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MODULE
3

LIMIT STATE DESIGN OF BEAMS

3.1 INTRODUCTION

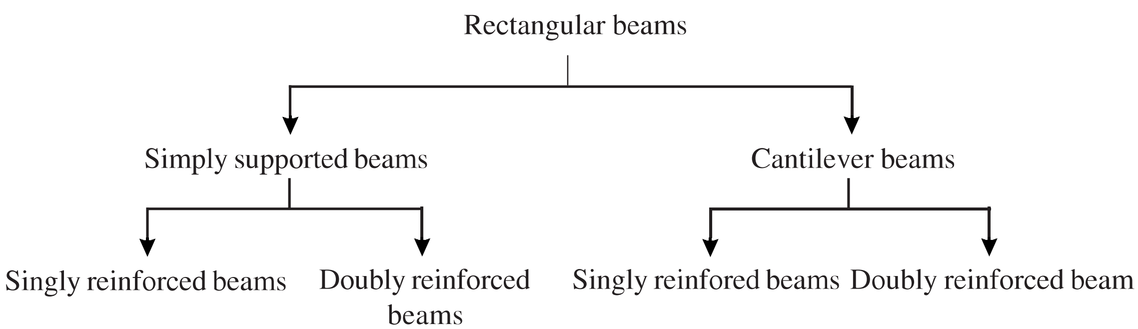
Beam is a horizontal member of structure carrying transverse loads. Beams carry the floor slab or roof slab and transfers all the loads including its self weight (Beam Self Weight) to the columns or walls.

RCC beam is subjected to bending moments and shear force, due to the vertical external load, bending compress the top fibres of the beam and elongates the bottom fibers. The strength of the RCC beam depends upon the composite action of concrete and steel.

There are different types of RC beams are constructing in Civil Engineering field interms of support conditions, i.e. Simply supported beams, cantilever beams, propped cantilever beams, over hanging beams, continuous beams and fixed beams

Following are the topics are discussed in this module

Design of Rectangular and Flanged beams



Flanged beams

- T - beams
  - Simply supported beams
    - (1) Singly reinforced beams
    - (2) Doubly reinforced beams
- L - beams
  - Simply supported beams
    - Singly reinforced beams

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3.2 CONCEPTS OF COMBINED BENDING AND TORSION

Reinforced concrete structural elements are subjected to torsion in addition to bending moments and shear forces depending on the loading and shape of the structural members.

Generally the torsion occurs in combination with the shear force and bending moment. Torsion is caused by eccentricity of loading to the axis of the structural element.

Generally all beams are subjected to some torsional moment. However in many cases these values are small. Normally torsion associated with flexure and shear develops in RC structures such as L - beams, curved beams, circular girders, Frame structure, Grid structure. A beam with cantilever slab, Ring beam of water tanks resting on columns.

Torsion produced in a beam is classified as

1. Primary torsion or Equilibrium torsion
2. Secondary torsion or compatibility torsion

1. Primary torsion / Equilibrium torsion

- Torsion which can be determined only by using the static equilibrium condition is called primary torsion. Torsion induced in beams curved in plan and subjected to gravity loads and also in beams where the transverse loads are Eccentric with respect to the shear centre.
- Example:- Small beam B_2 is cantilevered from Beam B_1. Beam B_1 is considered fixed at columns. Here negative moment of beam B_2 will be the torsional moment of beam B_1 (Refer fig. 3.1 (C))

2. Secondary torsion / Compatibility torsion

- Secondary torsion is induced in a structural member by rotation (twist) applied at one or more point along the length of member, through inter connected members, instead of by directly applied load.
- Twisting moment induced in this case is proportional to the torsional stiffness of the member.
- These moments are generally statistically indeterminate and their value is obtained using compatibility equation, this is the reason these torsion are called compatibility torsion.

3.2.1 Effect of torsional moment

Torsional moment induces shear stresses in the beam, because of the torsion, if beam fails in diagonal tension forming spiral cracks around the beam.

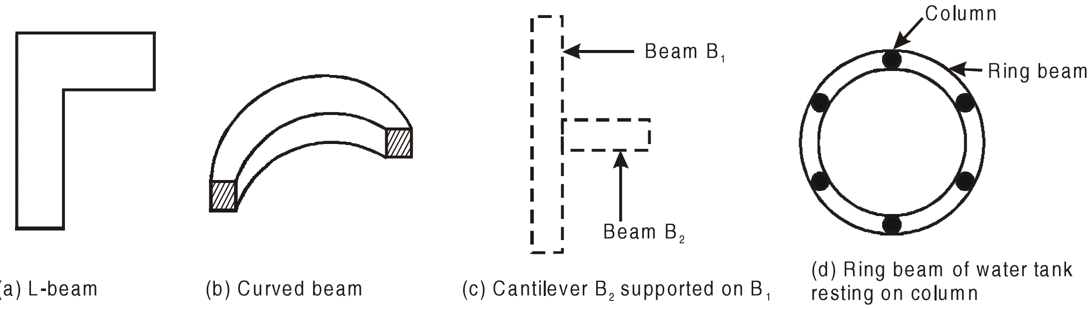

Fig. 3.1 : Types of Elements subjected to torsion

3.2.2 Design of beam section for Torsion

The simplified approach recommended by IS:456-2000 for the design of the rectangular beam section, subjected to torsion combined with flexure and shear, does not require determination of torsional reinforcement separately from that required for flexure and shear. Instead, the total longitudinal reinforcement is determined for a fictitious equivalent bending moment which is a function of flexural moment and torsion. Similarly, the transverse reinforcement is determined for a fictitious equivalent shear which is obtained from actual shear and torsion. In the flanged sections the contribution of flange is neglected i.e., the rectangular web portion alone is considered. Where the depth of web of a beam exceeds 450mm, the side face reinforcement shall be provided along the two faces. The total area of such reinforcement shall not be less than 0.1% of the web area and shall be distributed equally on two faces at a spacing not exceeding 300mm or web thickness whichever is less.

