \times 3}{8 \times 3.14} = 8.726$	$\frac{3}{2} = 1.5$	-61.649	-10.5975

$$\Sigma a = 85.655$$

$$\Sigma ax = 474.791$$

$$\Sigma ay = 452.1273$$

$$\therefore \bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{474.791}{85.655} = 5.543 \text{ mm}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{452.1273}{85.655} = 5.278 \text{ mm}$$

A hand-drawn diagram of a mechanical part on a grid background. The part has a total width of 180 mm (60 mm + 60 mm + 60 mm) and a total height of 60 mm (20 mm + 40 mm). The top edge features a semi-circular cutout on the left (40 mm radius), a flat section (80 mm), and a triangular cutout on the right (apex, 40 mm base). The part is divided into four numbered regions: ① (the main body), ② (a rectangular hole, 60 mm wide and 40 mm high), ③ (the triangular cutout), and ④ (the semi-circular cutout).

$$\Sigma a = 10172$$

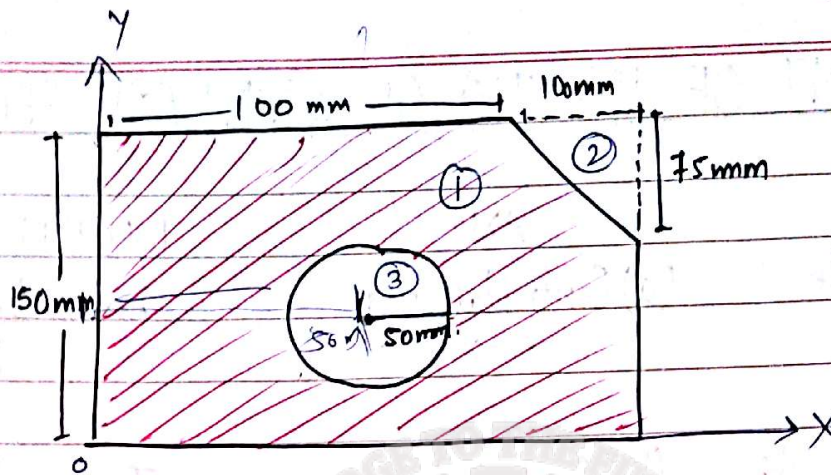
$$\Sigma ax = 593896.$$

$$\Sigma y = 202321.604$$

$$\therefore \bar{y} = \frac{\sum ay}{\sum a} = \frac{202321.604}{10172} = 19.89 \text{ mm}$$

~~$$\bar{x} = \frac{\sum ax}{\sum a} = 58.385 \text{ mm}$$~~

Q.11-

Ans-
□ ①Component
Rectangle

$$\text{Area (mm}^2\text{)} = 150 \times 200 = 30000$$

$$x \text{ (mm)} = \frac{200}{2} = 100$$

$$y \text{ (mm)} = \frac{150}{2} = 75$$

$$\Sigma ax = 3000000$$

$$ay = 2250000$$

△ ②

Triangle

$$\frac{1}{2} \times 100 \times 75 = 3750$$

$$x = 100 + \frac{2}{3} \times 100 = 166.66$$

$$y = 75 + \frac{2}{3} \times 75 = 125$$

$$\Sigma ax = 624975$$

$$ay = 468750$$

○ ③

Circle

$$3.14 \times (50)^2 = 7850$$

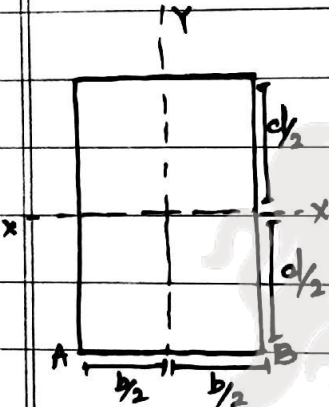
$$x = \frac{50}{2} = 100$$

$$\Sigma a = 41600$$

$$\Sigma ax$$

$$\Sigma ay =$$

• MOMENT OF INERTIA OF STANDARD SECTIONS:

SHAPE	AXIS	MOMENT OF INERTIA
(1) Rectangle	(a) Centroidal axis X-X	$I_{xx} = \frac{bd^3}{12}$
	(b) Centroidal axis Y-Y	$I_{yy} = \frac{db^3}{12}$
	(c) A-B	

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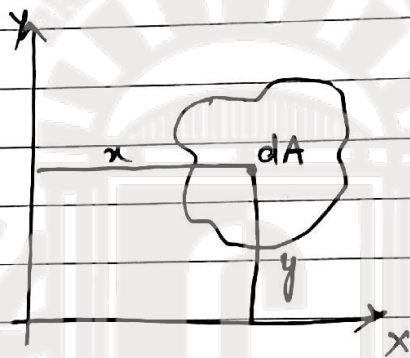
Difference b/w centroid & moment of inertia

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* Moment of Inertia signifies shape and distribution of area.

The word inertia represent the property of materials by virtue of which it resists any change in the state of rest or uniform motion or rotational motion.



