MODULE 2

LIMIT STATE ANALYSIS OF BEAMS

2.1 ANALYSIS OF BEAMS

Analysis means the preliminary work carried out before designing. During analysis we calculate loads coming on the structure, effects of different loads on the structure (Live load, dead loads, wind loads etc), values of bending moment and shear force.

In design of RC structures we use these values calculated in analysis to determine the size of the section and reinforcement details.

The aim of design is to decide the size or dimensions of the beam and provide suitable reinforcement. The section designed should perform safely during its life time.

Beam is a flexural member which resists loads mainly by bending.

In this module following types of beams are analysed.

- Singly reinforced beams } Rectangular beam (Simply supported and cantilever beams)
- Doubly reinforced beams } Rectangular beam (Simply supported and cantilever beams)
- T-beams - Flanged beams – Simply supported beam

***Note* :**

1. In designing R.C.C. beams, important basic rules are to be followed which are given in IS : 456-2000. Readers are requested to go through IS : 456-2000 before proceeding towards analysis and design problems.
2. **Simply supported beam :** It is a type of beam that has pinned support at one end and roller support at the other end. Depending on the load applied, it undergoes shearing and bending.
3. **Cantilever beam :** When a cantilever or slab is subjected to downward loads, it bends downwards with its convexity upwards. Due to this the upper fibres of the section are subjected to tensile stress and the lower fibres are subjected to compressive stress. Hence the main reinforcement (Tension) is provided at the upper face and not at the lower face.

The bending moment varies from a zero value at the free end to maximum value at the fixed end (Support). Hence the depth of the section and the steel reinforcement is varied as per requirements of BM. The reinforcement should be embedded in the support for a distance of 'L_d' from the edge of the supports.

where L_d = Development Length

2.2 REINFORCEMENT FOR BEAMS (CLAUSE 26.5, IS : 456-2000)

➢ **Minimum tension reinforcement: [clause 26.5.1.1 (a)]**

The minimum area of tension reinforcement shall not be less than that given by the following:

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

Where, A_s = minimum area of tension reinforcement,
b = breadth of beam or the breadth of the web of T-beam,
d = effective depth of beam
f_y = characteristic strength of reinforcement in N/mm^2.

➢ **Maximum tension reinforcement [clause 26.5.1.1, (b)]**

The maximum area of tension reinforcement shall not exceed 0.04bD.

where D = overall depth of beam

➢ **Compression reinforcement [clause 26.5.1.2]**

The maximum area of compression reinforcement shall not exceed 0.04bD. Compression reinforcement in beams shall be enclosed by stirrups for effective lateral restraint.

➢ **Side face reinforcement [caluse 26.5.1.3]**

When the depth of the web in a beam exceeds 750mm, it is a deep beam. So side face reinforcement should be provided along the two faces. The total area of such reinforcement shall not be less than 0.1% of the web area which shall be distributed equally or both faces. The spacing of side face reinforcement should not be more than 300 mm or web thickness whichever is less.

➢ **Transverse reinforcement in beams for shear (clause 26.5.1.4)**

The shear reinforcement in beams shall be taken around the outermost tension and compression bars.

2.3 ANALYSIS OF SINGLY REINFORCED BEAMS

The RC beams, in which the steel reinforcement is placed only on tension side are known as singly reinforced beams.

In singly reinforced beams the tension developed due to bending moment is mainly resisted by the steel reinforcement and the compression is resisted by the concrete alone.

Steps For Solving Analysis Problems

Type I Problems

Given : A_{st} or number of bars with diameter (ϕ) of bars , Size of beam, Grade of concrete and steel. If load to be calculated then span is given.

Required : Ultimate moment or Factored moment (M_u) or Moment of resistance (M) Total load (W) or super imposed load (W_{LL}).

Note : (i) Ultimate moment or factored moment = 1.5 × working moment

(ii) Ultimate load or factored load = 1.5 × working load

Steps:

1) Note down the values for $\frac{x_{u,\max}}{d}$ by Referring, IS : 456-2000

f_y (N/mm²)	$\frac{x_{u.\max}}{d}$
250	0.53
415	0.48
500	0.46

Where f_y = Yield strength of steel in N/mm²

2) Determine depth of neutral axis $\frac{x_u}{d}$ by referring, IS : 456-2000

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd}$$

Where f_{ck}= Characteristic strength of concrete 20 N/mm^2 for M20, 25N/mm^2 for M25.

b = Breadth of beam

$d = D$ - effective cover $\Rightarrow d = D - d^1$

d^1 = effective cover = clear cover + $\frac{\text{diameter}}{2}$

D = overall depth of beam

A_{st} = Area of tensile reinforcement = No. of bars $\times \frac{\pi \times \phi^2}{4}$

3) Compare $\frac{x_u}{d}$ and $\frac{x_{u,\max}}{d}$

➢ If $\frac{x_u}{d}$ less than $\frac{x_{u,\max}}{d}$, section is under reinforced or $M_{u_{\lim}} > M_u$.

Therefore calculate the Moment of resistance by the following expression (Refer IS : 456-2000)

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{bdf_{ck}}\right)$$

➢ If $\frac{x_u}{d}$ greater than $\frac{x_{u,\max}}{d}$, section is over reinforced or $M_{u_{\lim}} < M_u$.

Therefore calculate the Limiting Moment of resistance by the following expression

$$M_{u,\lim} = 0.36\frac{x_{u,\max}}{d}\left(1 - 0.42\frac{x_{u,\max}}{d}\right)bd^2 f_{ck}$$

or by substituting value of $\frac{x_{u,\max}}{d}$ in above equation we will get simplified formula

$$M_{u,\lim} = 0.148 f_{ck} bd^2 - \text{Fe250 (Mild steel)}$$

$$M_{u,\lim} = 0.138 f_{ck} bd^2 - \text{Fe415 (HYSD bars)}$$

$$M_{u,\lim} = 0.133 f_{ck} bd^2 - \text{Fe500 (HYSD bars)}$$

3. To Calculate Safe udl or live Load (i.e., unfactored load)

Unfactored moment, $M = \frac{M_u}{1.5}$

The Maximum BM for Simply supported beam carrying udl, $M = \frac{WL^2}{8}$

$$\therefore \quad W = \frac{8M}{L^2}$$

Here W = Total load

$\Rightarrow W = W_{DL} + W_{WL}$

W_{DL} = Self weight of beam or Dead load

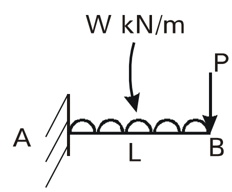
W_{LL} = Super imposed load or Live load

i.e., W_{DL} = Self weight of beam = $b \times D \times$ Density of RCC (ρ)

ρ = 25kN/m^3

Therefore Live load or superimposed load or safe udl, $W_u = W - W_{DL}$

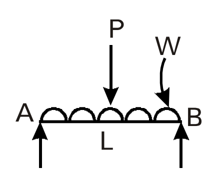
Note:



i) If cantilever beam carrying udl given, Then Maximum BM, $M = \frac{WL^2}{2}$

ii) If cantilever beam carrying udl (Self weight) + Point load at free end then

$$\text{Maximum } BM, M = \frac{WL^2}{2} + PL$$



iii) If Simply supported beam carrying udl (self weight) + Central point load given then

$$\text{Maximum } BM, M = \frac{WL^2}{8} + \frac{PL}{4}$$

Type II - Problems

Given: Factored or ultimate moment M_u or Moment of resistance (M), Total load (W), Span with type of support (L), size of beam, type of steel and concrete.

Required: Area of tension steel (A_{st}), or No. of bars

Steps:

1. If Load (W) and Effective span (L) is given

   Calculate Maximum bending moment

   $$M = \frac{WL^2}{8} \text{ for SS beam carrying udl [SS - simply supported beam]}$$

   $$M = \frac{WL^2}{2} \text{ for Cantilever beam carrying udl}$$

   Factored moment or Ultimate moment, $M_u = 1.5 \times M$

2. Calculate Area of steel (A_{st})

   $$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{bdf_{ck}}\right)$$

   Solve quadratic equation, find A_{st}

2.4 WORKED EXAMPLES ON ANALYSIS OF SINGLY REINFORCED BEAMS

1. **Find the depth of neutral axis of a singly reinforced RC beam of 230mm width and 450mm effective depth. It is reinforced with 4 bars of 16mm diameter. Use M20 concrete and Fe415 bars. Also comment on the type of beam.**

   Solution:

   Given: $b = 230mm,\ d = 450mm,\ f_{ck} = 20N/mm^2,\ f_y = 415\ N/mm^2$

   $$\therefore\ A_{st} = 4 \times \frac{\pi \times 16^2}{4} = 804.24mm^2$$

Step : 1 Depth of neutral axis (x_u)

(Refer IS : 456-2000, G-1.1 a)

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd} = \frac{0.87 \times 415 \times 804.24}{0.36 \times 20 \times 230 \times 450} = 0.389 \approx 0.39$$

Step : 2 Limiting value of $\frac{x_{u,\max}}{d}$

$$\therefore\ \frac{x_{u,\max}}{d} = 0.48 \text{ for Fe415 steel (IS : 456-2000)}$$

Since, $\frac{x_u}{d} < \frac{x_{u,\max}}{d}$,

$\therefore$ Section is under reinforced.

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2. **Find the depth of neutral axis of a singly reinforced RC beam of 250mm width and 500mm effective depth. It is reinforced with 4 bars of 20mm diameter. Use M20 concrete and Fe415 bars. Also check for type of section.**

   Solution:

   Given: $b = 250mm,\ d = 500mm,\ f_{ck} = 20N/mm^2,\ f_y = 415\ N/mm^2$

$$\therefore \quad A_{st} = 4\times\frac{\pi\times 20^2}{4} = 1256.64mm^2$$

Step : 1 Depth of neutral axis (x_u)

(Refer IS : 456-2000, G-1.1 a)

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87\times 415\times 1256.64}{0.36\times 20\times 250} = 0.50$$

Step : 2 Limiting value of $\frac{x_{u,\max}}{d}$

$\therefore \frac{x_{u,\max}}{d} = 0.48$ for Fe 415 steel

Since, $\frac{x_u}{d} > \frac{x_{u,\max}}{d}$

$\therefore$ Section is over reinforced.

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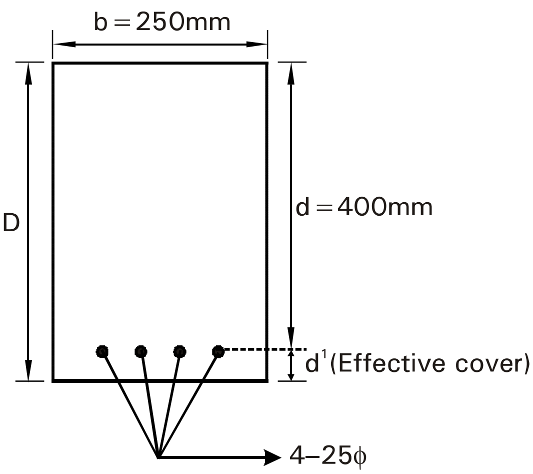
3. **A Singly reinforced concrete beam 250mm width is reinforced 4 bars of 25mm diameter at an effective depth of 400mm. If M20 grade concrete and Fe415 bars are used, compute Ultimate moment of resistance of the section.**

   Solution:

   Given : $b = 250$mm, $d = 400$mm, $f_{ck} = 20N/mm^2,\ f_y = 415\ N/mm^2$

$$A_{st} = 4\times\frac{\pi\times 25^2}{4} = 1963.50mm^2$$



Step : 1 Depth of neutral axis (x_u)

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd} = \frac{0.87 \times 415 \times 1963.50}{0.36 \times 20 \times 250 \times 400} = 0.984$$

Step : 2 Limiting value of $\frac{x_{u,\max}}{d}$

$$\therefore \frac{x_{u,\max}}{d} = 0.48 \text{ for Fe 415}$$

Since, $\frac{x_u}{d} > \frac{x_{u,\max}}{d}$, section is over reinforced. (Over reinforced sections are considered as Balance section or limiting section to calculate moment)

Step : 3 Calculation of ultimate moment of resistance (M_u)

Therefore Limiting Moment of resistance is calculated by using formula

Refer IS:456-2000, G-1.1(c)

$$M_{u,\lim} = 0.36 \frac{x_{u,\max}}{d}\left(1 - 0.42 \frac{x_{u,\max}}{d}\right) bd^2 f_{ck}$$

$$M_u = M_{u,\lim} = 0.36 \times 0.48 (1 - 0.42 \times 0.48) 250 \times 400^2 \times 20 = 110.4 \times 10^6 N - mm$$

$\boldsymbol{M_u}$ **= 110.40kN-m**

4. The Simply supported singly reinforced having 250mm wide and 500mm effective depth, provided with Fe415 steel and M20 grade concrete. Determine the limiting moment of resistance of beam.

Solution:

Given: $b = 250mm,\ d = 500mm,\ f_{ck} = 20N/mm^2\ f_y = 415\ N/mm^2$

Step : 1 Limiting value of $\frac{x_{u,\max}}{d}$

For Fe415 steel, $\frac{x_{u,\max}}{d} = 0.48$

Step : 2 Determine the Limiting moment of resistance $\left(M_{u_{\lim}}\right)$

$$M_{u,\lim} = 0.36\,\frac{x_{u,\max}}{d}\left(1-0.42\frac{x_{u,\max}}{d}\right)bd^2 f_{ck} \text{ or } M_{u_{\lim}} = 0.138 f_{ck} bd^2$$

$$M_{u,\lim} = 0.36\times0.48(1-0.42\times0.48)250\times500^2\times20 = 172.45\times10^6 N-mm = 172.45kN-m$$

5. Find the depth of neutral axis of singly reinforced beam of breadth 230mm reinforced with 4 bars of 16mm diameter. Concrete is of M20 grade and Fe415.

Solution:

Given: $b = 230mm,\ f_{ck} = 20N/mm^2,\ f_y = 415\ N/mm^2$

$$\therefore \text{ Area of steel, } A_{st} = 4\times\frac{\pi\times16^2}{4} = 804.24mm^2$$

$$\text{Depth of neutral axis, } x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87\times415\times804.24}{0.36\times20\times230} = 175.34mm$$

6. A rectangular RC beam having breadth 350mm is reinforced with 4 bars of 20mm diameter at an effective depth of 650mm. Use M20 concrete and Fe415 HYSD bars. Determine the Ultimate moment of resistance of the section.

Solution:

Given: b = 350mm, d = 650mm, f_{ck} = 20N/mm², f_y = 415 N/mm²

$$A_{st} = 4\times\frac{\pi\times20^2}{4} = 1256.64mm^2$$

Step : 1 Depth of neutral axis (x_u)

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd} = \frac{0.87\times415\times1256.64}{0.36\times20\times350\times650} = 0.27$$

Step : 2 Limiting value of $\frac{x_{u,max}}{d}$

$$\therefore \frac{x_{u,max}}{d} = 0.48 \text{ for Fe 415}$$

Since, $\frac{x_u}{d} < \frac{x_{u,max}}{d}$

0.27 < 0.48 section is under reinforced.

Step : 3 Calculation of ultimate moment of resistance (M_u)

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{bdf_{ck}}\right)$$

$$M_u = 0.87 \times 415 \times 1256.64 \times 650\left(1 - \frac{1256.64 \times 415}{350 \times 650 \times 20}\right) = 261109668.4 N - mm$$

= 261.10 kN.m

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7. **A rectangular section of 230mm × 500mm is used as a simply supported beam for a effective span of 6m.The beam consists of tensile reinforcement of 4000mm² and center of reinforcement is placed at 35mm from the bottom edge. What maximum total udl can be allowed on the beam. Use M20 concrete & Fe415 steel.**

Solution:

Given: D = 500mm, b = 230mm, A_{st} = 4000 mm²

Effective cover d' = 35mm, f_{ck} = 20N/mm²,

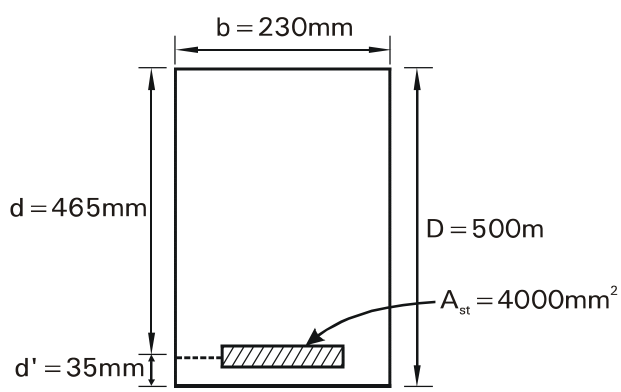
f_y = 415 N/mm², L = 6m

Effective depth, $d = D$ – effective cover = 500 – 35 = 465mm,

Step : 1 Depth of neutral axis (x_u)

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd}$$

$$= \frac{0.87 \times 415 \times 4000}{0.36 \times 20 \times 230 \times 465} = 1.87$$



Step : 2 Limiting value of $\frac{x_{u,max}}{d}$

For Fe 415, $\frac{x_{u,max}}{d} = 0.48$

Since, $\frac{x_u}{d} > \frac{x_{u,\max}}{d}$, section is over reinforced.

Step : 3 Calculation of ultimate moment of resistance

$$M_{u_{\lim}} = 0.138 f_{ck} b d^2$$

$$M_{u_{\lim}} = 0.138 \times 20 \times 230 \times 465^2 = 137.26 kN.m$$

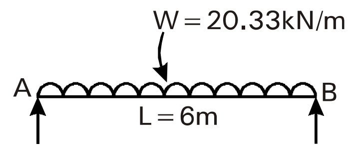
Unfactored moment, $M = \frac{M_u}{1.5} = \frac{137.26}{1.5} = 91.50 kN.m$

Step : 4 Calculation of maximum load on beam (*W*)

The Maximum BM for Simply supported beam carrying udl, $M = \frac{WL^2}{8}$ | W = Total load

$$\Rightarrow\ 91.50 = \frac{W \times 6^2}{8}$$

∴ Total udl, W = 20.33 kN/m

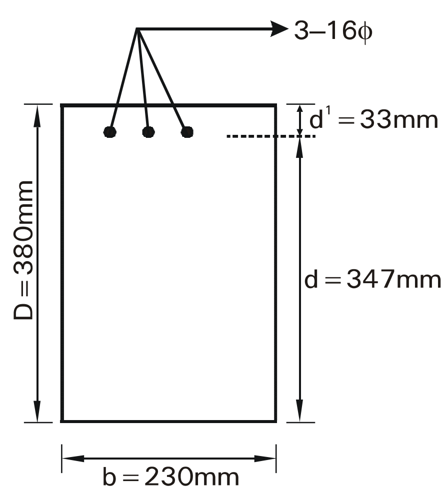


8. A cantilever R.C beam of span 2 m is rectangular in cross section 230 × 380 mm. It is reinforced with 3 – 16 dia bars on tension side. Determine the supermposed load on the beam in addition to its self weight. Use M20 concrete and Fe415 steel.

Solution:

Given: b = 230 mm, D = 380 mm, f_{ck} = 20 N/mm², f_y = 415 N/mm², L = 2m

$$A_{st} = \frac{3 \times \pi \times 16^2}{4} = 603.18 \text{mm}^2$$



Step : 1 Depth of neutral axis (x_u)

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 603.18}{0.36 \times 20 \times 230} = 131.50mm$$

Assuming clear cover = 25mm

$$d' = \text{clear cover} + \frac{\text{diameter}}{2} = 25 + \frac{16}{2} = 33mm$$

$$\therefore d = D - d' = 380 - 33 = 347 \text{ mm}$$

Step : 2 Limiting value of $\frac{x_{u,max}}{d}$

$$\frac{x_{u,\max}}{d} = 0.48$$

$$x_{u,max} = 0.48 \times d = 0.48 \text{ x } 347 = 166.56 \text{ mm.}$$

$$x_u < x_{u,max}$$

$\therefore$ The given section is under reinforced.

Step : 3 Calculation of ultimate moment of resistance

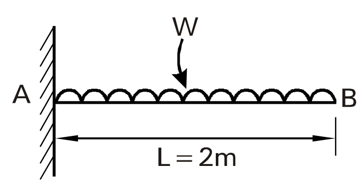
$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{bd\ f_{ck}}\right)$$

$$= 0.87 \times 415 \times 603.18 \times 347 \left(1 - \frac{603.18 \times 415}{230 \times 347 \times 20}\right)$$

$$= 64.94 \times 10^6 \text{ N-mm}$$

$$= 64.94 \text{ kN-m}$$

Here, W = Total load



Unfactored moment, $M = M_u / 1.5 = 64.94 / 1.5 = 43.29$ kN – m.

Step : 4 Calculation of superimposed load on beam (W_{LL})

Maximum BM, $M = \frac{WL^2}{2}$ —— For cantilever beam

$$W = \frac{2M}{L^2} = \frac{2 \times 43.29}{2^2} = 21.64 kN/m$$

W.K.T, $W = W_{LL} + W_{DL}$ | Density of RCC = 25 kN/m³

W_{DL} = Self weight of beam = $b \times D \times$ Density of RCC = 0.23 × 0.38 × 25 = 2.185 kN/m

$\therefore W_{LL} = W - W_{DL} = 21.64 - 2.185$

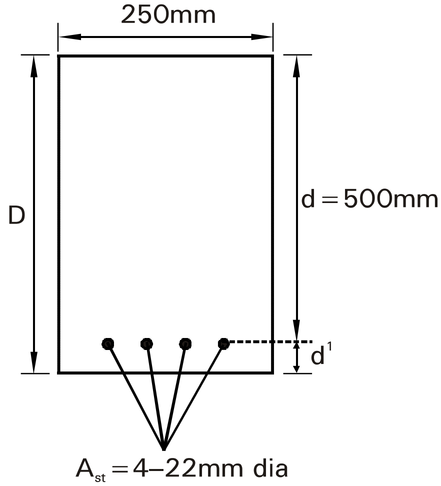
$W_{LL} = 19.46$ kN/m

9. A rectangular beam section 250 mm wide and 500 mm deep up to the center of tension steel consists of 4 No's 22 mm diameter bars. Find the position of the neutral axis and the safe Moment of resistance, if concrete is of M_{20} mix and steel is of Fe415 grade. Find what concentrated load it can carry at mid span, if the beam has an effective span of 6 m.

Solution:

Given : b = 250 mm, d = 500 mm, $f_{ck} = 20\ N/mm^2, f_y = 415\ N/mm^2$.