Refer IS: 456-2000 for calculation of reinforcements, in terms of

1. Shear and Torsion
2. Equivalent moment
3. Torsional reinforcement. - In the form of longitudinal bars
4. Transverse reinforcement (stirrups) - In the form of links or hoops
5. Side face reinforcement

The longitudinal reinforcement helps in reducing the crack width through dowel action and stirrups crossing the cracks resist shear due to vertical loads and torsion

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3.3 DESIGN PROBLEMS

Data : Clear span (l), Live load, type of supports, Grade of concrete and steel

Design : Cross sectional dimension (b, d and D), Area of steel reinforcement (Longitudinal steel and transverse steel [strirrups]), check for deflection control, Development length (if required)

Design steps : - Rectangular beams (Simply supported and cantilever beams)

1. **Selection of cross sectional dimensions**

(a) Effective depth of beam (d) - (IS:456-2000, clause 23.2.1)

Assume effective depth of beam,

$$d = \frac{\text{clear span}}{10 \text{ to } 20} = \frac{l}{10 \text{ to } 20} \text{ —— For simply supported beam}$$

$$d = \frac{l}{7} \text{ —— For cantilever beam}$$

Sl. No.	Span range	$\frac{\text{Span}}{\text{depth ratio}}\left(l/d\right)$	Loading type
1.	3 to 4 m	15 to 20	Light
2.	5 to 10 m	12 to 15	Medium to heavy
3.	> 10 m	10 to 12	Heavy

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(b) Overall depth of beam (D)

$$D = \text{Effective depth} + \text{Effective cover}$$

i.e., $$D = d + d'$$

$$\text{Effective cover, } d' = \text{Nominal cover} + \frac{\text{diameter}}{2}$$

or $$d' = 50 \text{ mm, Assume if not given in problem}$$

(c) Breadth / Widht of beam (b)

$$b = \frac{1}{3^{rd}} \text{ times of effective depth to } \frac{2}{3^{rd}} \text{ times of effective depth}$$

i.e., $$b = \frac{1}{3}\times d \quad \text{to} \quad \frac{2}{3}\times d$$

(Generally width of beams used, 150 mm, 200 mm, 230 mm, 250 mm and 300 mm)

2. **Calculation of Effective span (L) - (IS:456-2000, clause 22.2)**

(a) For simply supported beam

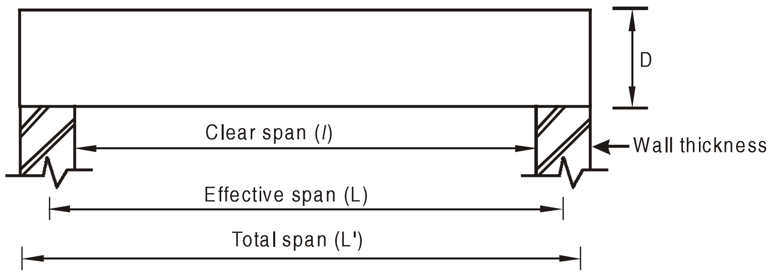

Fig. 3.2 : simply supported beam

Should be least of the following two :

(i) L = clear span (l) + bearing (wall thickness)

(ii) L = clear span (l) + effective depth (d)

(b) For cantilever beam

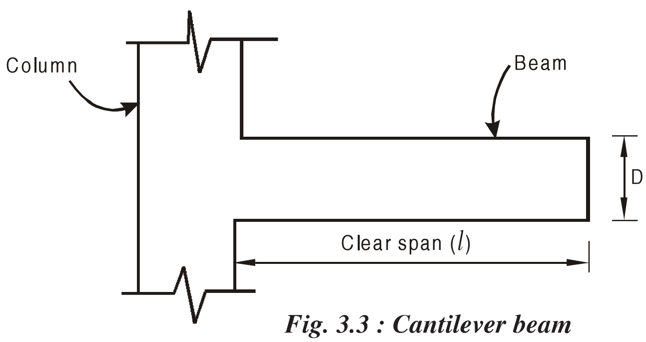

Fig. 3.3 : Cantilever beam

$$\text{Effective span } (L) = \text{clear span } (l) + \frac{\text{Effective depth}(d)}{2}$$

3. Load Calculation

1. Self weight of beam = $b \times D \times$ RCC density = ——— kN/m
2. Live load (Given) = ——— kN/m

Total load, W = ——— kN/m

Factored load, $W_u = 1.5 \times W$ = ——— kN/m

4. Calculation of maximum bending moment / Design moment

$$M_u = \frac{W_u L^2}{8}$$ ——— For simply supported beam carrying UDL

$$M_u = \frac{W_u L^2}{2}$$ ——— For cantilever beam carrying UDL

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5. Check for effective depth

Equating factored moment = Limiting moment of resistance

$M_{u_{lim}} = 0.148 f_{ck} bd^2$ ——— Fe 250 (Mild steel)