Consider a smaller area dA as shown in the figure of the lamina with x and y as reference axis. The product of $dA \times y$ represent the moment. If $dA \times y \times y$ it becomes moment of inertia.

$dA \times y$ represent 1st moment of area.

$dA \times y \times y$ represent 2nd moment of area.

The moment of inertia of entire lamina along x -axis is given by

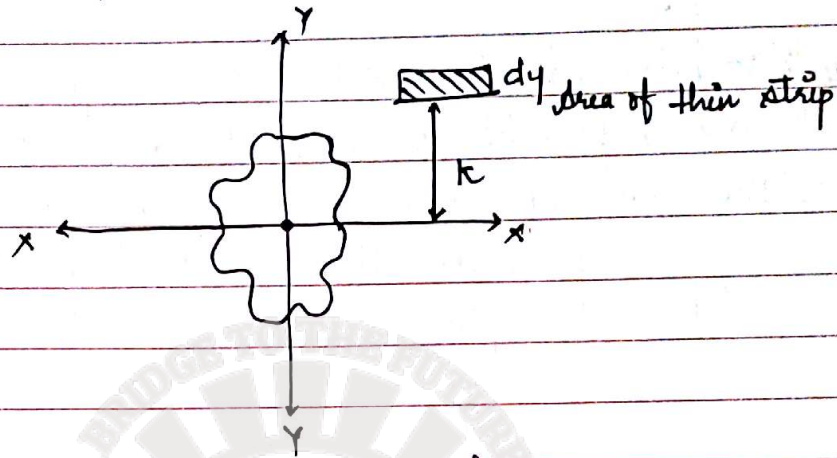
$$I_{xx} = \int dA \times y^2 \quad (\text{mm})^4$$

Similarly moment of inertia along y -axis is given by

$$I_{yy} = \int dA \times x^2 \quad (\text{mm})^4$$

The SI unit of moment of inertia is $(\text{mm})^4$

• RADIUS OF GYRATION :



Consider the area as shown in figure whose moment of inertia along x and y-axis are I_{xx} and I_{yy} .

In order to maintain the same moment of inertia along the reference axis, the area must be thrown to a new location such that the elementary strips maintain the constant distance k .

Then the distance k can be defined as the distance of area @ which if it is squeezed and placed so that there is no change in moment of inertia is called as the radius of gyration k i.e.

$$I_{xx} = \int y^2 dA$$

$$I_{yy} = \int x^2 dA$$

$$\therefore I_{xx} = k_{xx}^2 dA$$

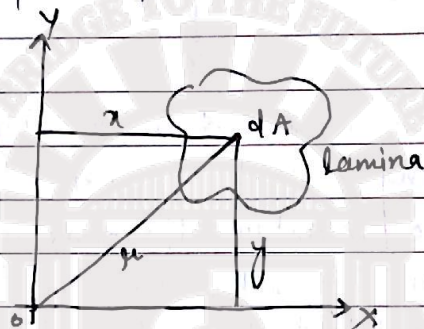
$$k_{xx} = \sqrt{\frac{I_{xx}}{A}} \quad \text{mm}$$

$$k_{yy} = \sqrt{\frac{I_{yy}}{A}} \quad \text{mm}$$

Imp

PERPENDICULAR AXIS THEOREM:-

Statement:- Moment of inertia of a given area about an axis I to the plane through a point is equal to the sum of moment of inertia of that area about any 2 I axis through that point. The axis will be in the plane of area.



Let x is the distance of dA from y -axis and y be the distance of dA from x -axis.

Then from fig.

$$r^2 = x^2 + y^2 \quad (\text{Pythagoras})$$

If z is the axis through a coordinate system at O , then according to the theorem we need to prove

$$I_{zz} = I_{xx} + I_{yy}$$

Proof: Let dA be an elemental area @ a distance r from O .

Then moment of inertia along z -axis is given by

$$I_{zz} = \sum dA \times r^2$$

We know that $r^2 = x^2 + y^2$

$$I_{zz} = \sum (x^2 + y^2) dA$$

$$= \sum x^2 dA + \sum y^2 dA$$

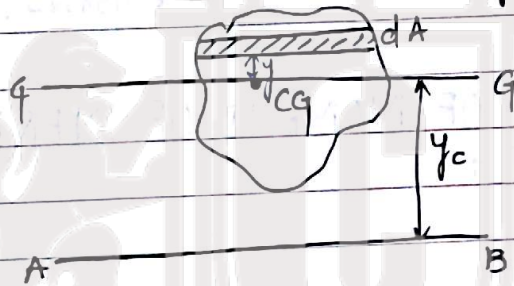
$$I_{zz} = I_{xx} + I_{yy} \quad // \text{ Hence proved.}$$

Imp • PARALLEL-AXIS THEOREM :-

Statement :- Moment of inertia of an area about any axis in a plane of area dA is equal to sum of moment of inertia about the centroidal parallel axis of the area and the product of the area and the square of distance of centroid of the area from the axis.