A_{st} = 4 – 22 mm dia, L = 6 m



$$A_{st} = 4 \times \frac{\pi \times 22^2}{4} = 1520.53 mm^2$$

Step : 1 Actual Neutral Axis or Depth of neutral axis (x_u)

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 1520.53}{0.36 \times 20 \times 250} = 305mm$$

Step : 2 Max Neutral Axis or Limiting value of neutral axis ($x_{u\,max}$)

$$x_{u,\,max} = 0.48 \times d = 0.48 \times 500 = 240\ mm.$$

$$x_u > x_{u,\,max}$$

$\therefore$ The given section is over reinforced.

Step : 3 Calculation of moment of resistance

For over reinforced section, $M_{u_{\lim}} = M_u$

$$M_{u_{\lim}} = 0.138 f_{ck} bd^2 \text{ for Fe 415}$$

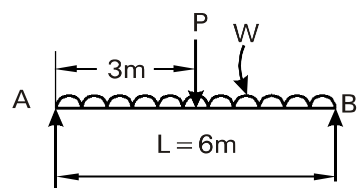
$$M_{u_{\lim}} = 0.138 \times 20 \times 250 \times 500^2$$

$$M_{u_{\lim}} = 172.50 \times 10^6 \ N - mm.$$

$$M_u = M_{u_{\lim}} = 172.50 \ kN - m.$$

Moment of resistance, $M = M_u / 1.5 = 172.50 / 1.5 = 115 \ kN - m.$

Step : 4 Calculation of concentrated load



Maximum BM for udl + point load, $M = \dfrac{WL^2}{8} + \dfrac{PL}{4}$

Assuming effective cover = 50mm, $\therefore D = d + d' = 500 + 50 = 500$ mm

Total load, $W = W_{DL} + W_{LL}$

$\therefore W_{DL}$ = self weight of beam = $b \times D \times RCC$ density = 0.25 × 0.55 × 25 = 3.44 kN/m

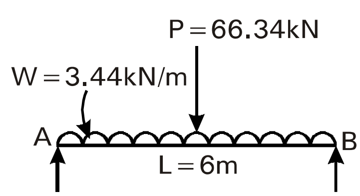
Here $W_{LL} = 0$ (Since point load is acting, W_{LL} becomes zero)

$\therefore W = W_{DL} = 3.44$ kN/m

$$\Rightarrow M = \frac{WL^2}{8} + \frac{PL}{4}$$

$$\Rightarrow 115 = \frac{3.44 \times 6^2}{8} + \frac{P \times 6}{4} = 15.48 + 1.5P$$

$\therefore P = 66.34$ kN



10. A rectangular beam of effective size 300 mm × 500 mm is used as a simply supported beam for effective span 6.5 m. Determine the superimposed load on the beam, if the maximum percentage of steel is provided, only on tension side ? Use M20 concrete and Fe 415 steel. Determine the amount of steel to be provided.

Solution:

Given : $d = 500$ mm, $f_{ck} = 20$ N/mm^2, $b = 300$ mm, $f_y = 415$ N/mm^2, $L = 6.5$ m

Step : 1 Limiting value of neutral axis

Since maximum percentage of steel is provided, beam is designed as a Balanced Section.

There fore $x_u = x_{u,lim}$

Limiting value of $\dfrac{x_{u,\max}}{d} = 0.48$ for *Fe* 415 steel

$$\therefore\ x_{u,max} = 0.48 \times d = 0.48 \times 500 = 240 \text{ mm}$$

Step : 2 Calculation of A_{st}

$$x_u = \frac{0.87 f_y A_{st}}{0.36\ f_{ck} b} \qquad \text{From IS : 456-2000,}$$

Note : Amount of steel for balance section can be obtained by equation compressive force to tensile force

put $x_u = x_{u\ max} = 240$ mm

$$A_{st} = \frac{0.36\ f_{ck} b\ x_{u\ \max}}{0.87\ f_y}$$

$$= \frac{0.36 \times 20 \times 300 \times 240}{0.87 \times 415}$$

$$\mathbf{A_{st} = 1435.81\ mm^2}$$

Step : 3 Calculate moment of resistance

$$M_u = M_{u\ lim} = 0.138 f_{ck}\ bd^2 \text{ – for } Fe \text{ 415}$$

$$= 0.138 \times 20 \times 300 \times 500^2$$

$$= 207 \times 10^6 \text{ N – mm} = 207 \text{ kN–m}$$

$$\text{Moment of resistance, } M = \frac{M_u}{1.5} = \frac{207}{1.5} = 138 \text{ kN–m}$$

Step : 4 Calculation of superimposed load on beam

$$\text{Maximum } BM,\ M = \frac{WL^2}{8}$$

$$\Rightarrow W = \frac{8M}{L^2} = \frac{8 \times 138}{6.5^2} = 26.13 \text{ kN/m}$$

$W = W_{DL} + W_{LL}$

W_{DL} = Self weight of beam

$\Rightarrow W_{DL} = b \times D \times RCC$ density

Assume $d' = 50$ mm

$\therefore D = d + d' = 500 + 50 = 550$ mm

$\therefore W_{DL} = 0.3 \times 0.55 \times 25 = 4.125$ kN/m

$W_{LL} = W - W_{DL} = 26.13 - 4.125 = 22$ kN/m

11. A reinforced concrete beam 250 mm wide and 600 mm effective depth reinforced with 1080 mm² tensile reinforcement. If M15 grade concrete and Fe 415 Steel used, Calculate ultimate moment of resistance.

Solution:

Given : $b = 250$ mm, $d = 600$ mm, $A_{st} = 1080$ mm^2, $f_{ck} = 15$ N/mm^2, $f_y = 415$ N/mm^2

Step : 1 Depth of neutral axis

$$\frac{x_u}{d} = \frac{0.87 f_y\ A_{st}}{0.36\ f_{ck}\ bd} = \frac{0.87 \times 415 \times 1080}{0.36 \times 15 \times 250 \times 600} = 0.48$$

Step : 2 Limiting neutral axis

$$\frac{x_{u,\max}}{d} = 0.48 \quad \text{---- for } Fe \text{ 415 steel}$$

$$\frac{x_u}{d} = \frac{x_{u,\max}}{d} = 0.48$$

$\therefore$ The section is balanced

Step : 3 Calculate ultimate moment of resistance

$$\begin{aligned} M_u &= 0.138 f_{ck} bd^2 \\ &= 0.138 \times 15 \times 250 \times 600^2 \\ &= 186300000 \text{ N–mm} \\ &= \mathbf{186.30\ kN–m} \end{aligned}$$

12. Find the moment of resistance of singly reinforced rectangular beam 230 × 500 mm effective depth reinforced with 4 bars of 16mm diameter. Use M20 grade concrete and Fe 415 steel.

Solution:

Given : $b = 230mm$, $d = 500mm$, $f_{ck} = 20N/mm^2$, $f_y = 415\ N/mm^2$

$$\text{Area of steel, } A_{st} = 4 \times \frac{\pi \times 16^2}{4} = 804.24 mm^2$$

Step : 1 Depth of neutral axis

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd} = \frac{0.87 \times 415 \times 804.24}{0.36 \times 20 \times 230 \times 500} = 0.35$$

Step : 2 Limiting neutral axis

$$\frac{x_{u,\max}}{d} = 0.48 \text{ For Fe415 steel}$$

$$\frac{x_u}{d} < \frac{x_{u,\max}}{d} \quad \therefore \text{The section is under reinforced section}$$

Step : 3 Calculate ultimate moment of resistance:

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{bdf_{ck}}\right)$$

$$M_u = 0.87 \times 415 \times 804.24 \times 500 \left(1 - \frac{804.24 \times 415}{230 \times 500 \times 20}\right) = 124.11 \times 10^6 N - mm = 124.11 kN - m$$

---

13. A singly reinforced concrete beam section 230 × 550 mm is reinforced with 4 bars of 25mm diameter with an effective cover of 40mm. The beam is simply supported over a span of 5m. Find the safe uniformly distributed load the beam can carry. Use M20 grade concrete and Fe415 steel.

Solution:

Given **:** b = 230*mm*, D = 550*mm*, f_{ck} = 20*N/mm*², f_y = 415 *N/mm*², L = 5m

Area of steel, $A_{st} = 4 \times \frac{\pi \times 25^2}{4} = 1963.5 mm^2$, $d = D - d' = 550 - 40 = 510$ mm

Step : 1 Depth of neutral axis

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd} = \frac{0.87 \times 415 \times 1963.5}{0.36 \times 20 \times 230 \times 510} = 0.84$$

Step : 2 Limiting neutral axis

$$\frac{x_{u,\max}}{d} = 0.48$$

$$\frac{x_u}{d} > \frac{x_{u,\max}}{d} \quad \therefore \text{The section is over reinforced section}$$

Step : 3 Calculate moment of resistance:

$$M_u = 0.138 f_{ck} bd^2 \text{ - Fe415}$$

$$M_u = 0.138 \times 20 \times 230 \times 510^2 = 165.11 \times 10^6 \, N-mm = 165.11 kN-m$$

Moment of resistance, $M = \frac{M_u}{1.5} = \frac{165.11}{1.5} = 110.07 kN.m$

Step : 4 Calculate safe udl on beam

$$M = \frac{WL^2}{8} \quad \Rightarrow W = \frac{8M}{L^2} = \frac{8 \times 110.07}{5^2} = 35.22 kN/m$$

$W = W_{DL} + W_{LL}$

W_{DL} = Self weight of beam = 0.23 × 0.55 × 25 = 3.16 kN/m

$\therefore W_{LL} = W - W_{DL}$ = 35.22 – 3.16 = 32.06 kN/m

---

14. Determine the area of reinforcement required for a singly reinforced concrete section having breadth of 300mm and overall depth 710mm to support Factored moment of 185kN-m. Adopt M-20 grade concrete and Fe415 grade steel. Effective cover to reinforcement is 35mm.

Solution:

Given : b = 300mm, D = 710 mm, M_u = 185kN-m,

$f_{ck} = 20 N/mm^2$, $f_y = 415 N/mm^2$

$\therefore d = D - d' = 710 - 35 = 675$ mm

Step : 1 Calculate limiting moment of resistance

$M_{u,lim} = 0.138 f_{ck} bd^2 = 0.138 \times 20 \times 300 \times 675^2 = 377.25 \times 10^6$ N.mm

$M_{u,lim}$ = 377.25 kN.m

Given, M_u = 185 kN.m

$\therefore M_{u,lim} > M_u$ or $M_u < M_{u,lim}$

The section is under reinforced

Step : 2 Calculation of Area of steel

W.K.T., $M_u = 0.87 f_y A_{st} d \left[1 - \frac{A_{st} f_y}{bdf_{ck}}\right]$

$$185 \times 10^6 = 0.87 \times 415 \times 675 A_{st} \left[1 - \frac{415 A_{st}}{300 \times 675 \times 20}\right]$$

$$24.97 \; A_{st}^2 - 243708.75 A_{st} + 185 \times 10^6 = 0$$

Solving quadratic equation, we get

A_{st} = 829.62 mm^2

15. Design the minimum effective depth required and the area of reinforcement for a rectangular beam having a width of 300mm to resist an moment of 150kN-m, using M-20 concrete and Fe-415HYSD bars.

Solution:

Given : $b = 300$mm, $M = 150kN\text{-}m$, $f_{ck} = 20N/mm^2$, $f_y = 415N/mm^2$

Step : 1 Calculate minimum effective depth

$$\text{Factored Moment, } M_u = M \times 1.5 = 150 \times 1.5 = 225 \text{ kN-m} = 225 \times 10^6 \text{ N-mm}$$

$$\text{Limiting moment } M_{u\,\lim} = 0.138 f_{ck}\, bd^2 \text{ For Fe415 steel}$$

$$\text{Equating } M_u = M_{u\,\lim}$$

$$M_u = 0.138 f_{ck}\, bd^2$$

$$\therefore d = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{225 \times 10^6}{0.138 \times 20 \times 300}}$$

Required effective depth $= 521.28mm$ Say $d = 530$mm

Step : 2 Calculate area of tensile reinforcement (A_{st})

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{bdf_{ck}}\right]$$

$$225 \times 10^6 = 0.87 \times 415 \times A_{st} \times 530 \left[1 - \frac{A_{st} \times 415}{300 \times 530 \times 20}\right]$$

$$24.97 A_{st}^2 - 191.35 \times 10^3 A_{st} + 225 \times 10^6 = 0$$

$$\therefore A_{st} = 1450.35 \text{ mm}^2$$

---

16. A singly reinforced rectangular beam is subjected to a bending moment of 75 kN-m at working loads. Calculate Area of steel. The materials used are M20 grade concrete and Fe415 steel. Provide effective depth 1.5 times the breadth.

Solution:

Given : $f_{ck} = 20N/mm^2$, $f_y = 415\ N/mm^2$, $M = 75kN\text{-}m$, $d = 1.5b$

Factored bending moment,

$$M_u = 1.5 \times 75 = 112.5\ kN\text{–}m$$

Step : 1 Calculate size of beam ($b \times d$)

Equating factored moment to the limiting moment of resistance

$$M_u = M_{u,lim} = 112.5 \times 10^6 \text{ N.mm}$$

$$M_{u,\lim} = 0.138 f_{ck} bd^2$$

$$112.5\times10^6 = 0.138\times20\times b\times(1.5b)^2$$

$$b^3 = \frac{112.5\times10^6}{6.21} = 18.11\times10^6$$

$$\therefore\ b = 262.63\text{mm, say } b = 270\text{mm}$$

$$\therefore\ d = 1.5b = 1.5\times270 = 405mm$$

Adopt 270 × 405 mm effective section.

Step : 2 Calculate area of tensile reinforcement (A_{st})

From IS:456-2000

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd}$$

For balanced section

$$\frac{x_u}{d} = \frac{x_{u,\max}}{d} = 0.48$$

$$\Rightarrow 0.48 = \frac{0.87\times415A_{st}}{0.36\times20\times270\times405}$$

$\therefore A_{st}$ = 1046.70 mm^2

17. If the ultimate load moment is 110 kN-m, what is the effective depth of singly reinforced concrete section, if the width of the beam is 230mm and concrete grade is M20 and type of steel is Fe415.

Solution:

Given : b = 230mm, M_u = 110KN-m, f_{ck} = 20N/mm^2, f_y = 415 N/mm^2

$M_{u\,\lim} = 0.138 f_{ck} bd^2$ - Fe415 steel

Equating $M_u = M_{u\lim}$

$$\therefore d = \sqrt{\frac{M_u}{0.138\times f_{ck}\times b}}$$

$$\therefore d = \sqrt{\frac{110\times10^6}{0.138\times20\times230}} = 416.27 \text{ mm say } 420 \text{ mm}$$

18. Calculate the area of reinforcement required for a simply supported reinforced concrete beam 230mm wide and 450mm effective depth to resist an ultimate moment of 80 kN-m. Assume M20 concrete and Fe415 steel.

Solution:

Given : $b = 230mm$, $d = 450mm$, $f_{ck} = 20 N/mm^2$, $f_y = 415\ N/mm^2$

Ultimate bending moment, $M_u = 80$KN-m

For Fe415 steel, limiting moment of resistance is given by,

Step : 1 Calculate limitng moment of resistence

$$M_{u,\lim} = 0.138 f_{ck} bd^2 \text{ - Fe415 (HYSD bars)}$$

$$M_{u,\lim} = 0.138 \times 20 \times 230 \times 450^2 = 128.54 \times 10^6\ N - mm \text{ or } 128.54 \text{ kN.m}$$

$$M_u < M_{u,lim}$$

As the ultimate moment to be resisted is less than the limiting moment, the section can be under reinforced section.

Step : 2 Calculate Area of Steel

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{bdf_{ck}}\right]$$

$$80 \times 10^6 = 0.87 \times 415 \times A_{st} \times 450 \left[1 - \frac{A_{st} \times 415}{230 \times 450 \times 20}\right]$$

$$32.57\ A_{st}^2 - 162.47 \times 10^3 A_{st} + 80 \times 10^6 = 0$$

$$A_{st} = 553.90 mm^2$$

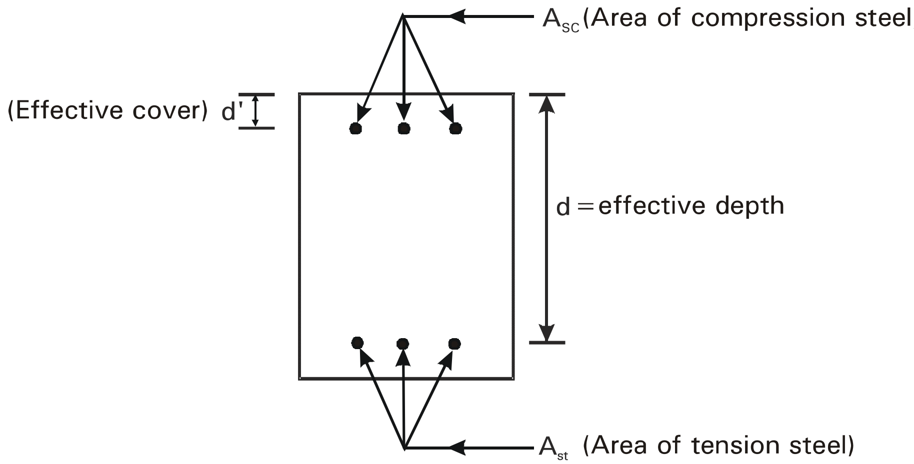
2.5 DOUBLY REINFORCED BEAMS

The R.C.C beam section in which steel reinforcement provided in both compression and tension zones are called Doubly reinforced beams.

R.C.C beams with compression reinforcement will be required in case where the depth of beam is restricted and Singly reinforced section is insufficient to resist the moment on the section.

Doubly reinforced beams, therefore, have moment of resistance more than the singly reinforced beams of the same depth for particular grades of steel and concrete. In many practical situations, architectural or functional requirements may restrict the overall depth of the beams.

It may be noted that even in so called singly reinforced beams there would be longitudinal hanger bars in compression zone for locating and fixing stirrups.



The necessity of doubly reinforced section arises due to following reasons:

1. When depth of section is restricted, the strength available from a singly reinforced section is inadequate.
2. When the external loads may occur on either face of the member i.e., the loads are alternating or reversing and may cause tension on both faces of the member.
3. When the loads are eccentric.
4. To reduce the deflection of the beam.
5. To resist the torsional moment.
6. In case of continuous beams or slab, the sections at supports are generally designed as doubly reinforced sections.
7. To improve the ductility of beams in earthquake regions.

2.6 ANALYSIS OF DOUBLY REINFORCED BEAM

A doubly reinforced beam has moment of resistance greater than that of limiting moment. Therefore a doubly reinforced beam subjected to factored moment can be analysed by considering it two sections as shown in fig. 2.1.

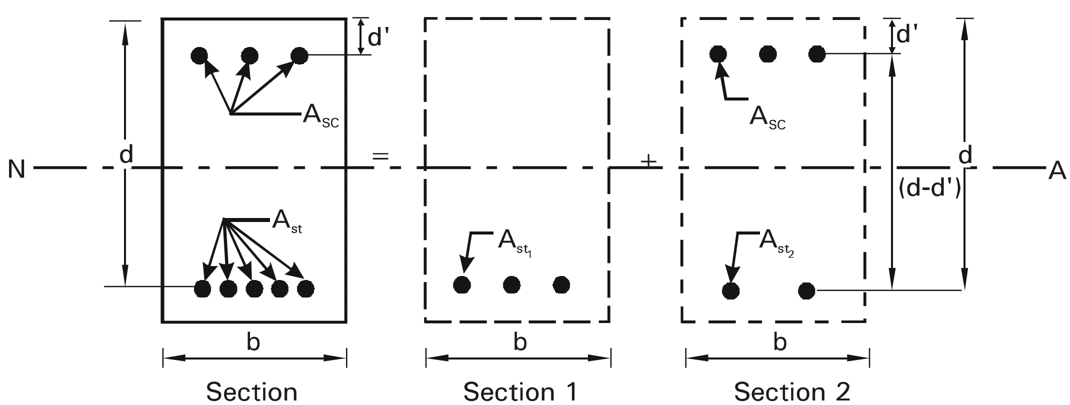
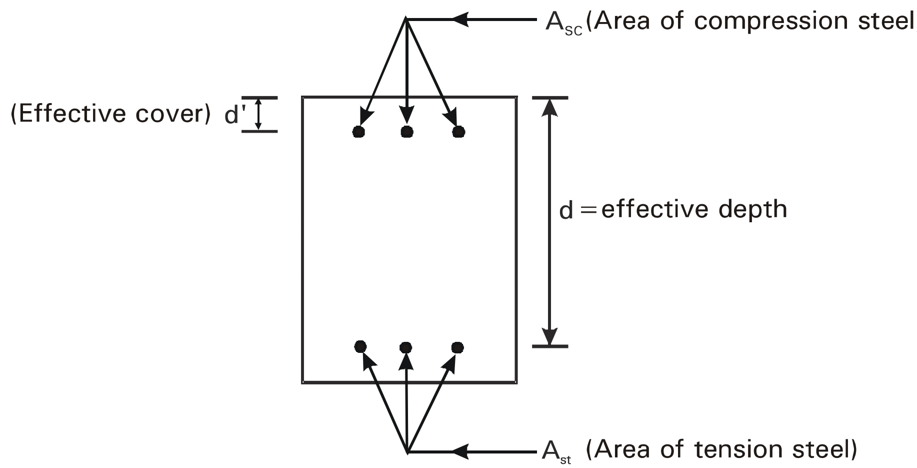


Fig. 2.1 : Doubly reinforced section



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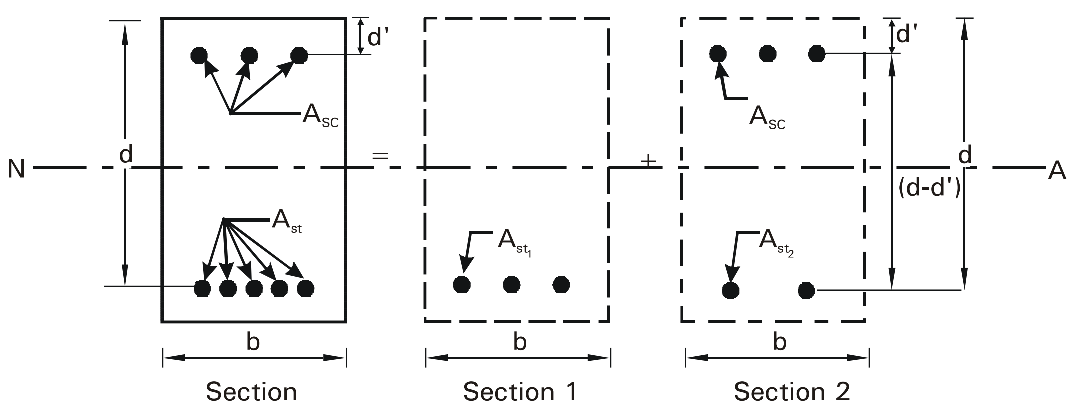


Fig. 2.1 : Doubly reinforced section

First section consists of a singly reinforced section to have moment of resistance of M_{u1} and reinforcement of A_{st1}.