$M_{u_{lim}} = 0.138 f_{ck} bd^2$ ——— Fe 415 (HYSD bars)

$M_{u_{lim}} = 0.133 f_{ck} bd^2$ ——— Fe 500 (HYSD bars)

$$\therefore d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck} b}}$$ ——— For Fe 415 steel

if $d_{req} < d_{provided}$, Hence design is safe

if $d_{req} > d_{provided}$, Design is not safe, revise the depth

6. Calculation of main reinforcement

(a) if $M_u < M_{u_{lim}}$ — section is under reinforced

Hence singly reinforced section is to be designed

Refer, IS : 456-2000, G-1.1(b)

$$M_u = 0.87 f_y A_{st} d\left[1 - \left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$$

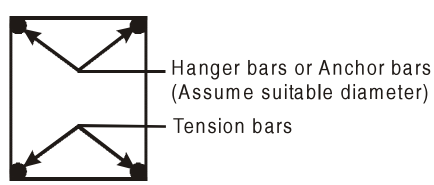
solve for '$A_{st_{required}}$'

Now, Assume suitable diameter of bars

(8mm, 10mm, 12mm, 16mm, 20mm, 25mm,......)

Area of one bar, $a_{st} = \frac{\pi}{4}\phi^2$

No. of bars = $\frac{A_{st}}{a_{st}}$ (Minimum number of bars should be two)



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Calculate '$A_{st\ provided}$' = No. of bars × a_{st}

(b) if $M_u > M_{u_{lim}}$ ——— section is over reinforced

Hence Doubly reinforced section is to be designed

(1) Area of compression reinforcement (A_{sc})

Refer IS : 456–2000, G-1.2

$$M_u - M_{u_{lim}} = f_{sc} A_{sc}(d - d')$$

$$\therefore A_{sc} = \frac{M_u - M_{u_{lim}}}{f_{sc}(d - d')}$$

Find 'f_{sc}'

$f_{sc} = \varepsilon_{sc} \times E_s$; where $\varepsilon_{sc} = \dfrac{0.0035(x_{u_{lim}} - d')}{x_{u_{lim}}}$

$f_{sc} = \dfrac{0.0035(x_{u_{lim}} - d')}{x_{u_{lim}}} \times E_s \not> 0.87 f_y$ ——— For *Fe* 250 (Mild steel)

where E_s = young's modulus of steel = 2×10^5 N/mm^2

Now, $\varepsilon_{sc} = \dfrac{0.0035(x_{u_{lim}} - d')}{x_{u_{lim}}}$

Refer IS : 456 – 2000, Fig 23 A, using graph, (referring ε_{sc} value)

Find 'f_{sc}' for *Fe* 415 and For Fe500 (Refer clause 2.7, Note 3a or 3b)

Now solve for '$A_{sc_{req}}$' (Area of compression reinforcement)

Assume suitable diameter of bars

(ϕ = 8mm, 10mm, 12mm, 16mm, 20mm, 25mm,......)

Area of one bar, $a_{sc} = \dfrac{\pi}{4}\phi^2$

No. of bars = $\dfrac{A_{sc}}{a_{sc}}$

Calculate $A_{sc_{provided}}$

(2) Area of tension reinforcement (A_{st})

Refer, IS:456:2000, G-1.1(a)

$\dfrac{x_u}{d} = \dfrac{0.87 f_y A_{st}}{0.36 f_{ck} bd}$, put $A_{st} = A_{st_1}$, $x_u = x_{u_{lim}}$

$\therefore \quad A_{st_1} = \dfrac{0.36 f_{ck} b\, x_{u_{lim}}}{0.87 f_y}$, Find '$A_{st_1}$'

Now, Refer, IS:456-2000, ANNEX - G, G - 1.2

$$A_{st_2} = \frac{A_{sc} f_{sc}}{0.87 f_y}, \qquad \text{Find } 'A_{st_2}'$$

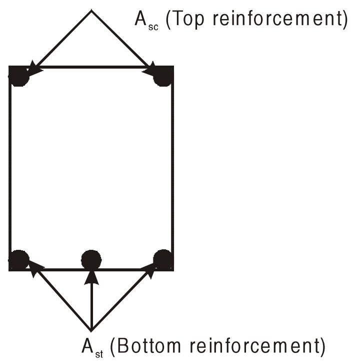
$\therefore$ Total tension reinforcement, $A_{st} = A_{st_1} + A_{st_2}$

Assume suitable diameter of bars

(ϕ = 8mm, 10mm, 12mm, 16mm, 20mm, 25mm,....)