Then according to the theorem wrt to figure we need to prove:-

$$I_{AB} = I_{GG} + Ay_c^2$$



Proof

I_{AB} = moment of inertia of the area about the axis AB.

I_{GG} = moment of inertia of the area about the axis GG which is || to AB.

y_c = distance of centroid G from the axis GG

Consider an elementary strip of area dA || to AB and it is at a distance of y from GG axis.

Then acc. to theorem then $I_{AB} = \sum (y + y_c)^2 \cdot dA$

$$\Rightarrow I_{AB} = \sum (y^2 + 2yy_c + y_c^2) dA$$

$$\Rightarrow I_{AB} = \sum y^2 dA + \sum 2yy_c dA + \sum y_c^2 \quad \text{--- (1)}$$

Now, $\sum y^2 dA$ = moment of inertia along the centroidal axis GG = I_{GG} .

$$\therefore \sum y^2 dA = I_{GG} \quad \text{--- (2)}$$

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● PROCEDURE TO SOLVE THE PROBLEMS :-

Component	Area	x	y	Ax	Ay	Ax ²	Ay ²	I _x	I _y
	ΣA			ΣAx	ΣAy	ΣAx ²	ΣAy ²	ΣI _x	ΣI _y

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A}$$

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A}$$

$$I_{1-1} = I_x + Ay^2$$

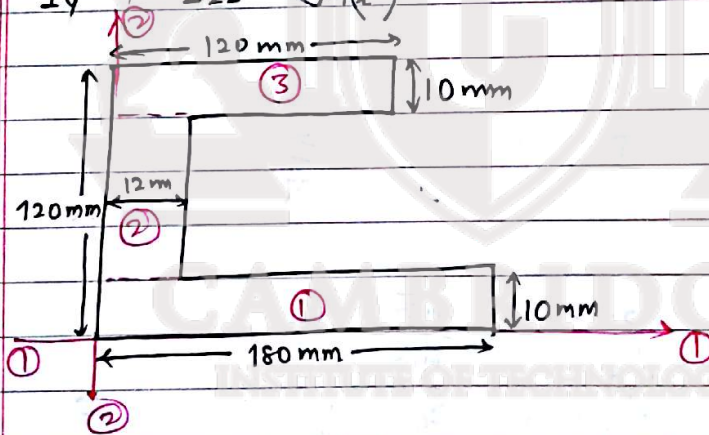
$$I_{1-1} = \Sigma I_x + \Sigma Ay^2$$

$$\bar{I}_x = I_{1-1} - \{Ay^2\} \Rightarrow I_{1-1} - \Sigma A \left(\frac{\Sigma Ay}{\Sigma A} \right)^2$$

$$I_{2-2} = \Sigma I_x + \Sigma Ax^2$$

$$\bar{I}_y = I_{2-2} - \Sigma A \left(\frac{\Sigma Ax}{\Sigma A} \right)^2$$

Q.1-



Ans.

	Area (mm ²)	x	y	Ax	Ay	Ax ²	Ay ²	I _x	I _y
①	180 × 10 = 1800	$\frac{180}{2}$ = 90	$\frac{10}{2}$ = 5	162000	9000	1458 × 10 ⁴	180 × 10 ³ 12		
②	100 × 12 = 1200	$\frac{12}{2}$ = 6	$\frac{10+100}{2}$ = 60	7200	72000	432 × 10 ⁴	$\frac{12 \times 100^3}{12}$ 10000		
③	120 × 10 = 1200	$\frac{120}{2}$ = 60	$\frac{10+100+10}{2}$ = 115	72000	138000	1587 × 10 ⁴	$\frac{120 \times 10^3}{12}$ 1000		

Component	Area	y	Ay	Ay^2	$I_x(\text{mm}^4)$	$I_y(\text{mm}^4)$
Rectangle	24×4 $= 96$	$\frac{4}{2} = 2$	192	384	$\frac{24^3 \times (4)}{12}$ $= 128$	$\frac{(24)^3 \times 4}{12}$ $= 4608$
Rectangle	4×2 $= 8$	$4 + \frac{2}{2} = 5$	40	200	$\frac{(4)(2)^3}{12}$ $= 5.33$	$\frac{(4)^3 \times 2}{12}$ $= 10.67$
Semicircle	$\frac{3.14 \times (6)^2}{2}$ $= 56.52$	$4 + 12 + \frac{4 \times 6}{3 \times 3.14}$ $= 16 + \frac{24}{9.42}$ $= 18.547$	1048.276	19442.383	$\frac{0.11 \times (6)^4}{8}$ $= 142.56$	$\frac{3.14 \times 6^4}{8}$ $= 508.68$
	$\Sigma A = 100.52$		$\Sigma Ay = 1720.276$	$\Sigma Ay^2 = 24626.383$	$\Sigma I_x = 846.56$	$\Sigma I_y = 5180.68$
	$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = 8.579$					
	$I_{xx} = \Sigma I_x + \Sigma Ay^2$					
	$I_{yy} = \Sigma I_y + \Sigma Ax^2$					