Second section consists of compression steel A_{sc} and additional tension steel A_{st2}. The moment of resistance for this section is M_{u2}.

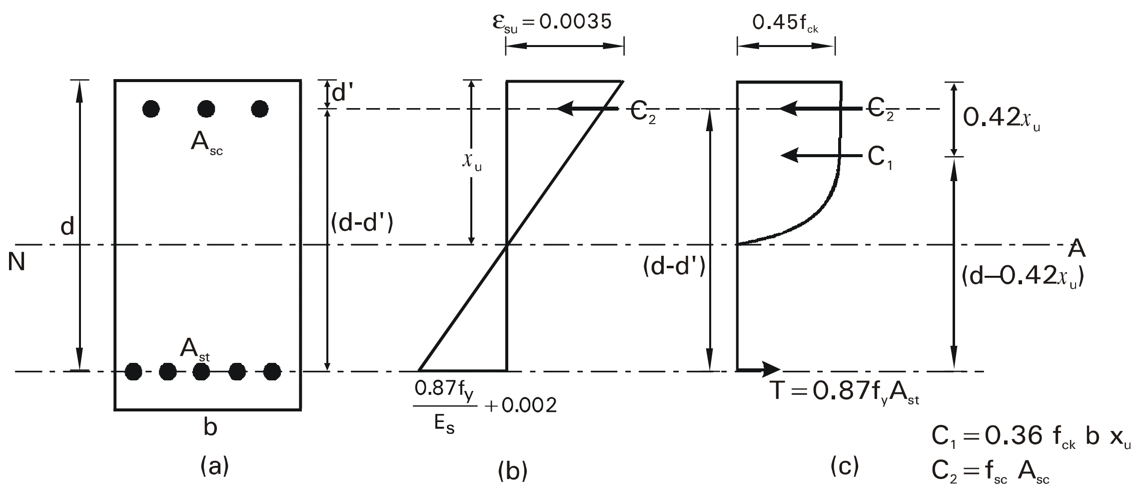


Fig : 2.2 Doubly reinforced section - strain diagram

From the fig. 2.2

M_{u1} corresponding to first section = $C_1 \times$ Lever arm

$$\therefore M_{u1} = 0.36 f_{ck} b x_u \left(d - 0.42 x_u\right)$$

$$\text{or } M_{u1} = 0.36 \frac{x_u}{d}\left(1 - 0.42\frac{x_u}{d}\right) f_{ck} b d^2$$

The additional moment of resistance, M_{u2} corresponding to second section = $C_2 \times$ Liver arm

$$\therefore M_{u2} = f_{sc} A_{sc}\left(d - d'\right)$$

$$M_u = M_{u1} + M_{u2}$$

$$M_u = 0.36 f_{ck} b x_u \left(d - 0.42 x_u\right) + f_{sc} A_{sc}\left(d - d'\right)$$

(Note: $x_u = x_{u\lim}$ for balanced section or over reinforced section)

For design of doubly reinforced beam - Refer IS : 456-2000, Clause G - 1.2

Applied moment M_u is usually larger than M_{ulim} given by equation

$M_{u\lim} = 0.36 \frac{x_{u\max}}{d}\left(1 - 0.42\frac{x_{u\max}}{d}\right) f_{ck} b d^2$ for a singly reinforced rectangular section.

The difference between the applied moment M_u and the limiting moment M_{ulim} is carried by additional tensile reinforcement A_{st2} and compression steel A_{sc}. Thus, a doubly reinforced section is equivalent to a singly reinforced balanced section and a section with additional tension and compression reinforcement.

Additional tension reinforcement given by,

$$A_{st2} = \frac{A_{sc} f_{sc}}{0.87 f_y}$$

The compression reinforcement given by,

$$A_{sc} = \frac{M_u - M_{u\lim}}{f_{sc}\left(d - d'\right)}$$

Where,

M_u = Ultimate flexural strength of doubly reinforced section.

M_{u_1} = Limiting or maximum moment of resistance of the singly reinforced section

M_{u_2} = Moment of resistances of the steel beam neglecting the Effect of concrete i.e., $M_{u_2} = f_{sc} A_{sc} (d - d')$

f_{sc} = The stress in the compression steel corresponding to the strain reached by it when the extreme concrete fibre reaches a strain of 0.0035

A_{sc} = Area of compression reinforcement

A_{st_1} = Area of tensile reinforcement for a singly reinforced section

A_{st_2} = Area of tensile reinforcement required to balance the compression reinforcement

A_{st} = Total area of tensile reinforcement i.e. $A_{st} = A_{st_1} + A_{st_2}$

d = Effective depth

d^1 = Depth of compression reinforcement from compressive force

b = Width of beam

E_s = Modulus of Elasticity of steel = 2×10^5 N/mm^2

ε_{sc} = Strain in compression steel

2.7 STEPS FOR SOLVING ANALYSIS PROBLEMS

Type I problems (To find M_u or M and W_u or W or W_{LL})

Given : A_{st} , A_{sc}, Size of beam, effective cover for compression steel (d'), Type of concrete and steel. If Load to be calculated then Span is given.

Required : Ultimate moment or Factored moment (M_u) or Moment of resistance (M), total load (W) or super imposed load (W_{LL})

Steps:

1. Calculate

$$A_{sc} = \text{No. of bars} \times \frac{\pi(\phi_c)^2}{4} \text{ in mm}^2$$

$$A_{st} = \text{No. of bars} \times \frac{\pi(\phi_t)^2}{4} \text{ in mm}^2$$

Where, ϕ_c = Dia of compression steel
ϕ_t = Dia of tenssion steel

2.Calculate $x_{u_{max}} = 0.53d$ for Fe 250

$x_{u_{max}} = 0.48d$ for Fe 415

$x_{u_{max}} = 0.46d$ for Fe 500

3. Stress in compression steel (f_{sc})

$$\varepsilon_{sc} = 0.0035 \frac{(x_{u,\max} - d')}{x_{u,\max}}$$ (Refer IS: 456-2000, Clause G.1.2)

Where ε_{sc} = strain in compression steel

Knowing the strain, the stress in compression steel (f_{sc}) can be obtained from stress strain curve of corresponding steel (IS: 456-2000, Fig 23 A).

Note: Calculation of f_{sc} for different grade of steel

1. For Mild Steel (Fe 250)

$$f_{sc} = \varepsilon_{sc} \times E_s \not> 0.87 f_y \qquad \text{Here } \varepsilon_{sc} = \frac{0.0035\left(x_{u_{max}} - d^1\right)}{x_{u_{max}}}$$

2. For Fe 415 HYSD bars
Using 'ε_{sc}' Note down the value of f_{sc} from IS: 456-2000, Fig 23 A

3. For Fe 500 HYSD bars : Use (a) or (b) for calculation of f_{sc}
   a. SP - 16, Table - A

Sl. No.	Stress level	Fe 500	
		Strain ε_{sc}	Stress f_{sc} (N/mm^2)
1.	$0.80 f_y$	0.00174	347.8
2.	$0.85 f_y$	0.00195	369.8
3.	$0.90 f_y$	0.00226	391.3
4.	$0.95 f_y$	0.00277	413.0
5.	$0.975 f_y$	0.00312	423.9
6.	$1.0 f_y$	0.00417	434.8
Linear interpolation may be done for intermediate values			

(or)

b. SP - 16 Table - F

Grade of Steel	$\frac{d'}{d}$			
	0.05	0.10	0.15	0.20
Fe 500	424 N/mm^2	412 N/mm^2	395 N/mm^2	370 N/mm^2

4. Calculate A_{st_2} and A_{st_1}

$$A_{st2} = \frac{A_{sc} f_{sc}}{0.87 f_y}$$

$$A_{st} = A_{st_1} + A_{st_2}$$

$$\therefore \; A_{st_1} = A_{st} - A_{st_2}$$

5. Depth of neutral exis

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st1}}{0.36 f_{ck} bd}, \text{ find } x_u$$

If, $x_u < x_{u,\max}$ section is under reinforced.

If $x_u > x_{u,\lim}$ section is over reinforced.

6. Calculate the Moment of resistance

$$\left.\begin{aligned} M_u - M_{u,\lim} &= f_{sc} A_{sc} (d - d') \\ M_u &= M_{u\lim} + f_{sc} A_{sc} (d - d') \\ \therefore \; M_u &= 0.36 \frac{x_u}{d}\left(1 - 0.42 \frac{x_u}{d}\right) bd^2 f_{ck} + f_{sc} A_{sc} (d - d') \end{aligned}\right\} \text{If } x_u < x_{u_{\max}}$$

or

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')$$

Put $x_u = x_{u,\lim}$ value and calculate Moment of resistance by the following expression,

$$\left.\begin{aligned} M_u &= 0.36 f_{ck} b x_{u,\lim} (d - 0.42 x_{u,\lim}) + f_{sc} A_{sc} (d - d') \text{ or} \\ M_u &= 0.148 f_{ck} bd^2 + f_{sc} A_{sc} (d - d') \text{ for } Fe\ 250 \\ M_u &= 0.138 f_{ck} bd^2 + f_{sc} A_{sc} (d - d') \text{ for } Fe\ 415 \\ M_u &= 0.133 f_{ck} bd^2 + f_{sc} A_{sc} (d - d') \text{ for } Fe\ 500 \end{aligned}\right\} \text{If } x_u > x_{u_{\max}}$$

7. To Calculate Safe udl of live Load : Follow the same steps as in singly reinforced beams

Type II Problems (Design of Doubly reinforced beam, A_{st} and A_{sc})

Data: Factored moment M_u or Moment of resistance (M), Total load (W), span with type of support (L), size of beam type of steel and concret.

Required : A_{st} & A_{sc} with No. of bars

Steps:

1. If Load (W) and Effective span (L) is given.

   Calculate Maximum bending moment, $M = \dfrac{WL^2}{8}$ for SS beam carrying udl

   $M = \dfrac{WL^2}{2}$ for Cantilever beam carrying udl

   Then, $M_u = 1.5 \times \text{M}$

2. Find the limiting moment of resistance,

   $M_{u_{\lim}} = 0.148 f_{ck} bd^2$ - Fe250

   $M_{u_{\lim}} = 0.138 f_{ck} bd^2$ - Fe415

   $M_{u_{\lim}} = 0.133 f_{ck} bd^2$ - Fe500

   If $M_u > M_{u,lim}$, Then beam is designed as doubly reinforced beam.

3. Area of compression steel,

   Find the f_{sc} value as explained in type I problem.

   $M_u - M_{u,\lim} = f_{sc} A_{sc} (d - d')$

   $A_{sc} = \dfrac{M_u - M_{u.\lim}}{f_{sc}(d - d')}$ Find A_{sc} with No. of bars

4. Area of tensile reinforcement,

   $A_{st1} = \dfrac{0.36 f_{ck} b x_{u,\max}}{0.87 f}$ Here $x_u = x_{u_{\max}}$

   $A_{st2} = \dfrac{A_{sc} f_{sc}}{0.87 f_y}$

   Total tension reinforcement A_{st}

   $A_{st} = A_{st1} + A_{st2}$

2.8 WORKED EXAMPLES ON ANALYSIS OF DOUBLY REINFORCED BEAMS

1. **A doubly reinforced beam section is 250 mm wide and 450 mm deep to the center of the tensile reinforcement. It is reinforced with 2 bars of 16 mm dia as compression reinforcement at an effective cover of 50mm and 4 bars of 25 mm dia as tensile steel. Using M-20 concrete and Fe-250 steel. Calculate the ultimate moment of resistance of the beam.**

   Solution:

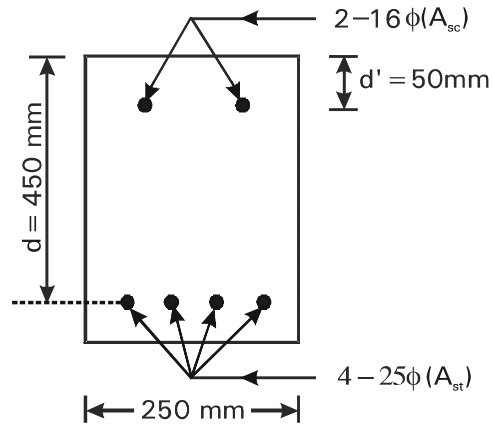
   Given : $b = 250$ mm, $d = 450$mm, $f_{ck} = 20 N/mm^2$, $f_y = 250 N/mm^2$

Step : 1 Calculate A_{sc} and A_{st}

$$A_{sc} = 2 \times \frac{\pi(16)^2}{4} = 402.12 mm^2$$

$$A_{st} = 4 \times \frac{\pi(25)^2}{4} = 1963.5 mm^2$$

$d' = 50mm$ (Given)



Step : 2 Calculate $x_{u\,max}$

$x_{u\,max} = 0.53 \times d = 0.53 \times 450 = 238.5$ mm

Step : 3 Stress in compression steel (f_{sc})

$f_{sc} = \varepsilon_{sc} \times E_s \ngtr 0.87 f_y \quad \because \; E_s = 2 \times 10^5$ N/mm²

$$= \left[\frac{0.0035(x_{u\max} - d')}{x_{u\max}}\right] \times 2 \times 10^5 = \left[\frac{0.0035(238.5 - 50)}{238.5}\right] \times 2 \times 10^5$$

$\therefore f_{sc} = 553.24 \ngtr 0.87 f_y$

Hence $f_{sc} = 0.87 f_y = 0.87 \times 250 = 217.5$ N/mm²

Step : 4 Calculate A_{st2} and A_{st1}

$$A_{st2} = \frac{f_{sc} A_{sc}}{0.87 f_y} = \frac{217.5 \times 402.12}{0.87 \times 250} = 402.12\,\text{mm}^2$$

$$A_{st1} = A_{st} - A_{st2} = 1963.5 - 402.12 = 1561.38\ \text{mm}^2$$

Step : 5 Depth of neutral axis

$$x_u = \frac{0.87 f_y A_{st1}}{0.36 f_{ck} b}$$

$$x_u = \frac{0.87 \times 250 \times 1561.38}{0.36 \times 20 \times 250} = 188.66\,\text{mm}$$

$x_u < x_{u\,\max}$

Hence the section is under reinforced

Step : 6 Calculate ultimate moment of resistance (M_u)

$$M_u = M_{u\lim} + f_{sc}\,A_{sc}\,(d - d')$$

$$= 0.36 f_{ck}\,bx_u\,(d - 0.42\,x_u) + f_{sc}\,A_{sc}\,(d - d')$$

$$= 0.36 \times 20 \times 250 \times 188.66\,(450 - 042 \times 188.66) + 217.5 \times 402.12\,(450\text{-}50)$$

$\therefore M_u = 160.89 \times 10^6$ N.mm or 160.89 kN.m

---

2. **A rectangular R.C. beam section is of 200mm × 500mm over all size. It is reinforced with 4 no's– 25mm dia bars in compression at an effective cover of 50 mm. Determine the area of tension reinforcement needed to make the beam section fully effective. What then would be the ultimate moment of resistance? Use M15 concrete and Fe –250 steel.**

Solution:

Given: b = 200 *mm*, D = 500 mm f_{ck} = 15 N/mm^2, f_y = 250N/mm^2, d' = 50 *mm* ,

d = 500 – 50 = 450 *mm,*

Step : 1 Calculate A_{sc}

$$A_{sc} = 4 \times \frac{\pi(25)^2}{4} = 1963.5 mm^2\ ,\ d' = 50\text{mm}$$

Step : 2 Calculate $x_{u\,\max}$

$x_{u,max}$ = 0.53d = 0.53 × 450 = 238.5 mm

Step : 3 Calculate $A_{st\,1}$

$x_u = x_{u\,\max}$ = 238.5mm (balanced section)

$$x_u = \frac{0.87 f_y A_{st1}}{0.36 f_{ck} b}$$

$$A_{st1} = \frac{0.36 f_{ck} x_u b}{0.87 f_y} = \frac{0.36 \times 15 \times 238.5 \times 200}{0.87 \times 250} = 1184.28 mm^2$$

Step : 4 Stress in compression steel (f_{sc})

$f_{sc} = 553.24 \not> 0.87\, f_y$ (Refer clause 2.8, Problem - 1)

$\therefore\ f_{sc} = 0.87 f_y = 0.87 \times 250 = 217.5\ N/mm^2$ (For Fe250 Steel)

Step : 5 Calculate A_{st2}

$$A_{st2} = \frac{f_{sc} A_{sc}}{0.87 f_y} = \frac{217.5 \times 1963.5}{0.87 \times 250} = 1963.5 mm^2$$

Step : 6 Calculate Area of Tension reinforcement (A_{st})

$$A_{st} = A_{st1} + A_{st2} = 1184.28 + 1963.5$$

$$A_{st} = 3147.78 mm^2$$

Step : 7 Calculate ultimate moment of resistance (M_u)

$f_{sc} = 553.24$ 0.87 fy (refer problem 1)

$$M_u = M_{u\,\lim} + f_{sc}\, A_{sc}\, (d - d')$$

$$M_u = 0.148 f_{ck} b d^2 + f_{sc} A_{sc} (d - d') \text{ ——— } for\ Fe\ 250$$

$$= 0.148 \times 15 \times 200 \times 450^2 + 217.5 \times 1963.5 (450 - 50) = 260.73 \times 10^6\ N - mm.$$

$$= 260.73 kN - m$$

---

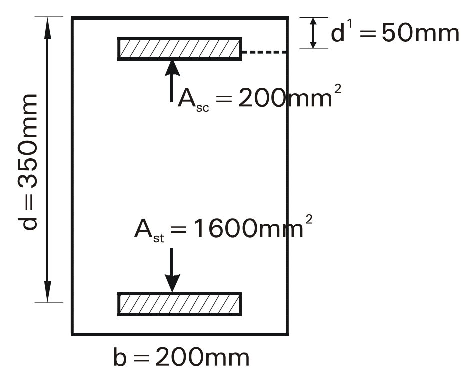
3. A doubly reinforced beam section is 200 mm wide and 350 mm deep to the center of the tensile reinforcement. It is provided with compression steel of area 200mm² at an effective cover of 50 mm and tensile steel of area 1600mm². Find the Ultimate Moment of resistance of the beam section. Take $f_{ck} = 20 N/mm^2$ & $f_y = 250 N/mm^2$.

Solution:

Given: $b = 200$ mm, $d' = 50 mm$,

$d = 350$ mm, $A_{sc} = 200 mm^2$, $f_{ck} = 20$ N/mm²,

$f_y = 250$ N/mm², $A_{st} = 1600 mm^2$, $M_u = ?$



Step : 1 Calculate $x_{u\,max}$

$x_{u\,max} = 0.53 \times d = 0.53 \times 350 = 185.5$ mm

Step : 2 Stress in compression steel (f_{sc})

$f_{sc} = \varepsilon_{sc} \times E_s \not> 0.87 f_y$ —— For Fe 250

$$= \left[\frac{0.0035(x_{u\max} - d')}{x_{u\max}}\right] \times 2 \times 10^5 = \left[\frac{0.0035(185.5 - 50)}{185.5}\right] \times 2 \times 10^5 = 511.32 \not> 0.87 f_y$$

$f_{sc} = 0.87 \times f_y = 0.87 \times 250 = 217.5$ N/mm^2

Step : 3 Calculate A_{st2} and A_{st1}

$$A_{st2} = \frac{f_{sc} A_{sc}}{0.87 f_y} = \frac{217.5 \times 200}{0.87 \times 250} = 200\,\text{mm}^2$$

$A_{st1} = A_{st} - A_{st_2} = 1600 - 200 = 1400$ mm^2

Step : 4 Depth of neutral axis

$$x_u = \frac{0.87 f_y A_{st1}}{0.36 f_{ck} b} = \frac{0.87 \times 250 \times 1400}{0.36 \times 20 \times 200}$$

$x_u = 211.45$ mm

$x_u > x_{u\,\max}$

$\therefore$ Section is over reinforced

Step : 5 Calculate ultimate moment of resistance (M_u)

$M_u = M_{u\,\lim} + f_{sc}\, A_{sc}\, (d - d')$

$$M_u = 0.148 f_{ck} b d^2 + f_{sc} A_{sc} (d - d')$$

$$= 0.148 \times 20 \times 200 \times 350^2 + 217.5 \times 200(350 - 50) = 85.57 \times 10^6\, N - mm.$$

$$= 85.57 kN - m$$

---

4. A doubly reinforced beam section is 250 mm wide 500mm deep to the center of the tensile reinforcement. It is reinforced with 2 bars of 16 mm dia as compression reinforcement at an effective cover of 40 mm, and 4 bars of 25 mm dia as tensile steel. Using of resistance M-20 concrete and Fe-415 steel, Calculate factored moment of resistance.