Area of one bar, $a_{st} = \frac{\pi}{4}\phi^2$

No.of bars = $\frac{A_{st}}{a_{st}}$, calculate $'A_{st_{provided}}'$ = No.of bars $\times$ a_{st}



✸ **Check of shear reinforcement - clause 26.5.1.1 (a)**

$$\frac{A_{st}}{bd} = \frac{0.85}{f_y} \Rightarrow A_{st_{min.}} = \frac{0.85\ bd}{f_y}$$

(i) If $A_{st} > A_{st_{min.}}$, Hence (ok)

(ii) If $A_{st} < A_{st_{min.}}$, Hence use $A_{st_{min}}$ for calculation of No.of bars

7. **Design of shear reinforcement**

- Design of shear force

$V_u = \frac{W_u L}{2}$ ——— For simply supported beam carrying UDL

$V_u = W_u L$ ——— For cantilever beam carrying UDL

- Shear stress / Nominal shear stress, $\tau_v = \frac{V_u}{bd}$ [clause 40.1]

- Percentage of tension steel reinforcement, $p_t = \dfrac{100A_{st_{prov}}}{bd}$ [Table 19]
- Calculation of 'τ_c'
  Refer IS:456 – 2000, Table No.19
  using p_t and f_{ck} value, note down the value of 'τ_c'
- Design shear strength $\tau_{c_{\max}}$
  Refer IS:456–2000, Table No.20
  using f_{ck} value, note down the value of $\tau_{c_{\max}}$
- Comparisons

(1) If $\tau_v < \tau_c$ — Shear reinforcement is not required. As per IS:456–2000, Minimum shear reinforcement should be provided in the form of stirrups

Refer, clause 26.5.1.6

$$\frac{A_{sv}}{bS_v} \geq \frac{0.4}{0.87f_y}$$

Spacing of stirrups should be least of the following three

1. $S_v = \dfrac{0.87 f_y A_{sv}}{0.4b}$

   Here, $A_{sv} = 2 \times \dfrac{\pi}{4}\phi^2$ [Area for 2-Legged vertical stirrups]

   Assume 6mm, 8mm or 10mm dia bars for vertical stirrups
   Find 'S_v'
2. 'S_v' should not be greater than 0.75 d
   i.e., $S_v \not> 0.75\ d$, if greater than 0.75 d
   then consider, S_v = 0.75 d
   Here d = Effective depth
3. 'S_v' should not be greater than 300 mm
   i.e., $S_v \not> 300$ mm, if greater than 300 mm
   then consder, S_v = 300 mm
   ∴ Provide 2L —— mm ϕ bars @ —— mm c/c

(2) if $\tau_v > \tau_c$ — shear reinforcement is required

(i) calculate shear carried by concrete, $V_{uc} = \tau_c bd$

(ii) calculate shear carried by stirrups, $V_{us} = V_c - V_{uc}$

i.e., $V_{us} = V_u - \tau_c bd$ [Refer, clause 40.4]

$\because\ V_u = V_{uc} + V_{us}$

$$\tau_v = \frac{V_u}{bd}$$

$$\tau_c = \frac{V_{uc}}{bd}$$

$$\Rightarrow V_{uc} = \tau_c bd$$

Spacing of stirrups should be least of the following three

$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$ [clause 40.4 (a)]

1. $S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$

$A_{sv} = 2 \times \frac{\pi}{4} \phi^2$ [Area for 2-Legged vertical stirrups]

Assume 6mm, 8mm or 10mm dia bars for vertical stirrups

Find 'S_v'

2. 'S_v' should not be greater than 0.75 d i.e., $S_v \ngtr 0.75d$

3. 'S_v' should not be greater than 300 mm i.e., $S_v \ngtr 300$ mm

∴ Provide 2L —— mm ϕ bars @ —— mm c/c

(3) if $\tau_v < \tau_{c_{max}}$ —— Design is safe

(4) if $\tau_v > \tau_{c_{max}}$ —— Design is not safe, revise the depth

8. Check for deflection control

(a) Note down the percentage of steel provided

$p_t = \frac{100 A_{st_{prov}}}{bd}$ —— For singly reinforced beam

$p_c = \frac{100 A_{sc_{prov}}}{bd}$ and p_t —— For doubly reinforced beam

(b) Stress in steel, Refer IS:456–2000, Fig 4,

$$f_s = 0.58 f_y \left[\frac{A_{st_{required}}}{A_{st_{provided}}} \right]$$

read out modification factor (k_t) from curve using f_s and p_t (Fig 4 of IS : 456–2000)

'k_t' is required for singly reinforced beam

read out modification factor (k_c) from curve, using p_c (Fig 5 of IS:456–2000)

k_t and k_c are required for doubly reinforced beam

✸ **For simply supported beam**

(1) $\left(\frac{L}{d}\right)_{max.} = 20 \times k_t \times k_c \times k_f$ $\left[\begin{array}{l} k_{f=1} \because \text{No flanged section} \\ k_{c=1} \because \text{No compression steel} \end{array}\right]$

Here, $k_c = 1$ and $k_f = 1$

$\therefore \left(\frac{L}{d}\right)_{max.} = 20 \times k_t \times 1 \times 1 = 20k_t$ —— For singly reinforced beam

Here, $k_f = 1$

$\therefore \left(\frac{L}{d}\right)_{max.} = 20 \times k_t \times k_c \times 1 = 20 \times k_t \times k_c$ —— For doubly reinforced beam

✸ **For cantilever beam**

$\left(\frac{L}{d}\right)_{max.} = 7k_t$ —— For singly reinforced beam

$\left(\frac{L}{d}\right)_{max.} = 7 \times k_t \times k_c$ —— For doubly reinforced beam

(2) $\left(\frac{L}{d}\right)_{actual}$ or $\left(\frac{L}{d}\right)_{provided}$ where, L = Effective span ; d = Effective depth

if $\left(\frac{L}{d}\right)_{max.} > \left(\frac{L}{d}\right)_{provided}$ —— Design is safe ; Hence Deflection Control is satisfied

if $\left(\frac{L}{d}\right)_{max.} < \left(\frac{L}{d}\right)_{provided}$ —— Design is not safe ; Hence Deflection control is not satisfied then, revise the depth