Solution:

Given : $b = 250$ mm, $d = 500$ *mm* , $d' = 40mm$, $f_{ck} = 20$ N/mm^2, $f_y = 415$N/mm^2

Step : 1 Calculate A_{sc} and A_{st}

$$A_{sc} = 2 \times \frac{\pi(16)^2}{4} = 402.12 mm^2$$

$$A_{st} = 4 \times \frac{\pi(25)^2}{4} = 1963.5 mm^2$$

Step : 2 Calculate $x_{u\,max}$

$x_{u\,max} = 0.48\,d = 0.48 \times 500 = 240$ mm

Step : 3 Stress in compression steel

$$\varepsilon_{sc} = \frac{00.35(x_{u\max} - d')}{x_{u\max}}$$

$$= \frac{0.0035(240-40)}{240}$$

$= 0.00291$

using ε_{sc}, note down the value of f_{sc} from IS:456–2000, Fig 23A

$\therefore f_{sc} = 0.84\,f_y$ corresponding to $f_y / 1.15$

$\therefore f_{sc} = 0.84 \times 415 = 348.6$ N/mm^2

Step : 4 Calculate A_{st_2} and A_{st_1}

$$A_{st_2} = \frac{f_{sc}A_{sc}}{0.87 f_y} = \frac{348.6 \times 402.12}{0.87 \times 415}$$

$\therefore\ A_{st_2} = 388.25$ mm^2

$A_{st_1} = A_{st} - A_{st_2} = 1963.5 - 388.25$

$= 1575.25$ mm^2

Step : 5 Depth of neutral axis

$$x_u = \frac{0.87 f_y A_{st1}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 1575.25}{0.36 \times 20 \times 250} = 315.96$$

$\therefore\ x_u > x_{u\,max}$, Hence section is over reinforced

Step : 6 Calculate Factored moment of resistance (M_u)

$M_u = M_{u\,lim} + f_{sc}\,A_{sc}$

$$M_u = 0.138 f_{ck} b d^2 + f_{sc} A_{sc} (d - d')$$

$$= 0.138 \times 20 \times 250 \times 500^2 + 348.6 \times 402.12(500-40) = 236.98 \times 10^6\ N-mm.$$

= 236.98 *kN-m*

---

5. **A doubly reinforced beam section is 250 mm wide, 500mm deep to the center of the tensile reinforcement. It is reinforced with 3 bars of 16 mm dia as compression reinforcement at an effective cover of 50 mm and 4 bars of 20 mm dia as tensile steel. Using M-20 concrete and Fe-500 steel, calculate the moment of resistance of the section.**

   Solution:

   Given: $b = 250$ mm, $d = 500\ mm$, $d' = 50mm$

Step : 1 Calculate A_{sc} and A_{st}

$$A_{sc} = 3 \times \frac{\pi(16)^2}{4} = 603.2 mm^2$$

$$A_{st} = 4 \times \frac{\pi(20)^2}{4} = 1256.64 mm^2$$

Step : 2 Calculate $x_{u\,max}$

$x_{u\ max} = 0.46\ d = 0.46 \times 500 = 230$ mm

Step : 3 Stress in compression reinforcement

$$\text{Now, } \frac{d'}{d} = \frac{50}{500} = 0.10$$

Refer, clause 2.7 Note - 3(b), SP-16, Table F

$$\text{For } \frac{d'}{d} = 0.1, f_{sc} = 412 N/mm^2$$

Step : 4 Calculate A_{st_2} and A_{st_1}

$$A_{st_2} = \frac{f_{sc} A_{sc}}{0.87 f_y} = \frac{412 \times 603.2}{0.87 \times 500} = 571.30 \text{ mm}^2$$

$$A_{st_1} = A_{st} - A_{st_2} = 1256.64 - 571.30 = 685.34 \text{ mm}^2$$

Step : 5 Depth of neutral axis (x_u)

$$x_u = \frac{0.87 f_y A_{st1}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 500 \times 685.34}{0.36 \times 20 \times 250}$$

$$\therefore x_u = 165.62 \text{ mm}$$

$$\Rightarrow x_u < x_{u,\ max}$$

The section is under reinforced.

Step : 6 Calculate Moment of reinforce of section (M)

$$M_u = M_{u\lim} + f_{sc} A_{sc}(d - d')$$

$$\text{Now, } M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + f_{sc} A_{sc}(d - d')$$

$$= 0.36 \times 20 \times 250 \times 165.62(500 - 0.42 \times 165.62) + 412 \times 603.2(500 - 50)$$

$$= 240.15 \times 10^6 N.mm.$$

$= 240.15\ kN\text{-}m$

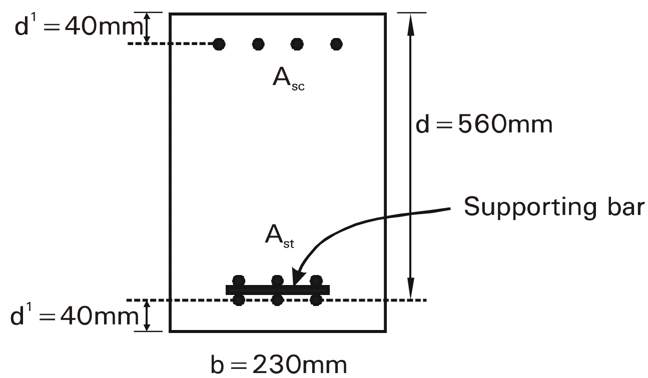
Moment of Resistance, $M = \frac{M_u}{1.5} = \frac{240.15}{1.5} = 160.1$ kN.m

6. A doubly reinforced beam of width of 230mm and 600mm total depth is reinforced with 4 bars of 16mm diameter as compression reinforcement and 6 bars of 20mm diameter as tension steel at an effective cover of 40mm on both sides. Find the super imposed load if beam is simply supported over an effective span of 6m.Use M20 grade concrete and Fe415.

Solution:

Given: $b = 230mm$, $D = 600\ mm$, $d' = 40mm$, $f_{ck} = 20N/mm^2$, $f_y = 415\ N/mm^2$, $L = 6m$

$\therefore d = 600 - 40 = 560$ mm



Step : 1 Calculate A_{st} and A_{sc}

$$A_{sc} = 4 \times \frac{\pi(16)^2}{4} = 804.2mm^2$$

$$A_{st} = 6 \times \frac{\pi(20)^2}{4} = 1885mm^2$$

Step : 2 Calculate $x_{u\,max}$

$x_{u\,max} = 0.48 \times d = 0.48 \times 560$

$= 268.80$ mm

Step : 3 Stress in compression steel (f_{sc})

$$\varepsilon_{sc} = \frac{0.0035(x_{u\max} - d')}{x_{u\max}} = \frac{0.0035(268.80 - 40)}{268.80} = 0.00279$$

using 'ε_{sc}' note down the value of f_{sc} from IS : 456 – 2000, Fig 23A

$\therefore f_{sc} = 0.83 f_y$ corresponding to $f_y/1.15$

$\therefore f_{sc} = 0.83 \times 415 = 344.45$ N/mm

Step : 4 Calculate A_{st_2} and A_{st_1}

$$A_{st_2} = \frac{f_{sc} A_{sc}}{0.87 f_y} = \frac{344.45 \times 804.2}{0.87 \times 415} = 767.22 \text{ mm}^2$$

$$A_{st_1} = A_{st} - A_{st_2} = 1885 - 767.22 = 1117.78 \text{ mm}^2$$

Step : 5 Depth of neutral axis

$$x_u = \frac{0.87 f_y A_{st1}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 1117.78}{0.36 \times 20 \times 230} = 243.70 \text{ mm}$$

$\therefore x_u < x_{u\max}$

Step : 6 Calculate ultimate Moment of resistance (M_u)

$M_u = M_{u\lim} + f_{sc} A_{sc} (d - d')$

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')$$

$$= 0.36 \times 20 \times 230 \times 243.70 (560 - 0.42 \times 243.70) + 344.45 \times 804.2 (560 - 40)$$

$\therefore M_u = 328.73 \times 10^6$ N.mm or 328.73 kN.m

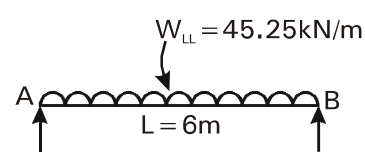
Step : 7 Calculation of super imposed load on beam

$$M = \frac{M_u}{1.5} = \frac{328.73}{1.5} = 219.15 \text{ kN.m}$$

For simply supported beam carrying UDL

$$M = \frac{WL^2}{8}$$

$$\therefore W = \frac{8M}{L^2} = \frac{8 \times 219.15}{6^2} = 48.7 \text{ kN/m}$$



W.K.T. $W = W_{DL} + W_{LL}$

W_{DL} = Self weight of beam = $0.23 \times 0.6 \times 25$

= 3.45 kN/m

$W_{LL} = W - W_{DL} = 48.7 - 3.45$

$W_{LL} = 45.25$ kN/m

7. Determine the moment of resistance of the beam having the following data. Size of beam = 300 × 550mm. Effective cover = 50mm. Tension reinforcement = 2500mm² and compression reinforcement = 500mm². Use M25 concrete and Fe500 steel.

Solution:

Given : b = 300 *mm*, D = 550 *mm*, $d' = 50mm$ d = 550 – 50 = 500*mm*, $A_{st} = 2500mm^2$,

$A_{sc} = 500mm^2$, f_{ck} = 25 N/mm², f_y = 500 N/mm²

Step : 1 Calculate $x_{u\,max}$

$x_{u\,max} = 0.46\,d = 0.46 \times 500 = 230$ mm

Step : 2 Stess in compression reinforcement (f_{sc})

$$\varepsilon_{sc} = \frac{0.0035\left(x_{u,\max} - d^1\right)}{x_{u,\max}} = \frac{0.0035(230-50)}{230}$$

$\therefore \varepsilon_{sc} = 0.002739$

Refer clause 2.7, Note 3(a), by using interpolation we get

$\therefore f_{sc} = 411.68$ N/mm²

Step : 3 Calculate A_{st_2} and A_{st_1}

$$A_{st_2} = \frac{f_{sc}A_{sc}}{0.87f_y} = \frac{411.68 \times 500}{0.87 \times 500} = 473.19mm^2$$

$$A_{st_1} = A_{st} - A_{st_2} = 2500 - 473.19 = 2026.81mm^2$$

Step : 4 Depth of neutral axis

$$x_u = \frac{0.87f_yA_{st_1}}{0.36f_{ck}b} = \frac{0.87 \times 500 \times 2026.81}{0.36 \times 25 \times 300} = 326.54mm$$

$x_u > x_{u\,max}$

The section is over reinforced.

Step : 5 Calculate moment of resistance

$M_u = M_{u\,\lim} + f_{sc}A_{sc}\,(d - d')$

$M_u = 0.133\,f_{ck}bd^2 + f_{sc}A_{sc}\,(d - d')$

$\therefore M_u = 0.133 \times 25 \times 300 \times 500^2 + 411.68 \times 500 \times (500 - 50)$

$\therefore M_u = 342$ kN.m

Moment of resistance, $M = \frac{M_u}{1.5} = \frac{342}{1.5} = 228kN.m$

8. A doubly reinforced rectangular beam of size 300mm × 600mm, simply supported at both the ends. The effective cover for both tension & compression steel is 35mm.The effective span is 6.0 m. The beam carries a super imposed load of 24 kN / m and superimposed dead load of 16 kN / m. Use M-20 grade of concrete and HYSD steel Fe-415. Determine tension and compression reinforcement. Draw the sketch showing reinforcements.

Solution:

Given : b = 300mm, D = 600mm, d'=35mm, L = 6.0m, Live load = 24kN/m,

Dead load = 16kN/m, f_{ck} = 20 N/mm² and f_y = 415 N/mm²

Step : 1 Load calculation

1) Self weight of beam = $b \times D \times 25 = 0.3 \times 0.6 \times 25 = 4.5$ kN/m
2) Super imposed dead load (Given) = 16 kN/m
3) Super imposed live load (Given) = 24 kN/m

W = 44.5 kN/m

Ultimate load (W_u) = 1.5 × 44.5 = 66.75 kN/m

Step : 2 Calculate ultimate moment of resistance (M_u)

$$M_u = \frac{W_u L^2}{8} = \frac{66.75 \times 6^2}{8} = 300.37 kN - m$$

Step : 3 Calculate limiting moment of resistance

$$d = D - 35 = 600 - 35 = 565mm$$

$$M_{u,\lim} = 0.138 f_{ck} bd^2 = 0.138 \times 20 \times 300 \times 565^2 = 264.32 \times 10^6 N - mm$$

$$= 264.32\ kN - m$$

$$\therefore M_u > M_{u,\lim}$$

The beam is designed as a doubly reinforced beam section.

Step : 4 Calculate Area of compression Reinforcement (A_{sc})

$$\varepsilon_{sc} = \frac{0.0035(x_{u\max} - d')}{x_{u\max}}$$

$$x_{u\,\max} = 0.48\ d = 0.48 \times 565 = 271.2\text{mm}$$

$$\therefore \varepsilon_{sc} = \frac{0.0035(271.2 - 35)}{271.2} = 0.00304$$

Now, using 'ε_{sc}' note down the value of f_{sc} from IS : 456-2000, Fig. 23A

$$f_{sc} = 0.85 f_y = 0.85 \times 415 = 352.75 \text{ N/mm}^2$$

Now, $$A_{sc} = \frac{M_u - M_{u,\lim}}{f_{sc}(d-d')} = \frac{(300.37-264.32)\times10^6}{352.75(565-35)} = 192.82mm^2$$

Step : 4 Calculate Area of Tension reinforcement (A_{st})

Since $M_u > M_{u\,\lim}$ considered as Balanced section

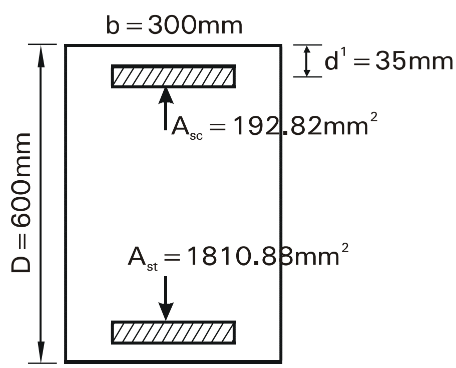
$$\Rightarrow x_u = x_{u\,\max} = 271.2 \text{ mm}$$

$$\Rightarrow x_u = \frac{0.87 f_y A_{st1}}{0.36 f_{ck} b}$$

$$A_{st1} = \frac{0.36 f_{ck} x_u b}{0.87 f_y} = \frac{0.36\times20\times271.2\times300}{0.87\times415} = 1622.5mm^2$$

$$A_{st2} = \frac{f_{sc} A_{sc}}{0.87 f_y} = \frac{352.75\times192.82}{0.87\times415} = 188.38mm^2$$

$$\therefore A_{st} = A_{st1} + A_{st2} = 1622.5 + 188.38 = 1810.88mm^2$$



9. A concrete beam has 350mm breadth and 700mm effective depth. Design the beam, if it is subjected to a factored BM of 600 kN.m, use M20 concrete and Fe 415 steel. Take d^1 = 50mm. Design stress - strain curve for Fe-415 steel are given below

SL. No.	Strain (ε_{sc})	stress (f_{sc}) N/mm²
1	0.00276	351.8
2	0.00380	360.9

Solution :

Given : $b = 350\text{mm}, d = 700\text{mm}, M_u = 600 \text{ kN.m}, f_{ck} = 20 \text{ N/mm}^2, f_y = 415 \text{ N/mm}^2, d^1 = 50\text{mm}$

Step : 1 Calculate $x_{u,\max}$

$$x_{u,\max} = 0.48 \times d = 0.48 \times 700 = 336 \text{ mm}$$

Step : 2 Stress in compression reinforcement (f_{sc})

$$\varepsilon_{sc} = \frac{0.0035\left(x_{u,\max} - d^1\right)}{x_{u,\max}} = \frac{0.0035\,(336 - 50)}{336} = 0.002979$$

Now using Design stress-strain curve for Fe 415 steel

$$f_{sc} = 353.71 \text{ N/mm}^2 \qquad \text{(By linear interpolation method)}$$

Step : 3 Calculate limiting moment of resistance

$$M_{u\,\lim} = 0.138 f_{ck} b d^2 = 0.138 \times 20 \times 350 \times 700^2 = 473.34 \text{ kN.m}$$

$$\therefore \quad M_u > M_{u\,\lim} \qquad \text{section is over reinforced}$$

The beam is designed as a doubly reinforced beam section

Step : 4 Calculate Area of compression reinforcement

$$A_{sc} = \frac{M_u - M_{u\,\lim}}{f_{sc}\left(d - d^1\right)} = \frac{(600 - 473.34) \times 10^6}{353.71 \times (700 - 50)} = 550.90 \text{ mm}^2$$

Assume 16 mm ϕ bars

$$\therefore \text{ Area of one bar } a_{sc} = \frac{\pi}{4} \times 16^2 = 201.06 \text{ mm}^2$$

$$\text{No. of bars} = \frac{A_{sc}}{a_{sc}} = \frac{550.90}{201.06} = 2.74, \quad \text{say } 3$$

Provide 3-16 mm ϕ bars @ compression reinforcement

Since $M_u > M_{u\,\lim}$, consider as Balanced section

$$\Rightarrow \quad x_u = x_{u\,\max} = 336 \text{ mm}$$

$$\therefore \quad x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$A_{st_1} = \frac{0.36 \times 20 \times 350 \times 336}{0.87 \times 415} = 2345.15 \text{ mm}^2$$

$$A_{st_2} = \frac{f_{sc} A_{sc}}{0.87 f_y} = \frac{353.71 \times 550.90}{0.87 \times 415} = 539.70 \text{ mm}^2$$

$$\therefore A_{st} = A_{st_1} + A_{st_2} = 2345.15 + 539.70 = 2884.85 \text{ mm}^2$$

Assume 25 mm ϕ bars

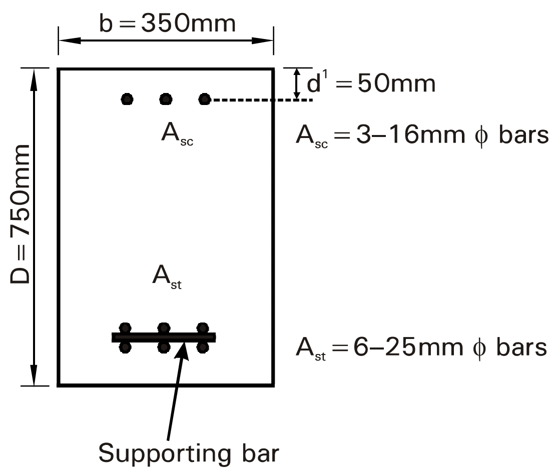
$$\text{Area of one bar, } a_{st} = \frac{\pi}{4} \times 25^2 = 490.87 \text{ mm}^2$$

$$\text{No. of bars} = \frac{A_{st}}{a_{st}} = \frac{2884.85}{490.87} = 5.87, \quad \text{say } 6$$

∴ Provide 6-25 mm ϕ bars @ tension reinforcement

Step : 5 Reinforcement details

| $D = d + 50 = 750$ mm



10. A RC beam of section 250mm × 500mm overall dimension is reinforced with 5 bars of 25mm diameter on tension side and 5 bars of 12mm diameter on compression side with an effective covers of 50mm for both. Determine the ultimate moment of resistance of the section Adopt M25 grade concrete and Fe 415 grade steel.

d^1/d	0.15	0.10
f_{sc} for Fe-415	342 N/mm²	353 N/mm²

Solution :

Given : $b = 250\text{mm}$, $D = 500\text{mm}$, $d^1 = 50\text{mm}$ (both side)

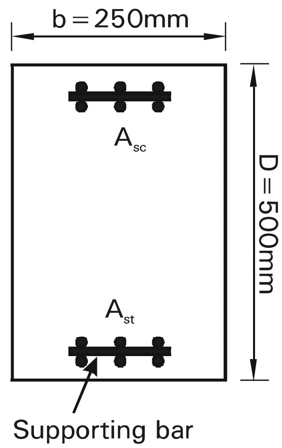
$$M_u = ?,\ f_{ck} = 25 \text{ N/mm}^2 \text{ and } f_y = 415 \text{ N/mm}^2$$

$$d = D - d^1 = 500 - 50 = 450 \text{ mm}$$

Step : 1 Calculate A_{sc} and A_{st}

$$A_{sc} = 5 \times \frac{\pi}{4} \times 12^2 = 565.48 \text{ mm}^2$$

$$A_{st} = 5 \times \frac{\pi}{4} \times 25^2 = 2454.36 \text{ mm}^2$$



Step : 2 Calculate $x_{u\max}$

$$x_{u\max} = 0.48 \times d = 0.48 \times 450 = 216 \text{ mm}$$

Step : 3 Stress in compression reinforcement

$$\frac{d^1}{d} = \frac{50}{450} = 0.11, \text{ using table,}$$

$$f_{sc} = 350.8 \text{ N/mm}^2 \quad \text{(By linear interpolation method)}$$

Step : 4 Calculate A_{st_2} and A_{st_1}

$$A_{st_2} = \frac{A_{sc} f_{sc}}{0.87 f_y} = \frac{565.48 \times 350.8}{0.87 \times 415} = 549.42 \text{ mm}^2$$

$$A_{st_1} = A_{st} - A_{st_2} = 2454.36 - 549.42 = 1904.94 \text{ mm}^2$$

Step : 5 Depth of neutral axis

$$x_u = \frac{0.87 f_y A_{st_1}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 1904.94}{0.36 \times 25 \times 250} = 305.67 \text{ mm}$$

$$\therefore \quad x_u > x_{u\,max}$$

The section is over reinforced

Step : 6 Calculate ultimate moment of resistance

$$M_u = M_{u\,lim} + f_{sc}A_{sc}\,(d - d^1)$$
$$= 0.138\ f_{ck}bd^2 + f_{sc}A_{sc}\,(d - d^1)$$
$$= 0.138 \times 25 \times 250 \times 450^2 + 350.8 \times 565.48\ (450 - 50)$$
$$M_u = 254 \times 10^6 \text{ N.mm} \quad \text{or} \quad 254 \text{ kN.m}$$

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11. Determine the ultimate moment (M_u) capacity of a beam b = 280 mm, d = 510 mm, d' = 50 mm, A_{st} = 2455 mm², A_{sc} = 402 mm², f_{ck} = 30 N/mm², f_y = 415 N/mm². Use f_{sc} = 353 N/mm².

Solution:

Given : $b = 280mm, d = 510mm, d' = 50mm, f_{ck} = 30N/mm^2, f_y = 415N/mm^2,$

$f_{sc} = 353N/mm^2, A_{st} = 2455mm^2, A_{sc} = 402mm^2$

Step : 1 Calculate $x_{u\,max}$

$$x_{u\,max} = 0.48d = 0.48 \times 510 = 244.8mm$$

Step : 2 Calculate A_{st_2} and A_{st_1}

$$A_{st_2} = \frac{f_{sc}A_{sc}}{0.87f_y} = \frac{353 \times 402}{0.87 \times 415} = 393.03 \text{ mm}^2$$

$$A_{st_1} = A_{st} - A_{st_2} = 2455 - 393.03 = 2061.97$$

Step : 3 Depth of neutral axis

$$x_u = \frac{0.87f_yA_{st_1}}{0.36f_{ck}b} = \frac{0.87 \times 415 \times 2061.97}{0.36 \times 30 \times 280}$$

$$x_u = 246.18 \text{ mm}$$

$$x_u > x_{u\,max}$$

$\therefore$ The section is Over reinforced.

Step : 4 Calculate ultimate moment of resistance (M_u)

$$M_u = M_{u,lim} + f_{sc}A_{sc}\,(d - d^1)$$
$$M_u = 0.138\ f_{ck}bd^2 + f_{sc}A_{sc}\,(d - d^1)$$
$$= 0.138 \times 30 \times 280 \times 510^2 + 353 \times 402\ (510 - 50)$$
$$= 366.78 \times 10^6 \text{ N.mm} \quad \text{or} \quad 366.78 \text{ kN.m}$$

2.9 T-BEAMS

In actual practice T-sections and L-sections are more common than the rectangular section since part of the RC slab, monolithic with the beam and participates with the structural behavior of the beam. For the same load and span T-beam and L-beam carries more moment of resistance than rectangular beams.