9. Development Length or Anchorage length at support

- For contilever beams only,
  Refer IS:456–2000, clause 26.2.1

$$L_d = \frac{\phi\sigma_s}{4\tau_{bd}} \quad \text{where, } \sigma_s = 0.87 f_y$$

$$\Rightarrow L_d = \frac{0.87 f_y \phi}{4\tau_{bd}}$$

'τ_{bd}' depends on grade of concrete [clause 26.2.1.1]

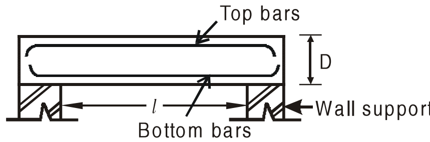
→ For deformed bars (Fe 415 and Fe 500), 'τ_{bd}' values shall be increased by 60%

For example for M_{20} concrete, $\tau_{bd} = 1.2$ N/mm^2

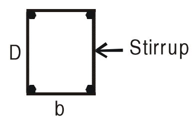
$\therefore L_d = \frac{0.87 f_y \phi}{4 \times 1.2 \times 1.6}$ —— [For deformed bars increased by 60% i.e., 1.6]

10. Reinforcement details

1. Longitudinal section



2. Cross section



3.4 WORKED EXAMPLES ON DESIGN OF SINGLY REINFORCED RECTANGULAR BEAMS [SIMPLY SUPPORTED AND CANTILEVER BEAMS]

1. A simply supported RC beam supports a service live load of 8 kN/m over a clear span of 3m. Support width is 200 mm. Adopt M_{20} grade concrete and Fe 415 grade steel. Design the beam for flexure and shear. Check the beam depth for control of deflection using empirical method. Sketch the reinforcement details.

Solution :

Given :

clear span, l = 3m

service live load = 8 kN/m

support width = 200 mm

$f_{ck} = 20$ N/mm^2 and $f_y = 415$ N/mm^2

Step : 1 Selection of cross - sectional dimensions

(a) Effective depth of beam (d)

$$d = \frac{\text{clear span}}{15}, \text{ For } l = 3, \frac{l}{d} = 15 \text{ to } 20$$

$$\text{i.e., } d = \frac{l}{15} = \frac{3000}{15} = 200 \text{ mm}$$

Say, $d = 250$ mm or 0.25 m [$\therefore d_{\text{provided}} = 250$ mm]

(b) Overall depth of beam (D)

$D = d + d'$ (Assume d' = 50 mm)

$\therefore D = 250 + 50 = 300$ mm or 0.3 m

(c) Width of beam (b)

$$b = \frac{1}{3} \times d \text{ to } \frac{2}{3} \times d$$

$\therefore\ b = \frac{2}{3} \times 250 = 166.66$ mm

Say, b = 200 mm or 0.2 m

$\therefore$ Beam size = $b \times D$ = 200 × 300 mm

Step : 2 Calculation of Effective span (L) : Should be least of the following two

(i) $L = l$ + support width = 3000 + 200 = 3200 mm or 3.2 m

(ii) $L = l$ + effective depth (d) = 3000 + 250 = 3250 mm or 3.25 m

$\therefore L = 3.2$ m

Step : 3 Load calculation

1. Self weight of beam = b × D × density of RCC
   = 0.2 × 0.3 × 25
   = 1.5 kN/m
2. Live load (Given) = 8.0 kN/m

Total load, W = 9.5 kN/m

$\therefore$ Factored load, $W_u = 1.5 \times W = 1.5 \times 9.5 = 14.25$ kN/m

Step : 4 Calculation of maximum bending moment

$$M_u = \frac{W_u L^2}{8} = \frac{14.25 \times 3.2^2}{8} = 18.24 \text{kN / m} \quad \text{or } 18.24 \times 10^6\ N.mm$$

Step : 5 Check for effective depth

$M_{u_{\lim}} = 0.138 f_{ck} bd^2$ —— For Fe 415 steel

Equating, $M_u = M_{u_{\lim}}$

$$\therefore\ d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{18.24 \times 10^6}{0.138 \times 20 \times 200}} = 181.77 \text{ mm}$$

$d_{req} < d_{provided} (250\text{mm})$, Design is safe, (ok)

Step : 6 Calculation of main reinforcement

$M_{u_{\lim}} = 0.138 f_{ck} bd^2$

$M_{u_{\lim}} = 0.138 \times 20 \times 200 \times 250^2 = 34.5 \text{kN.m}$ or 34.5×10^6 N.mm

$\therefore\ M_{u_{\lim}} > M_u$ or $M_u < M_{u_{\lim}}$ —— section is under reinforced

Hence singly reinforced section is to be designed

Refer IS:456–2000, G-1.1(b)

$$M_u = 0.87 f_y A_{st} d\left[1-\left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$$

$$18.24\times10^6 = 0.87\times415\times250 A_{st}\left[1-\left(\frac{415 A_{st}}{20\times200\times250}\right)\right]$$

$$37.45 A_{st}^2 - 90262.5 A_{st} + 18.24\times10^6 = 0$$

By solving quadratic equation, we get

A_{st} = 222.64 mm^2

- Check for minimum reinforcement – IS: 456-2000, clause 26.5.1.1(a)

$$\frac{A_{st_{\min}}}{bd} = \frac{0.85}{f_y}$$

$$\Rightarrow A_{st_{\min}} = \frac{0.85bd}{f_y} = \frac{0.85\times200\times250}{415} = 102.40mm^2$$