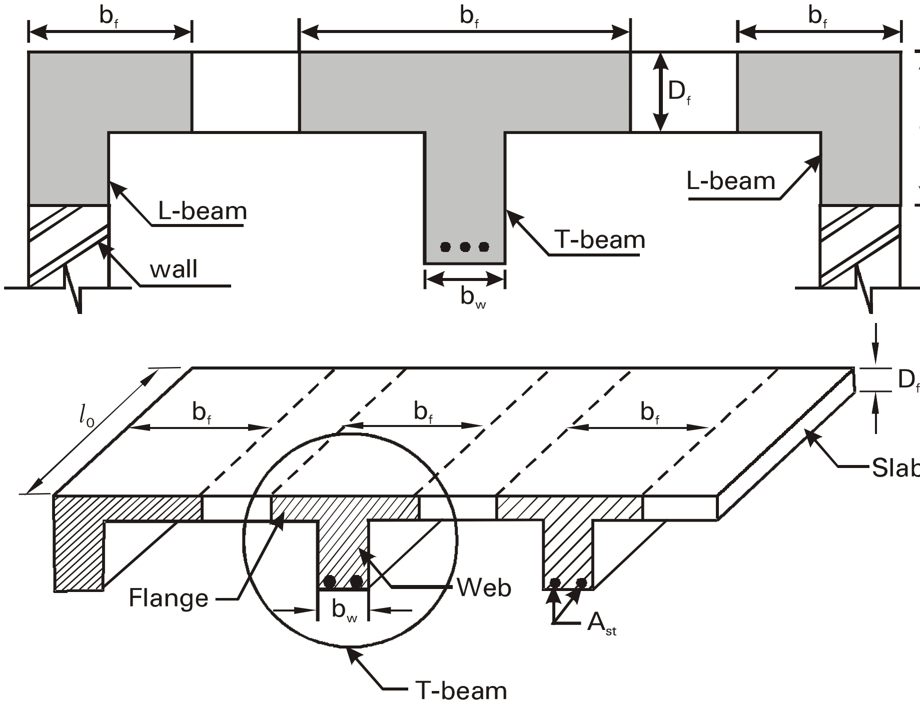


Fig. 2.3 : T-beam and L-beam

When a concrete slab is cast monolithically with and, connected to rectangular beams, a portion of the slab above the beam behaves structurally as a part of the beam in compression. The slab portions is called the flange and beam the web. If the flange projections are on either side of the rectangular web or rib, the resulting cross section resembles the T shape and hence is called a T-beam section. On the other hand, if the flange projects on one side, the resulting cross- section resembles an inverted L and hence is termed as L-beam. The fanged beams are shown in Fig. 2.3.

Advantages of T-beam

i) Beam and slab are casted manolithically hence, casting can be done at a time.

ii) Slab and beam combined together to carry more bending moment.

iii) For same section, T-beams have more moment of resistance (flexural strength) than that of rectangular beam.

2.10 EFFECTIVE WIDTH OF THE FLANGE

It is that portion of slab which acts integrally with the beam and extends on either side of the beam forming the compression zone. The effective width of flange depends upon the span of the beam , thickness of slab and breadth of the web. It also depends upon the type of loads and support conditions.

As per code (Clause 23.1.2 of IS:456-2000)

Effective flange width for T and L beams are as calculated follows:

a) For T- beams: $b_f = \frac{l_0}{6} + b_w + 6D_f$

b) For L-beams: $b_f = \frac{l_0}{12} + b_w + 3D_f$

c) For Isolated beams

i) For T- beams: $b_f = \frac{l_0}{\left(\frac{l_0}{b} + 4\right)} + b_w$

ii) For L-beams: $b_f = \frac{0.5l_0}{\left(\frac{l_0}{b} + 4\right)} + b_w$

where

b = actual width of the flange

b_f = effective width of the flange,

b_w = breadth of the web,

D_f = thickness of the flange,

l_0 = distance between point of zero moment

(for continuous beam, l_0 = 0.7 × (effective span of beam)

2.11 ANALYSIS OF T-BEAM

Analysis of T-beams is done by assuming the beam to be composed of two segments as shown in fig. 2.4.

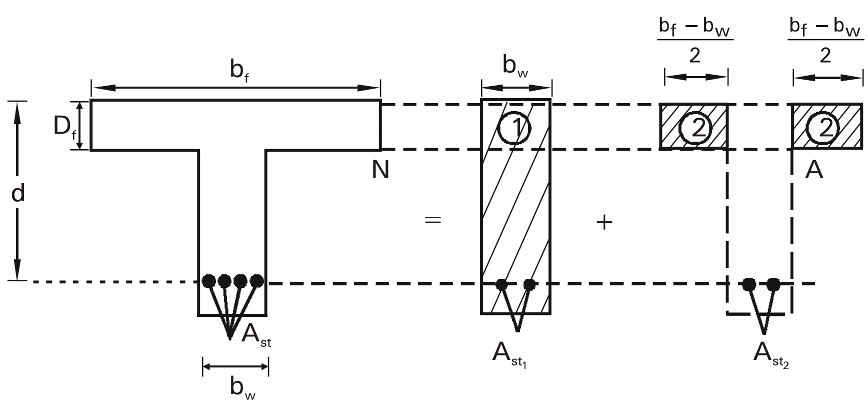


Fig. 2.4 : T-beam (C/S)

- First segment will be like a rectangular section and steel area A_{st1}.
- Second segment will be like a beam section having concrete section of area $[(b_f - b_w)D_f]$ and steel area of A_{st2}.

- Our consideration in design and analysis for depth of neutral axis $x_u > D_f$ will be ascertain the compressive force taken up by concrete in second segment and its line of action.
- If $x_u \leq D_f$, the beam can be thought of as a rectangular section of width b_f.

  The stress distribution for various values of x_u is as shown in fig. 2.5

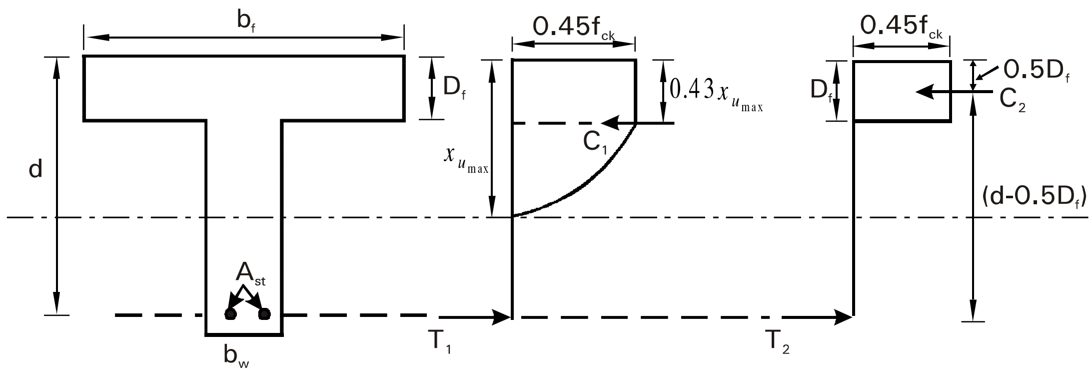


Fig. 2.5 : Stress block parameters (Flanged section)

Note : Derivation is not under the scope of this book

2.12 STEPS FOR CALCULATING DEPTH OF NEUTRAL AXIS AND MOMENT OF RESISTANT

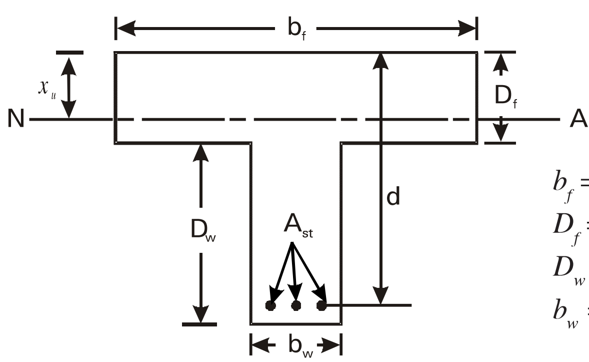
Given : b_f, d, A_{st}, D_f , grade of steel & grade of concrete, span for load calculation.

Required : Factored moment or moment of resistance load.

Case (i) Neutral axis lies with in the flange

Steps:

1. Calculate depth of neutral axis assuming neutral axis lies with in the flange



b_f = breadth of flange

D_f = Depth of flange or depth of slab

D_w = depth of web

b_w = breadth of web

Fig. 2.6 : Neutral axis lies with in the flange

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd}$$

Calculate x_u

If $x_u \leq D_f$ (Assumption is correct)

2. Note down the value of $\frac{x_{u,\max}}{d}$ from IS : 456 - 2000

   Calculate $x_{u,\,max}$

3. (a) If $x_u < x_{u,\,max}$ section is under reinforced calculate the Moment of resistance by the following expression

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b_f d f_{ck}}\right)$$

   (b) If $x_u > x_{u,\,max}$ section is over reinforced. Calculate the Moment of resistance by the following expression

$$M_{u,\lim} = 0.36 \frac{x_{u,\max}}{d}\left(1 - 0.42 \frac{x_{u,\max}}{d}\right) b_f d^2 f_{ck}$$

or $M_{u,\lim} = 0.148 f_{ck} b_f d^2$ —— Mild steel (Fe 250)

$M_{u,\lim} = 0.138 f_{ck} b_f d^2$ —— HYSD bars (Fe 415)

$M_{u,\lim} = 0.133\ f_{ck} b_f d^2$ —— HYSD bars (Fe 500)

4. Using Moment, calculate load, if required for support condition

Case (ii) Neutral axis lies below the flange

Steps:

1. Calculate neutral axis assuming Neutral axis (NA) lies with in flange. If $x_u > D_f$, assumption is wrong. NA lies below the flange.
2. Recalculate the value of x_u by using following relation $C_1 + C_2 = T$

   where

$$C_1 = 0.36 f_{ck} x_u b_w$$
$$C_2 = 0.45 f_{ck} (b_f - b_w) D_f$$
$$T = 0.87 f_y A_{st}$$
$$\Rightarrow C_1 + C_2 = T$$

$0.36 f_{ck} x_u b_w + 0.45 f_{ck} (b_f - b_w) D_f = 0.87 f_y A_{st}$ (Assume $\frac{D_f}{x_u} < 0.43$) and find x_u

If $x_u < D_f$ assumption is that $\frac{D_f}{x_u} > 0.43$

then recalculate x_u by using relation $C_1 + C_2 = T$

Where $C_1 = 0.36 f_{ck}\, x_u b_w$, $C_2 = 0.45 f_{ck}\,(b_f - b_w)\, y_f$,

$T = 0.87 f_y A_{st}$,

$y_f = (0.15\, x_u + 0.65\, D_f)$

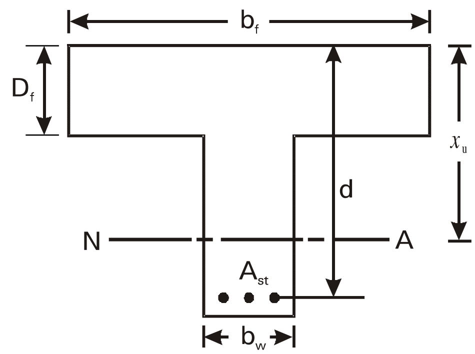


Fig. 2.7 : Neutral axis lies below the flange

3. If $x_u \geq x_{u,\,max}$ section is over reinforced or balanced.

a) If $\frac{D_f}{d} \leq 0.20$ use equation G.2.2, IS : 456-2000 for M_u calculation

$$M_u = 0.36\frac{x_{u,\max}}{d}\left(1 - 0.42\frac{x_{u,\max}}{d}\right) f_{ck} b_w d^2 + 0.45 f_{ck}\left(b_f - b_w\right) D_f \left(d - \frac{D_f}{2}\right)$$

b) If $\frac{D_f}{d} > 0.20$ use equation G.2.2.1, IS : 456-2000 for M_u calculation,

$$M_u = 0.36\frac{x_{u,\max}}{d}\left(1 - 0.42\frac{x_{u,\max}}{d}\right) f_{ck} b_w d^2 + 0.45 f_{ck}\left(b_f - b_w\right) y_f \left(d - \frac{y_f}{2}\right)$$

Where $y_f = (0.15 x_u + 0.65 D_f)$, but should not greater than D_f

4. If $x_u < x_{u,\,max}$ section is under reinforced

a) If $\frac{D_f}{x_u} \leq 0.43$ use equation G.2.2, IS : 456 – 2000 for M_u calculation with modification.

i.e., Replacing $x_{u,\,max}$ by x_u value.

$$M_u = 0.36\frac{x_u}{d}\left(1-0.42\frac{x_u}{d}\right)f_{ck}b_w d^2 + 0.45 f_{ck}\left(b_f - b_w\right)D_f\left(d-\frac{D_f}{2}\right)$$

b) If $\frac{D_f}{x_u} > 0.43$ use equation G.2.2.1, IS : 456 – 2000 for M_u calculation with modification.

i.e., Replacing $x_{u,\,max}$ by x_u value.

$$M_u = 0.36\frac{x_u}{d}\left(1-0.42\frac{x_u}{d}\right)f_{ck}b_w d^2 + 0.45 f_{ck}\left(b_f - b_w\right)y_f\left(d-\frac{y_f}{2}\right)$$

Where $y_f = (0.15x_u + 0.65D_f)$, but should not greater than D_f

2.13 STEPS FOR DETERMINATION OF AREA OF STEEL

Given : Dimensions of beam, Grade of concrete and steel, Factored moment.

Required : Area of tensile reinforcement.

Steps:

1. If load and span given

   Calculate Facfored moment $M_u = \frac{W_u L^2}{8}$

2. Calculate Moment of Resistance of flange only

   $$M_{uf} = 0.36 f_{ck} b_f D_f \left(d - 0.42 D_f\right)$$

3. If $M_{uf} \geq M_u$ then NA lies with in the flange. Hence design the T beam as " Rectangular Beam"

4. Case : 1 $x_u < D_f$

   The T - section is considered as rectangular section and the area of reinforcement is computed as

   $$M_u = 0.87 f_y A_{st}\, d\left(1-\frac{A_{st} f_y}{b_f d f_{ck}}\right)$$

   Find A_{st}

   Case : 2 $x_u > D_f, \frac{D_f}{d} \leq 0.2$ and $\frac{D_f}{x_u} < 0.43$

   For a known value of M_u, compute x_u

   $$C_1 = T_1$$

   $$0.36 f_{ck} b_w x_u = 0.87 f_y A_{st_w}$$

   $$\therefore \quad A_{st_w} = \frac{0.36 f_{ck} b_w x_u}{0.87 f_y}$$

Also $C_2 = T_2$

$$0.45\, f_{ck}\, (b_f - b_w)\, D_f = 0.87\, f_y\, A_{st_f}$$

$$\therefore \quad A_{st_f} = \frac{0.45 f_{ck}\left(b_f - b_w\right) D_f}{0.87 f_y}$$

Hence the total reinforcement in the T-section is obtained as

$$A_{st} = A_{st_w} + A_{st_f}$$

$$A_{st} = \left[\frac{0.36 f_{ck} b_w x_u}{0.87 f_y}\right] + \left[\frac{0.45 f_{ck}\left(b_f - b_w\right) D_f}{0.87 f_y}\right]$$

Case : 3 $x_u > D_f$, $\frac{D_f}{d} > 0.2$ and $\frac{D_f}{x_u} > 0.43$

For a known value of M_u, evaluate x_u by replacing $x_{u,\max}$ by x_u

Since $y_f = (0.15\, x_u + 0.65\, D_f)$

$$\therefore \quad A_{st_w} = \frac{0.36 f_{ck} b_w x_u}{0.87 f_y}$$

$$A_{st_f} = \frac{0.45 f_{ck}\left(b_f - b_w\right) y_f}{0.87 f_y}$$

$$\Rightarrow \quad A_{st} = A_{st_w} + A_{st_f}$$

2.14 WORKED EXAMPLES ON ANALYSIS OF T-BEAM

1. **Find the flange width of the following simply supported T-beam**

   Effective span = 6.0m

   C/C distance of adjacent panels = 3.0m

   Breadth of the web =350mm

   Thickness of slab = 100mm

   Solution:

   Given : $L = 6\text{m}$, $b_w = 350\text{mm}$, $D_f = 100\text{mm}$

   Since the beam is simply supported, the distance between the points of zero moments

   $$l_o = L = 6\text{m}$$

   Clear span of the slab to the left or right of the beam

   = C/C distance of adjacent panels – b_w = 3000 – 350 = 2650mm

- Effective width of the flange is the least of the following

a) $b_f = \frac{l_o}{6} + b_w + 6D_f$

$$b_f = \frac{6000}{6} + 350 + 6\times100 = 1950\text{mm}$$

b) b_f = b_w + Half of the clear distance to the adjacent beams on either side

$$b_f = 350 + \frac{2650}{2} + \frac{2650}{2} = 3000\text{mm}$$

Therefore, $b_f = 1950\text{mm}$

---

2. A singly reinforced slab 120 mm thick is supported by T-beam spaced 3 m c/c. The effective depth of beam 580mm and width of web 450 mm . Mild steel reinforcement 8 bars of 20mm dia have been provided in tension in two layers. The effective cover to steel bars in lower tier is 50 mm. The effective span of simply supported beams is 3.6 m. The grade of concrete is M-20. Determine the depth of neutral axis and the moment of resistance of T-beam section.

Solution:

Given : D_f= 120*mm*, d = 580*mm*, b_w= 450*mm*, effective cover = 50*mm*, L = 3.60*m*

$$f_{ck} = 20\text{N / mm}^2, f_y = 250\text{N / mm}^2$$

$$A_{st} = 8\times\frac{\pi(20)^2}{4} = 2513.27\text{mm}^2$$

Step : 1 Effective width of flange (b_f)

a) $b_f = \frac{l_o}{6} + b_w + 6D_f$

$$b_f = \frac{3.6}{6} + 0.45 + 6\times0.12 = 1.77m$$

b) $b_f = b_w$ + Half of the clear distance to the adjacent beams on either side

b_f = 0.45 + (3.0 – 0.45) = 3.0m

$\therefore$ b_f = 1.77m

Step : 2 Depth of Neutral axis

Assuming Actual Neutral Axis (x_u) lies within the flange ($i.e., x_u \le D_f$)

$$x_u = \frac{0.87 f_y A_{st}}{0.36 b_f f_{ck}} = \frac{0.87 \times 250 \times 2513.27}{0.36 \times 1770 \times 20} = 42.89mm < D_f\ (120mm)$$

Hence, Assumption is correct

Step : 3 Note down the value of $x_{u,\max}$

$$x_{u,\max} = 0.53d = 0.53 \times 580 = 307.4mm$$

$$\therefore x_u < x_{u,\max}$$

The section is under reinforced.

Step : 4 Calculate Moment of resistance

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{b_f d f_{ck}}\right)$$

$$= 0.87 \times 250 \times 2513.27 \times 580\left(1 - \frac{2513.27 \times 250}{1770 \times 580 \times 20}\right) = 307.34 \times 10^6\ N - mm \text{ or } 307.34 kN.m$$

$\therefore$ Moment of resistance, $M = \frac{M_u}{1.5} = 204.89$ kN.m

---

3. A T-beam of depth of 450 mm has a flange width of 1000 mm and depth of 120 mm. It is reinforced with 6 – 20mmϕ bars on tension side with a cover of 30 mm. If M-20 concrete and Fe-415 steel are used. Calculate moment of resistance of beam. Take b_w = 300mm.

Solution:

Given : $b_w = 300mm$, $b_f = 1000mm$, $D_f = 120mm$, clear cover $= 30mm$

$$\therefore \text{Effective cover} = 30 + \frac{20}{2} = 40mm \qquad \left| d = d^1 + \frac{\text{diameter}}{2} \right.$$

$$d = 450 - 40 = 410mm$$

$$A_{st} = 6 \times \frac{\pi(20)^2}{4} = 1885mm^2, f_{ck} = 20 \text{ N/mm}^2 \text{ and } f_y = 415 \text{ N/mm}^2$$

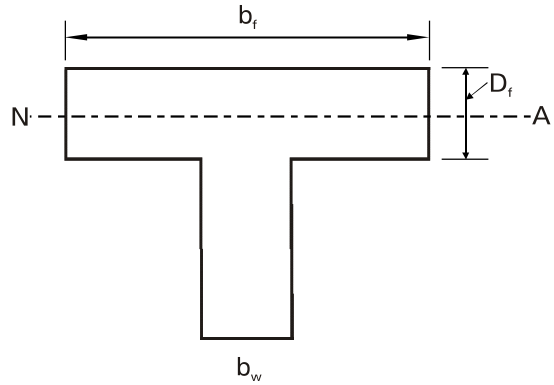
Step : 1 Depth of neutral axis (x_u)

Assuming Neutral Axis (x_u) lies within the flange ($i.e., x_u \le D_f$)

$$x_u = \frac{0.87 f_y A_{st}}{0.36 b_f f_{ck}} = \frac{0.87 \times 415 \times 1885}{0.36 \times 1000 \times 20} = 94.52mm < D_f\ (120mm)$$

Hence, Assumption is correct

Step : 2 Note down the value of $x_{u,max}$



$$x_{u,max} = 0.48d = 0.48\times410 = 196.8mm$$

$$\therefore x_u < x_{u,max}$$

The section is under reinforced.

Step : 3 Calculate Moment of resistance

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{b_f d f_{ck}}\right)$$

$$= 0.87\times415\times1885\times410\left(1 - \frac{1885\times415}{1000\times410\times20}\right) = 252.41\times10^6 N - mm \text{ or } 252.41 kN.m$$

$\therefore$ Moment of resistance, $M = \frac{M_u}{1.5} = \frac{252.41}{1.5} = 168.27kN.m$

---

4. **Calculate the Ultimate moment of resistance of a tee-beam having the following section properties. Use M20 and Fe 415 HYSD bars.**

   Width of flange = 1300mm

   Thickness of flange = 100mm

   Width of rib = 325mm

   Effective depth = 600mm

   Area of steel = 4000mm²

   Solution:

   Given : $b_w = 325mm$, $b_f = 1300mm$, $D_f = 100mm$, $d = 600mm$, $f_{ck} = 20N/mm^2$

   $f_y = 415N/mm^2$, $A_{st} = 4000mm^2$

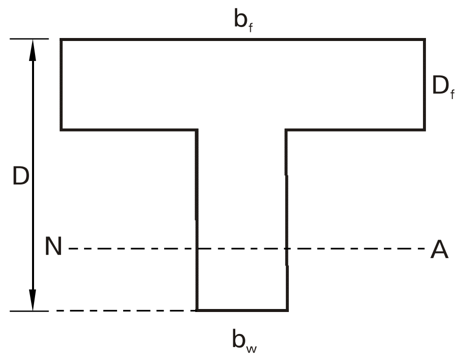
Step : 1 Depth of neutral axis (x_u)

Assuming Neutral Axis (x_u) lies within the flange ($i.e., x_u \le D_f$)

$$x_u = \frac{0.87 f_y A_{st}}{0.36 b_f f_{ck}} = \frac{0.87\times415\times4000}{0.36\times1300\times20} = 154.3mm > D_f\ (100mm)$$

Assumption is wrong, neutral axis lies below the flange.

$$\therefore \quad \frac{D_f}{d} = \frac{100}{600} = 0.166 < 0.2$$



Step : 2 Recalculate the value of 'x_u'

$$C_1 + C_2 = T$$

$$C_1 = 0.36 f_{ck} b_w x_u = 0.36 \times 20 \times 325 \times x_u = 2340 x_u$$

$$C_2 = 0.45 f_{ck} D_f \left(b_f - b_w\right) = 0.45 \times 20 \times 100 \times (1300 - 325) = 877500 N$$

$$T = 0.87 f_y A_{st} = 0.87 \times 415 \times 4000 = 1444200 N$$

$$2340 x_u + 877500 = 1444200$$

$$x_u = 242.18 mm$$

Step : 3 Note down the value of $x_{u,max}$

$$x_{u,\max} = 0.48 d = 0.48 \times 600 = 288 mm$$

$$\frac{x_u}{d} = \frac{242.18}{600} = 0.403$$

$\therefore$ $x_u < x_{u,\max}$, under reinforced section

$$\frac{D_f}{x_u} = \frac{100}{242.18} = 0.413 < 0.43$$

Step : 4 Calculate ultimate moment of resistance

Hence according to clause G-2.3 of IS:456-2000,Moment of resistance is computed by replacing x_u by $x_{u,\max}$

$$M_u = 0.36 \frac{x_u}{d}\left(1 - 0.42 \frac{x_u}{d}\right) f_{ck} b_w d^2 + 0.45 f_{ck} \left(b_f - b_w\right) D_f \left(d - \frac{D_f}{2}\right)$$

$$M_u = 0.36 \times 0.403 (1 - 0.42 \times 0.403) 20 \times 325 \times 600^2 + 0.45 \times 20 (1300 - 325) 100 \left(600 - \frac{100}{2}\right)$$

$$= 282025596.5 + 482625000$$

$$= 764.65 \times 10^6 \text{ N.mm} \quad \text{or} \quad 764.65 \text{ kN.m}$$

5. An isolated T-beam has a flange of 1200 × 100mm , width of rib is 250mm and effective depth is 600mm. Tension steel is 3500mm². Grade of concrete is M20 and steel grade is Fe415. Compute the ultimate moment of resistance. Span of SS beam = 8m. Also calculate the safe superimposed load the T-beam can carry, if effective cover = 50mm.

Solution:

Given : $b_w = 250mm$, $b = 1200mm$, $D_f = 100mm$, $d = 600mm$, $f_{ck} = 20N/mm^2$,

$f_y = 415N/mm^2$, $A_{st} = 3500mm^2$, $L = 8m$, $D = 600 + 50 = 650mm$

Step : 1 Calculate Effective flange width (b_f)

For Isolated T-beam Effective flange width (Refer IS:456-2000)

b = actual width of flange = 1200mm

l_0 = Span = 8m = 8000mm

$$b_f = \frac{l_0}{\frac{l_0}{b} + 4} + b_w = \frac{8000}{\frac{8000}{1200} + 4} + 250 = 1000mm$$

Step : 2 Depth of neutral axis (x_u)

Assuming Neutral Axis (x_u) lies within the flange ($i.e., x_u \le D_f$)

$$x_u = \frac{0.87 f_y A_{st}}{0.36 b_f f_{ck}} = \frac{0.87 \times 415 \times 3500}{0.36 \times 1000 \times 20} = 175.51mm > D_f\,(100mm)$$

Hence, Assumption is wrong , neutral axis lies below the flange.

$$\frac{D_f}{d} = \frac{100}{600} = 0.166 < 0.2$$

Step : 3 Recalculate the value of x_u

$$C_1 + C_2 = T$$

$$C_1 = 0.36 f_{ck} b_w x_u = 0.36 \times 20 \times 250 \times x_u = 1800 x_u$$

$$C_2 = 0.45 f_{ck} D_f \left(b_f - b_w\right) = 0.45 \times 20 \times 100 \times (1000 - 250) = 675000N$$

$$T = 0.87 f_y A_{st} = 0.87 \times 415 \times 3500 = 1263675N$$

$$1800 x_u + 675000 = 1263675$$

$$x_u = 327.04mm$$

Step : 4 Note down the value of $x_{u,\max}$

$$x_{u,\max} = 0.48d = 0.48 \times 600 = 288mm$$

$x_u > x_{u,\,max}$ (Over reinforced section)

$$\frac{D_f}{x_u} = \frac{100}{327.04} = 0.305 < 0.43$$

Step : 5 Calculate moment of resistance (working moment)

Hence according to clause G-2.2 of IS:456-2000, Moment of resistance is computed without any modification.

$$M_u = 0.36\frac{x_{u,\max}}{d}\left(1-0.42\frac{x_{u,\max}}{d}\right)f_{ck}b_w d^2 + 0.45 f_{ck}\left(b_f - b_w\right)D_f\left(d-\frac{D_f}{2}\right)$$

$$M_u = 0.36\times0.48\left(1-0.42\times0.48\right)20\times250\times600^2 + 0.45\times20\left(1000-250\right)100\left(600-\frac{100}{2}\right)$$

M_u = 619.58 × 10^6 *N-mm* = 619.58 *kN-m*

$$\text{Working moment} = M = \frac{M_u}{1.5} = \frac{619.58}{1.5} = 413.05 kN - m$$

Step : 6 Calculate safe super imposed on T-beam

Total load= Dead Load or self weight of beam + Live load

$$M = \frac{WL^2}{8} \Rightarrow W = \frac{8M}{L^2} = \frac{8\times413.05}{8^2} = 51.63 kN/m$$

Depth of web, $D_w = D - D_f$ = 650 – 100 = 550*mm*

W_{DL} = Self weigth of beam = $(b_f \times D_f + b_w \times D_w)\times density$ of RCC

$= (1.0\times0.10 + 0.25\times0.55)\times25 = 5.94 kN/m$

$\therefore W_{LL} = W - W_{DL}$ = 51.63 – 5.94 = 45.69 kN/m

6. Calculate the ultimate moment of resistance of a T - beam using following data;

i) Width of flange	**=**	**1500 mm**
ii) Depth of flange	**=**	**100 mm**
iii) Overall depth of beam	**=**	**600 mm**
iv) Width of rib	**=**	**300 mm**
v) Area of tension steel	**=**	**2455 mm²**
vi) Effective cover	**=**	**40 mm**

vii) f_{ck} = 15 Mpa, f_y = 415 Mpa

Solution:

Given : b_f = 1500 mm, D_f = 100 mm, D = 600 mm, b_w = 300 mm

A_{st} = 2455 mm^2, d^1 = 40 mm, d = 600 – 40 = 560 mm

f_{ck} = 15 N/mm^2, f_y = 415 N/mm^2

Step : 1 Depth of neutral axis (x_u)

Assuming depth of neutral axis lies with in the flange ($x_u \leq D_f$)

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f d}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f} = \frac{0.87 \times 415 \times 2455}{0.36 \times 15 \times 1500}$$

$$x_u = 109.42 \text{ mm} > D_f$$

Hence, Assumption is wrong

Step : 2 Recalculate the value of x_u

Now assuming neutral axis lies below flange and $\frac{D_f}{x_u} < 0.43$

$C_1 = 0.36 f_{ck} x_u b_w = 0.36 \times 15 x_u \times 300 = 1620 x_u$

$C_2 = 0.45 f_{ck} (b_f - b_w) D_f = 0.45 \times 15 (1500 - 300) \times 100 = 810000$ N

$T = 0.87 f_y A_{st} = 0.87 \times 415 \times 2455 = 886377.75$ N

$\Rightarrow C_1 + C_2 = T$

$$1620 x_u + 810000 = 886377.75$$

$$x_u = \frac{76377.75}{1620} = 47.14 \text{ mm}$$

$$x_u < D_f$$

Therefore assumption is that $\frac{D_f}{x_u} > 0.43$,

Step : 3 Again Recalculae the value of x_u

C_2 value changes, $C_2 = 0.45 f_{ck} (b_f - b_w) y_f$

$$y_f = 0.15 x_u + 0.65 D_f$$

$$C_2 = 0.45 \times 15 (1500 - 300) y_f$$

$$y_f = 0.15 \times x_u + 0.65 \times 100 = 0.15 x_u + 65$$

$$\Rightarrow C_2 = 0.45 \times 15 (1500 - 300) (0.15 x_u + 65)$$

$$= 8100 (0.15 x_u + 65) = 1215 x_u + 526500$$

Now using relation, $C_1 + C_2 = T$

$1620 x_u + 1215 x_u + 526500 = 886377.75$

$$2835 x_u = 359877.75$$

$$x_u = 126.94 \text{ mm} > D_f$$

$$\text{Now,} \quad \frac{D_f}{x_u} = \frac{100}{126.94} = 0.78 > 0.43$$

Hence, Assumption is correct.

Step : 4 Note down the value of $x_{u\,max}$

$$x_{u\,max} = 0.48\,d = 0.48 \times 560 = 268.80 \text{ mm}$$

$\therefore$ $x_u < x_{u\,max}$, section is under reinforced.

Step : 5 Calculate ultimate moment of resistance

Refer IS 456 - 2000, calculate moment of resistance using formula replacing $x_{u\,max} = x_u$

$$M_u = 0.36\,\frac{x_u}{d}\left(1 - 0.42\,\frac{x_u}{d}\right) f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) \times y_f \left(d - \frac{y_f}{2}\right)$$

$$y_f = 0.15\,x_u + 0.65\,D_f = 0.15 \times 126.94 + 0.65 \times 100$$

$$= 84.04 \text{ mm}$$

$$\frac{x_u}{d} = \frac{126.94}{560} = 0.23$$

$$\therefore M_u = 0.36 \times 0.23\,(1 - 0.42 \times 0.23) \times 15 \times 300 \times 560^2 + 0.45 \times 15 \times (1500 - 300) \times 84.04\left(560 - \frac{84.04}{2}\right)$$

$$= 105559905 + 352601417.5$$

$$= 458.16 \times 10^6 \text{ N–mm}$$

$$M_u = 458.16 \text{ kN–m}$$

---

7. Analyze a beam of T-shaped cross-section having an effective flange width of 1500 mm. flange -thickness of 100 mm, web width of 300 mm and an effective depth of 600 mm, to determine the limiting or ultimate moment of resistance of the beam for the cases of tension reinforcement of: (i) 5 × 22 mm dia bars, and (ii) 5 × 28 mm dia bars. The materials used are concrete mix of grade M20 and HYSD steel of grade Fe415.

***Solution**:*

Given : b_f=1500 mm, b_w = 300 mm, d = 600 mm. D_f = 100 mm, f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2.

***Case* 1: Area of tension steel 5 – 22ϕ**

$$A_{st} = 5 \times \frac{\pi(22)^2}{4} = 1900.66 mm^2$$

Step : 1 Depth of neutral axis (x_u)

Assuming neutral axis (x_u) lies within the flange ($i.e., x_u \leq D_f$)

$$x_u = \frac{0.87 f_y A_{st}}{0.36 b_f f_{ck}} = \frac{0.87 \times 415 \times 1900.66}{0.36 \times 1500 \times 20} = 63.54mm < D_f\ (100mm)$$

Hence, Assumption is correct

Step : 2 Note down the value of $x_{u,\max}$

$$x_{u,\max} = 0.48d = 0.48 \times 600 = 288mm$$

$$\therefore x_u < x_{u,\max}$$

The section is under reinforced.

Step : 3 Calculate ultimate moment of Resistance,

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b_f d f_{ck}}\right)$$

$$= 0.87 \times 415 \times 1900.66 \times 600 \left(1 - \frac{1900.66 \times 415}{1500 \times 600 \times 20}\right) = 393.69 \times 10^6 N - mm$$

$$M_u = 393.69 kN - m.$$

***Case* 2: Area of tension steel 5 – 28ϕ**

$$A_{st} = 5 \times \frac{\pi (28)^2}{4} = 3078.76 mm^2$$

Step : 1 Depth of neutral axis (x_u)

Assuming neutral axis (x_u) lies within the flange ($i.e., x_u \le D_f$)

$$x_u = \frac{0.87 f_y A_{st}}{0.36 b_f f_{ck}} = \frac{0.87 \times 415 \times 3078.76}{0.36 \times 1500 \times 20} = 102.92mm > D_f\ (100mm)$$

Hence, Assumption is wrong

Neutral axis lies below the flange,

$$\frac{D_f}{d} = \frac{100}{600} = 0.166 < 0.2 \text{ \& assuming } \frac{D_f}{x_u} < 0.43$$

Step : 2 Recalculate the value of x_u

$$C_1 + C_2 = T$$

$$C_1 = 0.36 f_{ck} b_w x_u = 0.36 \times 20 \times 300 \times x_u = 2160 x_u$$

$$C_2 = 0.45 f_{ck} D_f \left(b_f - b_w\right) = 0.45 \times 20 \times 100 \times (1500 - 300) = 1080000N$$

$$T = 0.87 f_y A_{st} = 0.87 \times 415 \times 3078.76 = 1111586N$$

$$\Rightarrow 2160 x_u + 1080000 = 1111586$$

$$x_u = 14.62mm < D_f$$

$\therefore$ Assumption is that $\frac{D_f}{x_u} > 0.43$

Step : 3 Again Recalculate the value of x_u

C_2 value changes, $C_2 = 0.45 f_{ck} (b_f - b_w) y_f$

Where, $y_f = (0.15x_u + 0.65D_f)$, but should not greater than D_f

$$y_f = (0.15x_u + 0.65D_f) = 0.15x_u + 0.65\times100 = 0.15x_u + 65$$

$$\Rightarrow C_2 = 0.45 f_{ck} \left(b_f - b_w\right) y_f = 0.45\times20\times(1500-300)\times\left(0.15x_u + 65\right)$$
$$= (1620x_u + 702000)N$$

$$C_1 + C_2 = T$$

$$C_1 = 2160x_u$$
$$T = 1111586N$$
$$2160x_u + 1620x_u + 702000 = 1111586$$
$$3780x_u = 409586$$
$$x_u = 108.35mm > D_f$$

Now $\frac{D_f}{x_u} = \frac{100}{108.35} = 0.92 > 0.43$, Hence Assumtion is correct

Step : 4 Note down the value of $x_{u,\max}$

$$x_{u,\max} = 288mm$$

$\therefore\ x_u < x_{u,\max}$

The section is under reinforced.

Step : 5 Calculate ultimate moment of resistance

Hence according to clause G-2.2.1 of IS:456-2000,Moment of resistance is computed by

$$M_u = 0.36\frac{x_u}{d}\left(1-0.42\frac{x_u}{d}\right)f_{ck}b_w d^2 + 0.45 f_{ck}\left(b_f - b_w\right)y_f\left(d - \frac{y_f}{2}\right) \qquad \left|\frac{x_u}{d} = \frac{108.35}{600} = 0.18\right.$$

$$y_f = (0.15x_u + 0.65D_f) = 0.15x_u + 0.65\times100 = 0.15\times108.35 + 65 = 81.25 < D_f$$

$$M_u = 0.36\times0.18\left(1-0.42\times0.18\right)20\times300\times600^2 + 0.45\times20$$

$$\left(1500-300\right)81.25\left(600 - \frac{81.25}{2}\right)$$

$$= 129386419.2 + 490851562.5 = 620.23 \times 10^6 N.mm$$

or $M_u = 620.23 kN.m$

8. A T - beam of flange width 1400 mm, flange thickness 100 mm, rib width 250 mm has an effective depth of 500 mm. The beam is reinforced with 4 bars of 20 mm diameter. Find the ultimate moment of resistance. Use M20 concrete and Fe415 steel. Use limit state method.

Solution:

Given : b_f = 1400 mm, D_f = 100 mm, b_w = 250 mm, d = 500 mm,

f_{ck} = 20 N/mm², f_y = 415/mm²

$$A_{st} = \frac{4 \times \pi \times 20^2}{4} = 1256.63 \text{ mm}^2$$

Step : 1 Depth of neutral axis

Assuming x_u lies within the flange

$$x_u = \frac{0.87 f_y A_{st}}{0.36 b_f f_{ck}} = \frac{0.87 \times 415 \times 1256.63}{0.36 \times 1400 \times 20}$$

$$x_u = 45.01 \text{ mm} < D_f \text{ (100 mm)}$$

Hence, Assumption is correct.

Step : 2 Note down the value of $x_{u,max}$

$$x_{u,max} = 0.48\, d = 0.48 \times 500 = 240 \text{ mm}$$

$$\therefore\ x_u < x_{u,\ max}$$

The section is under reinforced.

Step : 3 Calculate ultimate moment of resistance

$$M_u = 0.87 f_y A_{st}\, d \left(1 - \frac{A_{st} f_y}{b_f d\, f_{ck}}\right)$$

$$= 0.87 \times 415 \times 1256.63 \times 500 \left(1 - \frac{1256.63 \times 415}{1400 \times 500 \times 20}\right)$$

$$= 218.40 \times 10^6 \text{ N} - \text{mm}$$

$$\mathbf{M_u = 218.40\ kN\text{–}m}$$

9. Determine the area of tensile reinforcement required in as flanged beam having the following sectional dimensions to support a factored moment of 300 kN-m.

Width of flange b_f = 750*mm*

Width of rib or web b_w = 300*mm*

Thickness of flange D_f = 120*mm*

Effective depth d = 600*mm*

***M* – 20 grade concrete and Fe415 HYSD bars.**

Solution:

Given : M_u = 300 kN.m, b_f = 750 mm, b_w = 300 mm, D_f = 120 mm, d = 600 mm
f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2

Step : 1 Caluclate Moment of Resistance of flange

$$M_{uf} = 0.36 f_{ck} b_f D_f \left(d - 0.42 D_f\right) = 0.36 \times 20 \times 750 \times 120 (600 - 0.42 \times 120) = 356.14 \times 10^6 \, N - mm$$

M_{uf} = 356.14 kN.m

$\therefore M_u$ = 300 kN.m (Given)

Since $M_{uf} \geq M_u$ then NA lies with in the flange, $\therefore x_u < D_f$

Hence design the T beams as "Rectangular Beam", $b = b_f$

Step : 2 Calculate Area of Tensile reinforcement (A_{st})

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{b_f d f_{ck}}\right]$$

$$300 \times 10^6 = 0.87 \times 415 \times A_{st} \times 600 \left[1 - \frac{A_{st} \times 415}{750 \times 600 \times 20}\right]$$

$$9.98 A_{st}^2 - 216630 A_{st} + 300 \times 10^6 = 0$$

Solving, $A_{st} = 1486.67 mm^2$

---

10. Determine the area of tensile reinforcement required for T - beam having following details.

Effecitve flange width – 2400 mm

Depth of flange – 150 mm

Width of web – 300 mm

Effective depth – 750 mm

Working moment – 800 kN – m

Type of concrete M20

Type of steel – Fe 415 HYSD bars

Solution:

Given : b_f = 2400 mm, D_f = 150 mm, b_w = 300 mm, d = 750 mm, f_{ck} = 20 N/mm^2,
f_y = 415 N/mm^2, M = 800 KN–m

Factored moment, M_u = 800 × 1.5 = 1200 kN – m

Step : 1 Calculate moment of resistance of flange

$$M_{uf} = 0.36 f_{ck} b_f D_f (d - 0.42 D_f)$$

$$= 0.36 \times 20 \times 2400 \times 150\,(750 - 0.42 \times 150)$$

$$= 1780.70 \times 10^6 \text{ N} - \text{mm}$$

$$M_{uf} = 1780.70 \text{ kN} - \text{m}$$

Since $M_u < M_{uf}$, Neutral axis lies with in the flange,

$\therefore$ $x_u < D_f$, hence design the T - beam as Rectangular beam, $b = b_f$

Step : 2 Calculate Area of Tensile reinforcement (A_{st})

$$M_u = 0.87\, f_y\, A_{st}\, d \left[1 - \frac{f_y A_{st}}{b_f d\, f_{ck}}\right]$$

$$1200 \times 10^6 = 0.87 \times 415 \times A_{st} \times 750 \left[1 - \frac{415 \times A_{st}}{2400 \times 750 \times 20}\right]$$

$$3.12\, A_{st}^2 - 270.78 \times 10^3\, A_{st} + 1200 \times 10^6 = 0$$

$$\therefore \quad A_{st} = 4684.49 \text{ mm}^2$$

---

11. A T-beam has the following cross sectional details

Effective width of flange = 1800 mm

Thickness of flange = 150 mm

Width of rib = 300 mm

Effective depth = 1000 mm

Calculate the limiting or balanced moment capacity of the section and the corresponding area of tension reinforcement. Assume f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2

Solution :

Given : b_f = 1800 mm, D_f = 150 mm, b_w = 300 mm, d = 1000 mm

f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2

Step : 1 Note down the value of $x_{u,max}$

$$x_{u,max} = 0.48\, d = 0.48 \times 1000 = 480 \text{ mm}$$

For limiting value,

$$x_u = x_{u,max} = 480 \text{ mm}$$

$x_u > D_f$ Hence N-A lies outside the flange

$$\therefore \quad \frac{D_f}{x_u} = \frac{150}{480} = 0.31 < 0.43$$

$$\frac{D_f}{d} = \frac{150}{1000} = 0.15 < 0.2$$

Step : 2 Calculate ultimate moment of resistance

$$M_u = 0.36\,\frac{x_{u,\max}}{d}\left(1-0.42\,\frac{x_{u,\max}}{d}\right)f_{ck}b_w d^2 + 0.45 f_{ck}\left(b_f - b_w\right)D_f\left(d-\frac{D_f}{2}\right)$$

$= 0.36 \times 0.48\ (1 - 0.42 \times 0.48)\ 20 \times 300 \times 1000^2$

$+ 0.45 \times 20\ (1800 - 300) \times 150\ (100 - 0.5 \times 150)$

$= 827.78 \times 10^6 + 1873.12 \times 10^6$

$\therefore M_u = 2700.90 \times 10^6$ N.mm or 2700 kN.m

Step : 3 Calculate Area of tension reinforcement

$$A_{st} = A_{st_w} + A_{st_f}$$

$$= \frac{0.36 f_{ck} b_w x_u}{0.87 f_y} + \frac{0.45 f_{ck}\left(b_f - b_w\right)D_f}{0.87 f_y}$$

$$= \frac{0.36\times20\times300\times480+0.45\times20(1800-300)\times150}{0.87\times415}$$

$$\therefore\ A_{st} = 8480.26\ \text{mm}^2$$

2.15 SHEAR DESIGN

Shear stresses generate in beams due to bending or twisting. The two types of shear stress are called flexural shear stress and torsional shear stress, respectively.