$\therefore \quad A_{st} > A_{st_{\min}}$, Hence (ok)

Now Assume 12 mm ϕ bars

$\therefore$ Area of one bar, $a_{st} = \frac{\pi}{4}\phi^2 = \frac{\pi}{4}\times12^2 = 113.09mm^2$

No. of bars = $\frac{A_{st}}{a_{st}} = \frac{222.64}{113.09} = 1.96$ say 2 No's

$\therefore \quad A_{st_{provided}} = 2\times113.09 = 226.18mm^2$ | Compression reinforcement - Top; Tension reinforcement - Bottom

$\therefore$ Provide 2-12 mm ϕ bars at tension reinforcement and 2-10 mm ϕ bars (Hanger bars) at compression reinforcement (Assume)

Step : 7 Design of shear reinforcement

- Design of shear force

$$V_u = \frac{W_u L}{2} = \frac{14.25\times3.2}{2} = 22.8kN$$

- Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{22.8\times10^3}{200\times250} = 0.45N/mm^2$

- Percentage of tension steel reinforcement, $p_t = \frac{100A_{st_{prov}}}{bd} = \frac{100\times226.18}{200\times250} = 0.45\%$

- Calculation of τ_c

  Refer IS:456–2000, Table No-19

  using $p_t = 0.45\%$ and $f_{ck} = 20$ N/mm^2

p_t(%)	τ_c(N/mm^2)
0.25	0.36
0.45	?
0.50	0.48

By using interpolation, we get

$\therefore\ \tau_c = 0.45$ N/mm^2

- Design shear strength '$\tau_{c_{\max}}$'

  Refer table - 20

  using $f_{ck} = 20$ N/mm^2

  $\therefore\ \tau_{c_{\max}} = 2.8$ N/mm^2

- Comparisons

(a) Since $\tau_v = \tau_c$ —— Hence, shear reinforcement is taken by concrete

$\therefore$ provide minimum shear reinforcement as per IS:456–2000, clause 26.5.1.6

Spacing of stirrups should be least of the following three

1. $S_v = \dfrac{0.87 f_y A_{sv}}{0.4b}$

   Assume $2L$ – vertical stirrups of 8 mm ϕ bars

   $$\therefore\ A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

   $$\Rightarrow S_v = \frac{0.87 \times 415 \times 100.53}{0.4 \times 200} = 453.70 \text{ mm}$$

2. $S_v \not> 0.75\, d$, $\therefore\ S_v = 0.75 \times 250 = 187.5$ mm
3. $S_v \not> 300$ mm, $\therefore\ S_v = 300$ mm

   $\therefore\ S_v = 187.5$ mm, say $S_v = 175$ mm

   $\therefore$ Provide 2L – 8 mm ϕ bars @ 175 mm c/c

(b) $\tau_{c_{\max}} > \tau_v$, Design is safe (ok)

Step : 8 Check for deflection control

(a) Note down the percentage of steel provided

$p_t = 0.45$ %

(b) Stress in steel, Refer IS:456–2000, Fig 4

$$f_s = 0.58 f_y \left(\frac{A_{st_{req}}}{A_{st_{prov}}} \right) = 0.58 \times 415 \times \frac{222.64}{226.18} = 236.93 \text{ N/mm}^2$$

read out the modification factor (k_t) from curve using

$f_s = 236.93$ N/mm^2 and $p_t = 0.45\%$ (Fig 4 of IS:456–2000)

$\therefore k_t = 1.3$

1. $\left(\frac{L}{d}\right)_{max.} = 20 \times k_t \times k_c \times k_f$

$k_c = 1$ and $k_f = 1$ ($\because$ No compression steel and no flanged section)

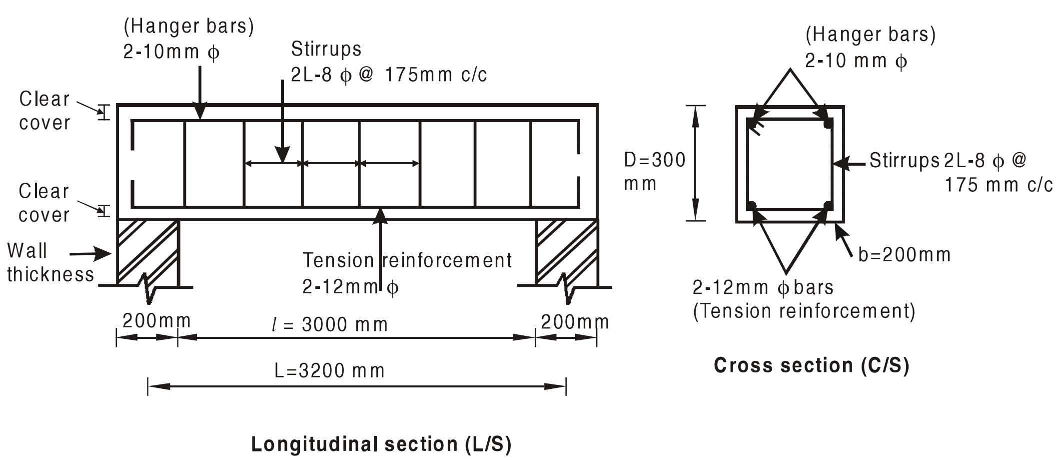
$$\therefore \left(\frac{L}{d}\right)_{max.} = 20 \times 1.3 \times 1 \times 1 = 26$$

2. $\left(\frac{L}{d}\right)_{provided} = \frac{3200}{250} = 12.8$

$$\left(\frac{L}{d}\right)_{max.} > \left(\frac{L}{d}\right)_{provided}$$

Hence deflection control is satisfied, (ok)

Step : 9 Reinforcement details



2. Design a reinforced beam of clear span of 5m to support a working live load of 15 kN/m. Adopt M_{20} grade concrete and Fe-415 steel-sketch the reinforcement details.

Solution :

Given : clear span, l = 5m

working live load = 15 kN/m

support width = 300 mm (Assume)

f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2

Step : 1 Selection of cross-sectional dimensions

(a) Effective depth of beam (d)

$$d = \frac{\text{clear span}}{12 \text{ to } 15} \text{ for span range 5 to 10 m}$$

$$\therefore\ d = \frac{\text{clear span}}{12} = \frac{l}{12} = \frac{5000}{12} = 416.66\,\text{mm}$$

say d = 450 mm or 0.45 m [$\therefore\ d_{\text{provided}}$ = 450 mm]

(b) Overall depth of beam (D)

$D = d + d'$ (Assume d' = 50 mm)

$\therefore\ D$ = 450 + 50 = 500 mm or 0.5 m

(c) Width of beam (b)

$$b = \frac{1}{3} \times d \text{ to } \frac{2}{3} \times d$$

$$\therefore\ b = \frac{2}{3} \times 450 = 300 \text{ mm or } 0.3\text{m}$$

$\therefore$ Beam size = $b \times D$ = 300 × 500 mm

Step : 2 Calculation of effective span (L)

should be least of the following two

(i) $L = l$ + support width = 5000 + 300 = 5300 mm or 5.3 m

(ii) $L = l$ + effective depth (d) = 5000 + 450 = 5450 mm or 5.45 m

$\therefore\ L$ = 5.3 m

Step : 3 Load calculation

1. Self weight of beam = $b \times D \times RCC$ density
   = 0.3 ×0.5 × 25
   = 3.75 kN/m
2. Live Load (Given) = 15 kN/m

   Total load, W = 18.75 kN/m

   $\therefore$ Factored load, W_u = 1.5 × W = 1.5 × 18.75 = 28.12 kN/m

Step : 4 Calculation of Maximum bending moment

$$M_u = \frac{W_u L^2}{8} = \frac{28.12 \times 5.3^2}{8} = 98.73 \text{ kN.m or } 98.73 \times 10^6 \text{ N.mm}$$

Step : 5 Check for effective depth

$M_{u_{\lim}} = 0.138 f_{ck} bd^2$ —— For Fe 415 steel

Equating, $M_u = M_{u_{\lim}}$

$$\therefore\ d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{98.73\times10^6}{0.138\times20\times300}} = 345.31\text{mm}$$

$d_{req} < d_{prov}$ (450 mm), Design is safe, (ok)

Step : 6 Calculation of main reinforcement

$M_{u_{\lim}} = 0.138 f_{ck} bd^2 = 0.138\times20\times300\times450^2 = 167.67\text{kN.m}$ or 167.67×10^6 N.mm

$\therefore\ M_{u_{\lim}} > M_u$ —— section is under reinforced

Hence singly reinforced section is to be designed

Refer IS : 456 – 2000, G-1.1(b)

$$M_u = 0.87 f_y A_{st} d\left[1-\left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$$

$$98.73\times10^6 = 0.87\times415\times450 A_{st}\left[1-\left(\frac{415 A_{st}}{20\times300\times450}\right)\right]$$

$$24.97 A_{st}^2 - 162472.5 A_{st} + 98.73\times10^6 = 0$$

$\therefore\ A_{st} = 678.40\ \text{mm}^2$

- Check for minimum reinforcement - clause 26.5.1.1 (a)

$$\frac{A_{st_{\min}}}{bd} = \frac{0.85}{f_y}$$

$$A_{st_{\min}} = \frac{0.85bd}{f_y} = \frac{0.85\times300\times450}{415} = 276.50 mm^2$$

$\therefore\ A_{st} > A_{st_{\min}}$, Hence (ok)

Now Assume 20 mm ϕ bars

$$\therefore\ \text{Area of one bar, } a_{st} = \frac{\pi}{4}\phi^2 = \frac{\pi}{4}\times20^2 = 314.15\text{mm}^2$$

$$\therefore\ \text{No. of bars} = \frac{A_{st}}{a_{st}} = \frac{678.40}{314.15} = 2.16 \text{ say } 3$$

$$\therefore\ A_{st_{provided}} = 3\times314.15 = 942.45\text{mm}^2$$

∴ provide 3 – 20 mm ϕ bars at tension reinforcement (bottom) and 2-12 mm ϕ bars (Hanger bars) at compression reinforcement (Assume)

Step : 7 Design of shear reinforcement

- Design of shear force

$$V_u = \frac{W_u L}{2} = \frac{28.12 \times 5.3}{2} = 74.51\text{kN}$$

- Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{74.51 \times 10^3}{300 \times 450} = 0.55 \text{ N/mm}^2$