Bending is usually accompanied by bending shear. Shear thus produced is accompanied by diagonal tension and compression.

As concrete is weak in tension, large diagonal tension stress can be produce cracking. If shear stress is large, steel in the form of vertical stirrups or bent up bars should be provided.

RC members are generally subjected to maximum shear forces normally near the support sections of simply supported flexural members.

a) Flexural shear

The shear associated with change of bending moment along the span is known as flexural shear, or simply shear.

Flexural shear force present in beam is given by = $\frac{dM}{dx} = V$

The horizontal and vertical shear stresses are to be accounted for in the designs of beams.

Exact analysis of shear in a reinforced concrete beam is quite complex, several experimental studies have been conducted to understand the various modes of failure. Which could occur due to possible combination of shear and bending moment acting at a given section.

b) Punching shear

The shear associated with the possibility of punching a thin member by concentrated load is called punching near.

A slab carrying a concentrated wall load, a slab supported directly by columns (called flat slab) without beam or a footing slab carrying a concentrated column load are subjected to punching shear.

For the member subjected to both the above types of shear, the flexural shear is referred to as one-way shear, whereas punching shear is called two way shear. A footing slab carrying concentrated column load is subjected to both these shears.

c) Torsional shear

When a member is subjected to torsion, it is subjected to torsional shear.

- The beams are usually subjected to flexural shear, and sometimes to torsion shear also.
- The slabs are the plate elements and usually subjected to flexural shear. However, sometimes they are subjected all the types of shears as in case of restrained two-way slabs and the flat slabs.
- Usually the shear failures of shallow- RCC beams may not lead to immediate failure, however it considerably reduces its flexural strength and thus there is a state of impending shear failure. Hence the shear design is considered as limit state of collapse.

- If the shear failures take place before flexural failures, they are brittle and occur without warning. If the flexural failure takes place prior to shear failure, the ductile failure of the beam is ensured.

2.15.1 Types of shear failure

Different modes of failure are:

a) **Diagonal Tension failure:** Which occur under large shear force and less bending moment. Such cracks are normally at 45^0 with horizontal.

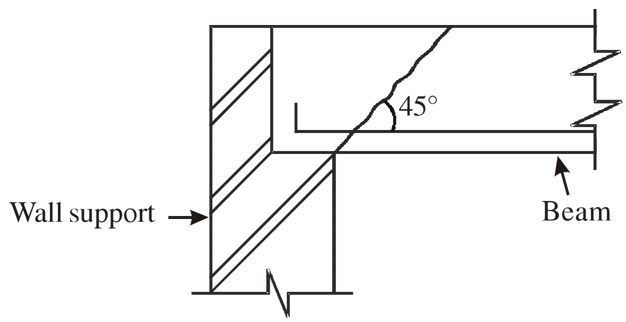


Fig. 2.8 : Diagonal Tension

b) **Flexural shear failure:** Which occurs under large bending moment and less shear force. Which occurs normally at closer to 90° with horizontal. When flexural crack occurs in combination with a diagonal tension crack (as in usually the case) the crack is some times called flexural shear crack.

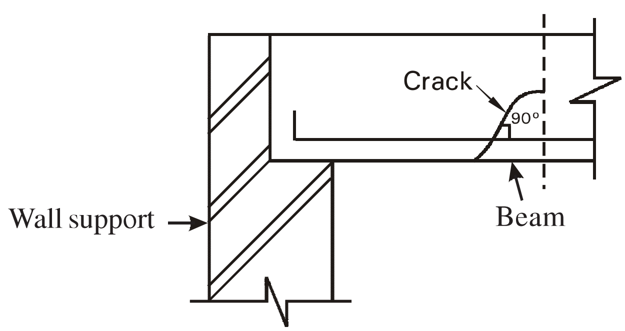


Fig. 2.9 : Flexural shear

It should be noted that it is the flexure crack that usually forms first and due to increased shear stress at the tip of the crack this flexural crack extends into diagonal tension crack.

c) **Diagonal compression failure:** Which occurs under large shear force. It is characterized by the crushing of concrete. Normally it occurs in beams which are reinforced against heavy shear.

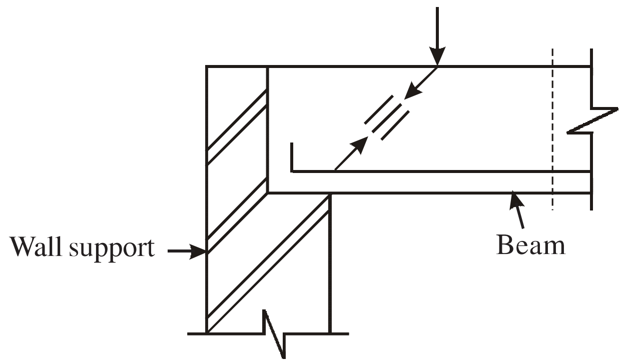


Fig. 2.10 : Diagonal compression

d) Shear tension failure :

Due to inadequate anchorage of the longitudinal bars, the diagonal cracks propagate horizontally along the bars.

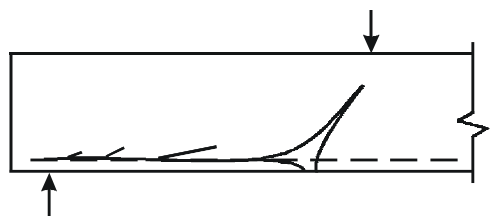


Fig. 2.11 : Shear tension failure

e) Web crushing failure

The concrete in the web crushes due to inadequate web thickness.

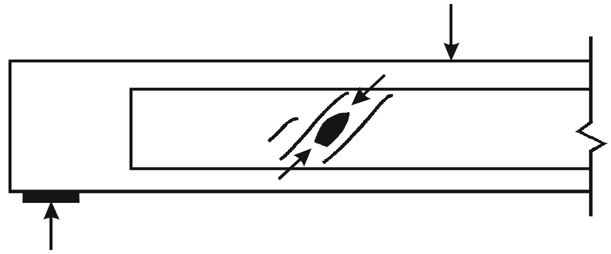


Fig. 2.12 : Web crushing failure

f) Arch rib failure

For deep beams, the web may buckle and subsequently crush. There can be anchorage failure or failure of the bearing.

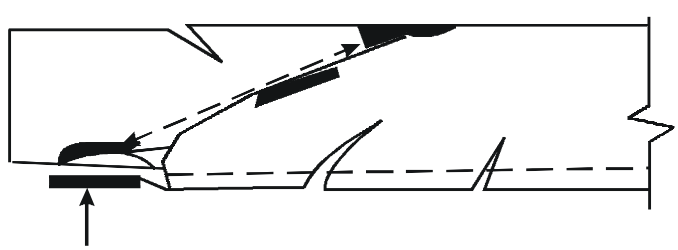


Fig. 2.13 : Arch rib failure

The objective of design for shear is to avoid shear failure. The beam should fail in flexure at its ultimate flexural strength. Hence, each mode of failure is addressed in the design for shear. The design involves not only the design of the stirrups, but also limiting the average shear stress in concrete, providing adequate thickness of the web and adequate development length of the longitudinal bars.

2.15.2 Shear Reinforcement

The types of shear reinforcement that can be provided to resist diagonal tension are

- A system of vertical stirrups.
- A system of inclined stirrups placed at right angles to diagonal tension crack.
- Main tensile steel bent up bar.

a) Vertical Stirrups

These are the steel bars vertically placed around the tensile reinforcement at suitable spacing along the length of the beam. Their diameter varies from 6 mm to 16 mm. The free ends of the stirrups are anchored in the compression zone of the beam to the anchor bars (hanger bar) or the compressive reinforcement. Depending upon the magnitude of the shear force to be resisted the vertical stirrups may be one legged, two legged, four legged and so on as shown in Fig.2.14. It is desirable to use closely spaced stirrups for better prevention of the diagonal cracks. The spacing of stirrups near the supports is less as compared to spacing near the mid span since shear force is maximum at the supports.

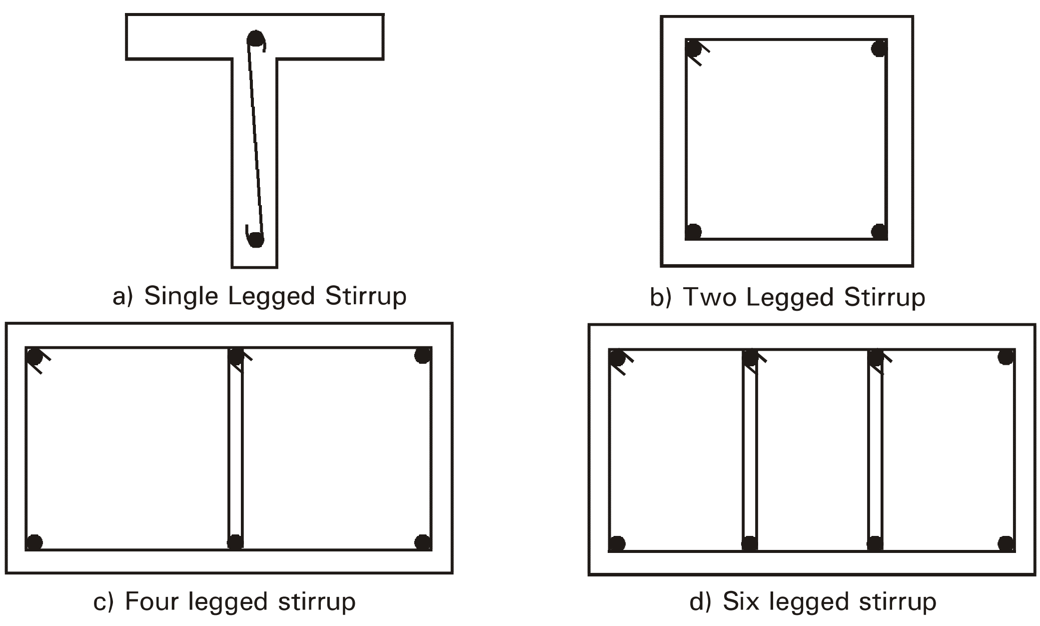


Fig. 2.14 : Shear reinforcement with different legs

b) Inclined Stirrups

Inclined stirrups are also provided generally at 45° for resisting diagonal tension as shown in Fig. 2.15. They are provided throughout the length of the beam.

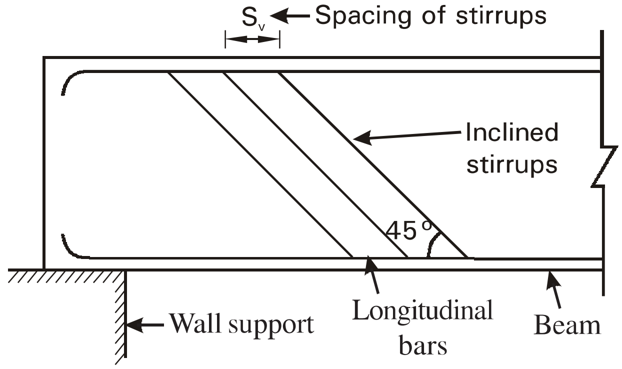


Fig. 2.15 : Beam with inclined stirrups

c) Bent up Bars along with vertical Stirrups

Some of the longitudinal bars in a beam can be bent up near the supports where they are not required to resist bending moment (Bending Moment is very less near the supports). These bent up bars resist diagonal tension. Equal number of bars are to be bent on both sides to maintain symmetry. The bars can be bent up at more than one point uniformly along the length of the beam. These bars are usually bent at 45° as shown in Fig. 2.16. This system is used for heavier shear forces. The total shear resistance of the beam is calculated by adding the contribution of bent up bars and vertical stirrups. The contribution of bent up bars is not greater than half of the total shear reinforcement.

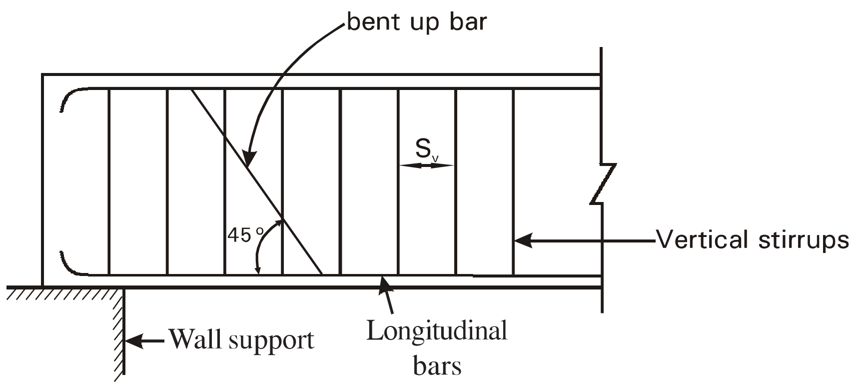


Fig. 2.16 : Beam with bent up bars along with vertical stirrups

2.15.3 Nominal shear stress (IS : 456 - 2000, Clause 40.1)

The nominal shear stress in beams of uniform depth shall be obtained by the following equation:

$$\tau_v = \frac{V_u}{bd}$$

V_u = Shear force due to design loads,

b = breadth of the member (for flanged beam, breadth of web),

d = effective depth.

τ_v = Nominal shear stress

2.15.4 Design shear strength of concrete

Depending upon the grade of concrete and the percentage of tensile steel, the design shear strength of concrete (τ_c) in beams without shear reinforcement is given in Table 19 of IS: 456-2000.

2.15.5 Maximum shear stress($\tau_{c\ max}$)

If the shear strength of concrete beam(τ_c) is less than that of nominal shear stress(τ_v), i.e. $\tau_c < \tau_v$ coming on the beam, then shear reinforcement is to be provided. The nominal shear stress in the beams with shear reinforcement shall not exceed maximum shear stress (τ_{max}) given in Table 20 of IS : 456-2000. If nominal shear stress is greater than the maximum shear stress then the section is to be redesigned.

As per IS : 456-2000, Table 20, maximum shear stress in concrete given below:

Grade of concrete	M15	M20	M25	M30	M35	M40 and above
$\tau_{c\max}$ (N/mm^2)	2.5	2.8	3.1	3.5	3.7	4.0

2.15.6 Minimum shear reinforcement (IS: 456 : 2000, Clause 26.5.1.6) [i.e., $\tau_v < \tau_c$]

When the nominal shear (τ_v) is less than the shear strength (τ_c), then no shear reinforcement is to be designed. But in such cases minimum shear reinforcement is to be provided in the form of stirrups such as

$$\frac{A_{sv}}{bS_v} \geq \frac{0.4}{0.87 f_y}$$

A_{sv} = Total cross sectional area of stirrup legs effective in shear,

S_v = Spacing of stirrups along the length of the member,

b = breadth of beam or breadth of web of the flanged beam,

f_y = Characteristic compressive strength of stirrup reinforcement in N/mm^2 which shall not be greater than 415N/mm^2.

The minimum shear reinforcement is provided for the following:

- Any sudden failure of beams is prevented if concrete cover bursts and the bond to the tension steel is lost.

- Brittle shear failure is arrested which would have occurred without shear reinforcement.
- Tension failure is prevented which would have occurred due to shrinkage, thermal stresses and internal cracking in beams.
- To hold the reinforcement in place when concrete is poured.
- Section becomes effective with the tie effect of the compression steel.

2.15.7 Maximum spacing of stirrups [IS : 456 - 2000, Clause 26.5.1.5]

The maximum spacing of vertical stirrups shall not exceed 0.75d or 300mm whichever is less. In case of inclined stirrups at 45^0, the maximum spacing is limited to d or 300mm whichever is less.

2.15.8 Design of shear reinforcement [Clause 40.4]

When τ_v is more than the τ_c, i.e., $\tau_v > \tau_c$ shear reinforcement is to be designed and can be provided in the following forms:

i) Vertical stirrups,

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

ii) Inclined stirrups or a series of bars bent up at different cross sections.

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}(\sin\alpha + \cos\alpha)$$

iii) Single bar or single group of parallel bars, all bent-up at the same cross section

$$V_{us} = 0.87 f_y A_{sv} \, \text{Sin}\alpha$$

Where τ_c = Design shear strength of the concrete

α = Angle between the inclined stirrup or bent up bar and the axis of the member, not less than 45°

2.15.9 Procedure for Design of shear reinforcement (Using IS : 456 - 2000)

Steps

1. Design of shear force, $V_u = \frac{W_u L}{2}$ – For simply supported beam carrying UDL

   $V_u = W_u L$ – For cantilever beam carrying UDL

2. Nominal shear stress/shear stress, $\tau_v = \frac{V_u}{bd}$ [Clause 40.1]

3. Percentage of steel reinforcement, $p_t = \dfrac{100A_{st_{provided}}}{bd}$ [Table 19]
4. Design shear strength of concrete (τ_c) – [Table 19]
   Using p_t and f_{ck} value, Note down the value of 'τ_c' from Table 19 of IS : 456 - 2000
5. Maximum shear stress $\left(\tau_{c_{max}}\right)$ – [Table 20]

   Using f_{ck} value, Note down the value of '$\tau_{c_{max}}$' from table 20 of IS : 456 - 2000.
6. Comparisons

   (i) If $\tau_v < \tau_c$ or $\tau_c > \tau_v$

   Shear reinforcement is not required. As per IS : 456 - 2000, Minimum shear reinforcement should be provided in the form of stirrups.

   Refer, clause 26.5.1.6 (Minimum shear reinforcement)

$$\frac{A_{sv}}{bS_v} \geq \frac{0.4}{0.87 f_y} \Rightarrow S_v = \frac{0.87 f_y A_{sv}}{0.4b}$$

   Spacing of stirrups should be least of the following three

   1. $S_v = \dfrac{0.87 f_y A_{sv}}{0.4b}$

      Assume φ of bars (eg. 6 mm, 8 mm, 10 mm......)

      ∴ Assume 2 Legged or 4 Legged or 6 legged dia stirrups

      ∴ $A_{sv} = 2 \times \dfrac{\pi}{4}\phi^2$ – For 2-L diameter stirrups

      $A_{sv} = 4 \times \dfrac{\pi}{4}\phi^2$ – For 4-L diameter stirrups

      Find 'S_v'

   2. 'S_v' should not be greater than $0.75d$ – clause 26.5.1.5

      ∴ $S_v \ngtr 0.75d$, if greater than $0.75d$

      then consider, $S_v = 0.75d$ – For vertical stirrups

      $S_v = d$ – For Inclined stirrups (@ 45º angle)

   3. 'S_v' should not be greater than 300mm – clause 26.5.1.5

      i.e., $S_v \ngtr$ 300mm, if greater than 300mm

      then consider, S_v = 300mm

      ∴ Provide 2L or 4 L ____ mm φ bars @ ____ mm c/c

   (ii) If $\tau_v > \tau_c$ or $\tau_c < \tau_v$

   Shear reinforcement is provided or required.

a. Calculate shear carried by concrete, $V_{uc} = \tau_c bd$ – Clause 40.4

W.K.T, $\tau_v = \dfrac{V_u}{bd}$

Similarly, $\tau_c = \dfrac{V_{uc}}{bd} \Rightarrow V_{uc} = \tau_c bd$

b. Calculate shear carried by stirrups, – Clause 40.4

$V_{us} = V_u - V_{uc}$ $\quad\quad |V_u = V_{uc} + V_{us}$

$\Rightarrow V_{us} = V_u - \tau_c bd$

Spacing of stirrups should be least of the following three

→ **For Vertical Stirrups**	**For Inclined Stirrups**
$V_{us} = \dfrac{0.87 f_y A_{sv} d}{S_v}$	$V_{us} = \dfrac{0.87 f_y A_{sv} d}{S_v}(\sin\alpha + \cos\alpha)$
1. $\Rightarrow S_v = \dfrac{0.87 f_y A_{sv} d}{V_{us}}$	1. $\Rightarrow S_v = \dfrac{0.87 f_y A_{sv} d}{V_{us}}(\sin\alpha + \cos\alpha)$
Clause 40.4	Clause 40.4
2. $S_v \not> 0.75\, d$ or $S_v = 0.75d$	2. $S_v \not> d$, or $S_v = d$
3. $S_v \not> 300$mm or $S_v = 300$mm	3. $S_v \not> 300$mm, or $S_v = 300$mm

Provide 2L or 4 L —— mm ϕ bars @ —— mm c/c

(iii) If $\tau_v < \tau_{c_{max}}$ or $\tau_{c_{max}} > \tau_v$, Design is safe

(iv) If $\tau_v > \tau_{c_{max}}$ or $\tau_{c_{max}} < \tau_v$, Design is not safe. Revise the section (depth)

2.16 WORKED EXAMPLES ON ANALYSIS AND DESIGN OF SHEAR REINFORCEMENT

1. **An Simply supported RCC beam subjected to a design shear force of 180kN. Size of beam 250 × 600mm depth to the centre of reinforcement. It is reinforced with 4 bars of 20mm diameter. Use M20 concrete and Fe415 steel. Comment on its shear design.**

Solution:

Given : $V_u = 180kN = 180 \times 10^3 N$, $b = 250mm$, $d = 600mm$, $f_{ck} = 20 N/mm^2$, $f_y = 415\ N/mm^2$

$$A_{st} = 4 \times \frac{\pi \times 20^2}{4} = 1256.64 mm^2$$

Step : 1

Nominal shear stress, $\tau_v = \dfrac{V_u}{bd} = \dfrac{180 \times 10^3}{250 \times 600} = 1.2\text{N/mm}^2$

Step : 2

$$\text{Percentage of steel} = \frac{100 \times A_{st}}{bd} = \frac{100 \times 1256.64}{250 \times 600} = 0.84\%$$

Step : 3

Design shear strength of concrete (τ_c),

Refer table19 of IS: 456 - 2000, using p_t and f_{ck} value, note down the value of τ_c.

% – – – – – – – – – – τ_c

By interpolation, we have

0.75 – – – – – – – – –0.56

0.84 – – – – – – – – – – ?

1.0 – – – – – – – – – – 0.62

$$\tau_c \text{ for } 0.84 = 0.56 + \frac{(0.62 - 0.56)}{(1.0 - 0.75)} \times (0.84 - 0.75) = 0.58 N/mm^2$$

$$\tau_c = 0.58 N/mm^2$$

Step : 4

Maximum shear stress ($\tau_{c_{max}}$)

Refer Table 20 using f_{ck} value, Note down the value of $\tau_{c_{max}}$

$\therefore$ $\tau_{c_{max}}$ = 2.8 N/mm^2

Step : 5

Comparisons or Comment

(i) $\tau_v > \tau_c$ – Shear reinforcement is required

(ii) $\tau_{c_{max}} > \tau_v$ – Design is safe

---

2. A RCC simply supported beam 230mm × 450mm effective carries a udl of 65kN/m over a effective span of 7m. The beam is reinforced with 1% steel on tension side, Use M20 concrete and load factor = 1.5. Comment on the shear design of the beam.

Solution:

Given : W = 65*kN/m*, b = 230*mm*, d = 450*mm*, f_{ck} = 20*N/mm*^{2},

p_t = 1%, load factor = 1.5, L=7*m*

Factored load, W_u = 1.5 × W = 1.5 × 65 = 97.5 kN

Step : 1

$$\text{Design of shear force, } V_u = \frac{W_u L}{2} = \frac{97.5 \times 7}{2} = 341.25\text{kN} \text{ – For simply supported beam}$$

Step : 2

$$\text{Nominal shear stress, } \tau_v = \frac{V_u}{bd} = \frac{341.25 \times 10^3}{230 \times 450} = 3.29\text{N/mm}^2$$

Step : 3

Percentage of steel, p_t = 1% (Given)

Step : 4

Design shear strength of concrete (τ_c),

using p_t and f_{ck}, Note down the value of τ_c from Table 19 of IS : 456 -2000

$\therefore\ \tau_c = 0.62$ N/mm^2

Step : 5

Maximum shear stress ($\tau_{c_{max}}$)

Using 'f_{ck}' Note down the value of $\tau_{c_{max}}$

$\therefore\ \tau_{c_{max}} = 2.8$N/mm^2

Step : 6

Comparisons or Comment

(i) $\tau_v > \tau_c$ – Shear reinforcement is required

(ii) $\tau_v > \tau_{c_{max}}$ – Design is not safe, Hence the section is to be re-designed

---

3. An RCC simply supported beam 300 × 600mm carries a udl of 16kN/m over a span of 5m. The beam is reinforced with 1.5% of steel on tension side with an effective cover of 50mm. Calculate the nominal shear stress and design shear strength of concrete. Check shear reinforcement required or not. Use M20 concrete. Take load factor = 1.5.

Solution:

Given: $W = 16kN/m$, $b = 300mm$, $D = 600mm$, $f_{ck} = 20N/mm^2$, $p_t = 1.5\%$, Load factor = 1.5, $d^l = 50mm$, $L = 5m$

Effective depth, $d = 600 - 50 = 550$mm, Factored load, $W_u = 1.5\times W = 1.5 \times 16 = 24$kN/m

Step : 1

$$\text{Design of shear force, } V_u = \frac{W_u L}{2} = \frac{24\times 5}{2} = 60\text{kN}$$

Step : 2

$$\text{Nominal shear stress, } \tau_v = \frac{V_u}{bd} = \frac{60\times 10^3}{300\times 550} = 0.36\text{N/mm}^2$$

Step : 3

Percentage of steel, p_t = 1.5% (Given)

Step : 4

Design shear strength of concrete (τ_c)

Refer Table19 of IS:456 - 2000, using p_t = 1.5% and f_{ck} = 20N/mm^2

$\therefore\ \tau_c = 0.72$N/mm^2

Step : 5

Maximum shear stress ($\tau_{c_{max}}$)

$\therefore\ \tau_{c_{max}} = 2.8\text{N/mm}^2$

Step : 6

Comparisons

(i) $\tau_c > \tau_v$ – Entire Shear taken by concrete, shear reinforcement is not required. But IS code recommends that Nominal or Minimum shear reinforcement must be provided as per clause 26.5.1.6

(ii) $\tau_{c_{max}} > \tau_v$ – Design is safe.

---

4. A simply supported R.C.C. beam is 250 mm wide and 500 mm effective depth and is reinforced with 4 - bars of 22 mm diameter as tensile steel, if the beam is subjected to a factored shear of 65 kN at the support , find the nominal shear stress at the support and design the shear reinforcement. Use M20 concrete and Fe 250 steel. Use limit state method.

Solution:

Given: $b = 250mm$, $d = 500\ mm$, $V_u = 65\text{ kN} = 65 \times 10^3\text{ N}$

$f_{ck} = 20\ N/mm^2$, $f_y = 250\ N/mm^2$, $A_{st} = 4\times\frac{\pi}{4}\times 22^2 = 1520.53\text{mm}^2$

Step : 1

$$\text{Nominal shear stress, } \tau_v = \frac{V_u}{bd} = \frac{65\times10^3}{250\times500} = 0.52\text{ N/mm}^2$$

Step : 2

$$\text{Percentage of steel, } \frac{100A_{st}}{bd} = \frac{100\times1520.53}{250\times500} = 1.21\%$$

Step : 3

Design shear strength of concrete (τ_c)

Using $p_t = 1.21\%$ and $f_{ck} = 20\text{N/mm}^2$, Note down the value of τ_c

$\therefore\ \tau_c = 0.66\text{N/mm}^2$ (By interpolation method)

Step : 4

Maximum shear stress ($\tau_{c_{max}}$)

$\therefore\ \tau_{c_{max}} = 2.8\text{N/mm}^2$

Step : 5

Comparisons

(i) $\tau_v < \tau_{c_{max}}$ Design is safe.

(ii) $\tau_c > \tau_v$ – Shear reinforcement is not required. ∴ Provide minimum shear reinforcement as per IS : 456 - 2000 Clause 26.5.1.6

∴ Spacing of stirrups should be least of the following three

$$S_v = \frac{0.87 f_y A_{sv}}{0.4b}$$

Assume 2L - 8mm ϕ bars vertical stirrups

$$\therefore A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53\text{mm}^2$$

1) $S_v = \dfrac{0.87 f_y A_{sv}}{0.4 \times b} = \dfrac{0.87 \times 250 \times 100.53}{0.4 \times 250} = 218.65\text{mm}$

2) S_v should not be greater than $0.75d = 0.75 \times 500 = 375$ mm

3) S_v should not be greater than 300 mm, ∴ $S_v = 300$mm

∴ Provide $2L - 8$ ϕ stirrups at 200 mm c/c as shear reinforcement.

---

5. A simply supported R.C.C. beam is 300 × 475 mm effective depth and it is reinforced with 4–16 mm ϕ as tensile reinforcement. If the beam is subjected to total load of 33 kN/m over a effective span of 6m. Two bars are bent up to a 45° at a distance of 0.75 m from the face of the support. Design shear reinforcement for beam. Use Limit state method. take f_{ck} = 15 MPa, and f_y = 250 MPa.

Solution:

Given : $b = 300$ mm, d = 475 mm, $W = 33$kN/m, $L = 6$m, $f_{ck} = 15\text{N/mm}^2$,
$f_y = 250\text{N/mm}^2$, $\alpha = 45°$
Factored $W_u = 1.5 \times W = 1.5 \times 33 = 49.5$kN/m

Step : 1

Design of shear force, $V_u = \dfrac{W_u L}{2} = \dfrac{49.5 \times 6}{2} = 148.5\text{kN}$

Step : 2

Nominal shear stress, $\tau_v = \dfrac{V_u}{bd} = \dfrac{148.5 \times 10^3}{300 \times 475} = 1.04\text{N/mm}^2$

Step : 3

Percentage of steel

At support 2 bars are bent up, No. bars available at tension zone = 2

$$\therefore A_{st} = 2 \times \frac{\pi \times 16^2}{4} = 402.12 \text{ mm}^2$$

$\therefore$ percentage of steel $= \dfrac{100 A_{st}}{bd} = \dfrac{100 \times 402.12}{300 \times 475} = 0.28\%$

Step : 4

Design shear strength of concrete (τ_c)

Refer, Table 19 of IS: 456–2000 using $p_t = 0.28\%$ and $f_{ck} = 15\text{N/mm}^2$

$$\tau_c = 0.36 \text{ N/mm}^2$$

→ For vertical stirrups along with bent up bars

(a) Shear strength carried by stirrups

$V_{us} = V_u - V_{uc} = V_u - \tau_c bd = 148.5 \times 10^3 - 0.36 \times 300 \times 475 = 97.2 \times 10^3$ N

Assume 2L - 8mm dia stirrups

$$\therefore A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

(b) Shear strength of bent up bars

$$(V_{us})_{bent} = 0.87 f_y (A_{st})_{bent} \times \sin\alpha$$

$$(A_{st})_{bent} = 2 \times \frac{\pi}{4} \times 16^2 = 402.12\text{mm}^2$$

$$\therefore (V_{us})_{bent} = 0.87 \times 250 \times 402.12 \times \sin 45 = 61.84 \times 10^3 \text{ N}$$

- If $(V_{us})_{bent} < \dfrac{V_{us}}{2}$ then,

$$(V_{us})_{stirrups} = V_{us} - (V_{us})_{bent}$$

- If $(V_{us})_{bent} > \dfrac{V_{us}}{2}$ then, $(V_{us})_{bent} = \dfrac{V_{us}}{2}$

$$(V_{us})_{stirrups} = V_{us} - \frac{V_{us}}{2}$$

Now, $(V_{us})_{bent} > \dfrac{V_{us}}{2}$ $\qquad \left| \dfrac{V_{us}}{2} = \dfrac{97.2}{2} = 48.6 \text{ kN} \right.$

$\therefore (V_{us})_{stirrups} = 97.2 - 48.6 = 48.6$ kN

Spacing of stirrups should be least of the following three

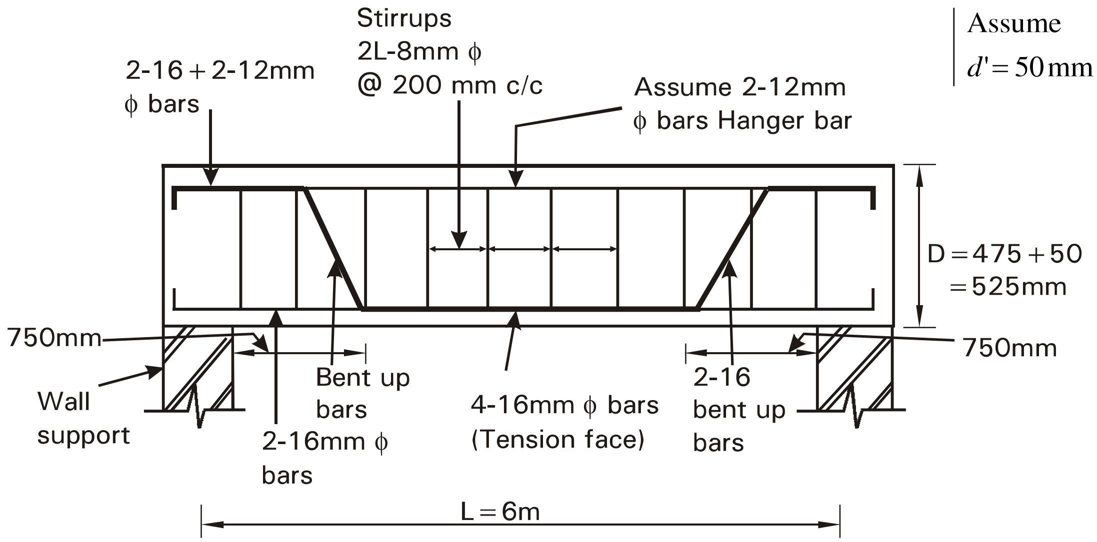
→ For vertical stirrups along with bent up bars

1. $S_v = \dfrac{0.87 f_y A_{sv} d}{(V_{us})_{stirrups}} = \dfrac{0.87 \times 250 \times 100.53 \times 475}{48.6 \times 10^3} = 213.7\text{mm}$

2. $S_v \not> 0.75\,d$, $\therefore S_v = 0.75 \times 475 = 356.25$mm
3. $S_v \not> 300$mm, $\therefore S_v = 300$mm

Say $S_v = 200$mm

$\therefore$ Provide 2L – 8mm ϕ bars @ 200 mm c/c



6. **A 230mm wide and 550mm deep RC beam is reinforced with 2L - 10mm inclined stirrups at 200mm c/c with $\alpha = 60°$. Longitudinal steel consists of 4 bars of 25mm with a Effective cover of 50mm. Use f_{ck} = 25N/mm^2 and f_y = 415N/mm^2. Calculate the strength of the section in shear.**

Solution :

Given, $b = 230$mm, $D = 550$mm, $S_v = 200$mm, $\alpha = 60°$, $d^1 = 50$mm, $f_{ck} = 25$N/mm^2,

$f_y = 415$N/mm^2, $d = 550 - 50 = 500$mm, $A_{st} = 4 \times \frac{\pi}{4} \times 25^2 = 1963.5$mm^2

Step : 1

Percentage of tension reinforcement, $p_t = \frac{100 A_{st}}{bd} = \frac{100 \times 1963.5}{230 \times 500} = 1.7\%$

Step : 2

Design shear strength of concrete (τ_c)

Using p_t and f_{ck},

$\therefore \tau_c = 0.77$ N/mm^2

Step : 3

Shear strength of concrete, $V_{uc} = \tau_c bd = 0.77 \times 230 \times 500 = 88.55$kN

Step : 4

Strength due to shear reinforcement - Inclined Stirrups

$$V_{us} = \frac{0.87 f_y A_{st\,in} d}{S_v}(\sin\alpha + \cos\alpha) \qquad A_{st_{in}} = \text{Area of inclined stirrups}$$

$$A_{st_{in}} = 2\times\frac{\pi}{4}\times 10^2 = 157.07 \text{ mm}^2$$

$$\therefore\ V_{us} = \frac{0.87\times 415\times 157.07\times 500}{200}(\sin 60^\circ + \cos 60^\circ) = 193.66\text{kN}$$

Step : 5

Strength of the section in shear

$$V_u = V_{uc} + V_{us} = 88.55 + 193.66 = 282.21\text{kN}$$

---

7. A RC beam of rectangular section with a width of 250mm and effective depth of 500mm is reinforced with 4 bars of 25mm diameter as tension reinforcement. Two of the tension bars are bent up at 45° near the support section. The beam is also provided with 2L-8mm ϕ @ 200mm c/c. Use M25 grade concrete and Fe415, grade steel. Determine the ultimate shear strength of the support section.

Solution:

Given, b = 250mm, d = 500mm, S_v = 200mm, f_{ck} = 25N/mm², f_y = 415N/mm²

$$\text{Total } A_{st} = 4\times\frac{\pi}{4}\times 25^2 = 1963.5\text{mm}^2$$

2 bars bent up @ support section - $A_{st_b} = 2\times\frac{\pi}{4}\times 25^2 = 981.74\text{mm}^2$

$$A_{sv} = 2\times\frac{\pi}{4}\times 8^2 = 100.53\text{mm}^2$$

Step : 1

Percentage of steel (at support), $p_t = \frac{100A_{st}}{bd} = \frac{100\times 981.74}{250\times 500} = 0.78\%$

Step : 2

Design shear strength of concrete (τ_c)

Using p_t and f_{ck},

$\therefore\ \tau_c = 0.57$ N/mm²

Step : 3

Shear strength of concrete, $V_{uc} = \tau_c bd = 0.57 \times 250 \times 500 = 71.25\text{kN}$

Step : 4

Strength resisted by vertical stirrups

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v} = \frac{0.87 \times 415 \times 100.53 \times 500}{200} = 90.74\text{kN}$$

Step : 5

Strength due to shear reinforcement - bent up bars

$$V_{us_{bent}} = 0.87 f_y A_{st_b} \sin\alpha$$

$$= 0.87 \times 415 \times 981.74 \times \sin 45°$$

$$= 250.63\text{kN}$$

Step : 6

Total shear resistance of support section or Design strength of the section

$$V_u = V_{uc} + V_{us} = V_{us_{bent}}$$

$$= 71.25 + 90.74 + 250.63$$

$$= 412.62\text{kN}$$

---

8. A R.C beam 250 × 550 mm is reinforced with 3 bars of 20 mm diameter on the tension side. 8 mm diameter 2 legged stirrups are provided at a spacing of 250 mm C/C. Assume effective cover = 50 mm. Use M20 grade concrete and Fe - 415 steel. Calculate the shear strength of the section.

Solution:

Given : b = 250 mm, S_v = 250 mm, D = 550 mm, f_{ck} = 20 N/mm², f_y = 415 N/mm²

$$A_{st} = \frac{3 \times \pi \times 20^2}{4} = 942.5 \text{ mm}^2$$

$$A_{sv} = 2 \times \frac{\pi \times 8^2}{4} = 100.53 \text{ mm}^2$$

Effective depth d = 550 – 50 = 500 mm

Step : 1

Percentage of tension reinforcement, $p_t = \frac{100\ A_{st}}{bd} = \frac{100 \times 942.5}{250 \times 500} = 0.75\%$

Step : 2

Design shear strength of concrete (τ_c)

Refer, Table 19 of IS : 456 - 2000, using p_t and f_{ck}

$\therefore \tau_c = 0.56\text{N/mm}^2$

Step : 3

Shear taken by concrete, $V_{uc} = \tau_c\ bd = 0.56 \times 250 \times 500 = 70\text{kN}$

Step : 4

Shear taken by stirrups, $V_{us} = \dfrac{0.87\ f_y\ A_{sv}\, d}{S_v}$

$$= \frac{0.87 \times 415 \times 100.53 \times 500}{250} = 72.59 \text{ kN}$$

Step : 5

Total shear resistance of support section, $V_u = V_{uc} + V_{us} = 70 + 72.59 = 142.59\text{kN}$

---

9. A simply supported beam 300 × 1010 mm effective has a span of 7 m carrying a udl of 45 kN/m throughout the span & provided with Tensile steel is 6-22 mm φ bars. If concrete M20 & Fe415 steel are used. Design shear reinforcement. $\tau_{c\,max}$ = 2.8 N/mm², τ_c = 0.56 N/mm² for 0.75% steel.

Solution:

Given:

b = 300mm, d = 1010mm, L = 7m, W = 45kN/m, f_{ck} = 20N/mm², f_y = 415N/mm²,

τ_{cmax} = 2.8N/mm², τ_c = 0.56N/mm² for p_t = 0.75%

$$A_{st} = 6 \times \frac{\pi \times 22^2}{4} = 2280.79mm^2$$

Factored load, $W_u = 1.5 \times 45 = 67.5$ kN/m

Step : 1

Design of Shear force, $V_u = \dfrac{W_u L}{2} = \dfrac{67.5 \times 7}{2} = 236.25\text{kN} = 236.5 \times 10^3\text{N}$

Step : 2

Nominal shear stress, $\tau_v = \dfrac{V_u}{bd} = \dfrac{236.25 \times 10^3}{300 \times 1010} = 0.78 \text{ N/mm}^2$

Step : 3

Comparisons

(i) $\tau_v > \tau_c$ - Shear reinforcement is required

$\tau_{c\,max} > \tau_v$ - Design is safe

Step : 4 Calculation of spacing

Shear to be taken by stirrups, $V_{us} = V_u - \tau_c bd = 236.5 \times 10^3 - 0.56 \times 300 \times 1010 = 66820N$

Providing 8mm dia 2 legged stirrups

$$A_{sv} = 2 \times \frac{\pi \times \phi^2}{4} = 2 \times \frac{\pi \times 8^2}{4} = 100.53\text{mm}^2$$

Spacing of stirrups should be least of the following three

i) $S_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87 \times 415 \times 100.53 \times 1010}{66820} = 548.62mm$

ii) $S_v \not> 0.75\, d$, $\therefore S_v = 0.75 \times 1010 = 757.5$ mm

iii) $S_v \not> 300$mm, $\therefore S_v = 300$mm

$\therefore$ Provide 2L - 8mm ϕ vertical stirrups @ 300mmc/c

10. Design the shear reinforcement for a T - beam with following data:

Flange width = 2100mm

Thickness of flange = 150mm

Overall depth = 750mm

Effective cover = 50mm

Longitudinal bars = 4 - 25 mm ϕ bars

Web width = 300mm

Effective length for simply supported T - beam = 5m

Working load = 40kN/m

f_{ck} = 20N/mm^2 and f_y = 415N/mm^2

Solution:

Given:

$b_f = 2100$mm, $D_f = 150$mm, $D = 750$mm, $d' = 50$mm, $b_w = 300$mm, $L = 5$m, $W = 40$kN/m,

$f_{ck} = 20$N/mm^2, $f_y = 415$ N/mm^2, $d = D - d' = 750 - 50 = 700$mm

$$A_{st} = 4 \times \frac{\pi}{4} \times 25^2 = 1963.5\text{mm}^2$$

Factored load, $W_u = 1.5 \times W = 1.5 \times 40 = 60$ kN/m

Step : 1

Design of Shear force, $V_u = \frac{W_u L}{2} = \frac{60 \times 5}{2} = 150$ kN

Step : 2

Nominal shear stress, $\tau_v = \frac{V_u}{b_w d} = \frac{150 \times 10^3}{300 \times 700} = 0.71\text{N/mm}^2$

Step : 3

Percentage of steel, $p_t = \frac{100\, A_{st}}{b_w\, d} = \frac{100 \times 1963.5}{300 \times 700} = 0.93\%$

Step : 4

Design shear strength of concrete τ_c

Using $p_t = 0.93\%$ and $f_{ck} = 20\text{N/mm}^2$, from IS : 456-2000, Table 19

$\therefore \tau_c = 0.60\text{N/mm}^2$ (By interpolation method)

Step : 5

Maximum shear stress ($\tau_{c\,max}$)

$$\therefore \quad \tau_{c\,max} = 2.8\text{N/mm}^2$$

Step : 6

Comparisons

(i) $\tau_v > \tau_c$ - Hence shear reinforcement is required

(a) Shear carried by concrete, $V_{uc} = \tau_c\, b_w d = 0.6 \times 300 \times 700 = 126\text{kN}$

(b) Shear carried by stirrups, $V_{us} = V_u - V_{uc} = 150 - 126 = 24\text{kN}$

Spacing of stirrups should be least of the following three

→ For vertical stirrups

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

Assume 2L - vertical stirrups of 8 mm dia bars

$$\therefore A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53\text{mm}^2$$

1. $S_v = \dfrac{0.87 \times 415 \times 100.53 \times 700}{24 \times 10^3} = 1058.64\text{mm}$
2. $S_v \ngtr 0.75\, d, \quad \therefore S_v = 0.75 \times 700 = 525\text{mm}$
3. $S_v \ngtr 300\text{mm}, \quad \therefore S_v = 300\text{mm}$

Say $S_v = 300\text{mm}$

$\therefore$ Provide 2L - 8mm ϕ bars @ 300 mm c/c