- Percentage of tension steel reinforcement, $p_t = \frac{100 A_{st_{prov}}}{bd} = \frac{100 \times 942.45}{300 \times 450} = 0.698\%$

$$\therefore \quad p_t \approx 0.7\ \%$$

- Calculation of τ_c

  Refer IS : 456 – 2000, Table No. 19, using $p_t = 0.7\%$ and $f_{ck} = 20$ N/mm²

  ∴ $\tau_c = 0.54$ N/mm² (by interpolation)

- Design shear strength '$\tau_{c_{max}}$', Refer table No.-20

  using $f_{ck} = 20$ N/mm², $\tau_{c_{max}} = 2.8$ N/mm²

- Comparisons

(a) $\tau_v > \tau_c$ —— Hence shear reinforcement is required

(i) Calculate shear carried by concrete, $V_{uc} = \tau_c bd = 0.54 \times 300 \times 450 = 72900$N

$$\therefore V_{uc} = 72.9 \text{ kN}$$

(ii) Calculate shear carried by stirrups, $V_{us} = V_u - V_{uc} = 74.51 - 72.9 = 1.61$ kN

Spacing of stirrups should be least of the following three

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v} \quad \text{[clause 40.4(a)]}$$

1. $S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$

   Assume 2L – vertical stirrups of 8mm ϕ bars

   $\therefore A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53\text{mm}^2$

   $\Rightarrow S_v = \frac{0.87 \times 415 \times 100.53 \times 450}{1.61 \times 10^3} = 10144.94\text{ mm}$

2. $S_v \ngtr 0.75\,d$, ∴ $S_v = 0.75\,d = 0.75 \times 450 = 337.5$ mm
3. $S_v \ngtr 300$ mm ∴ $S_v = 300$ mm

say $S_v = 300$ mm

∴ Provide 2L – 8mm φ bars @ 300 mm c/c

(b) $\tau_{c_{max}} > \tau_v$ —— Design is safe, (ok)

Step : 8 Check for deflection control

(a) Note down the percentage of steel provided, $p_t = 0.7$ %

(b) Stress in steel, Refer IS : 456 – 2000, Fig 4

$$f_s = 0.58 f_y \left[\frac{A_{st_{req}}}{A_{st_{prov}}} \right] = 0.58 \times 415 \times \frac{678.40}{942.45} = 173.26 \text{N/mm}^2$$

read out the modification factor (k_t)

∴ $k_t = 1.4$

1. $\left(\frac{L}{d}\right)_{max.} = 20 \times k_t \times k_c \times k_f$

$k_c = 1$ and $k_f = 1$ (∵ No compression steel and no flanged section)

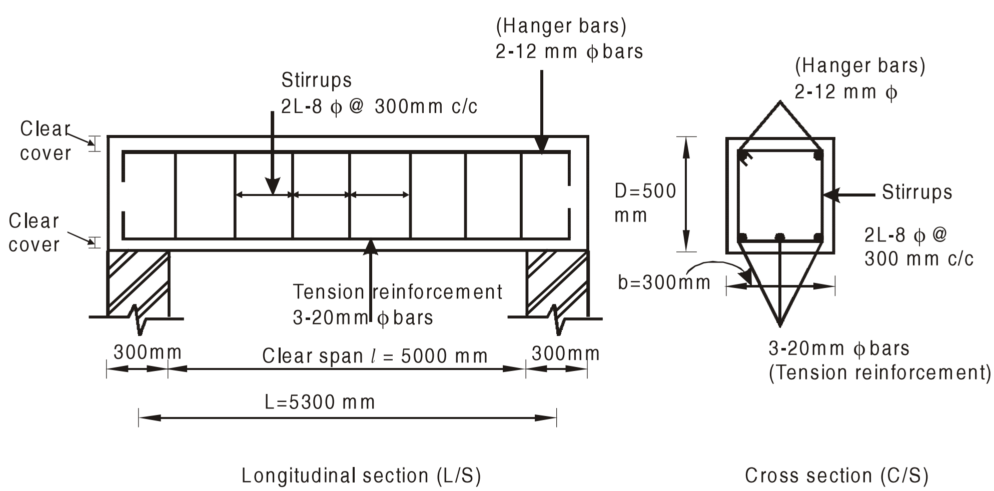
∴ $\left(\frac{L}{d}\right)_{max.} = 20 \times 1.4 \times 1 \times 1 = 28$

2. $\left(\frac{L}{d}\right)_{provided} = \frac{5300}{450} = 11.77$

$\left(\frac{L}{d}\right)_{max.} > \left(\frac{L}{d}\right)_{provided}$

Hence deflection control is satisfied (ok)

Step : 9 Reinforcement details



3. Design a cantilever beam of clear span 3.25m, service load is 15 kN/m. Use M20 concrete and Fe 415 steel. Sketch the reinforcement details.

Solution :

Given : clear span, l = 3.25 m, live load = 15 kN/m

f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2

Step : 1 Selection of cross - sectional dimensions

(a) Effective depth of beam (d)

$$d = \frac{\text{clear span}}{7} = \frac{l}{7} = \frac{3250}{7} = 464.28\text{mm}$$

say, d = 500 mm or 0.5 m [$\therefore d_{\text{provided}}$ = 500 mm]

(b) Overall depth of beam (D)

$D = d + d'$ (Assume d' = 50 mm)

$\therefore$ D = 500 + 50 = 550 mm or 0.55m

(c) Width of beam (b)

$$b = \frac{1}{3} \times d \quad \text{to} \quad \frac{2}{3} \times d$$

$$\therefore\ b = \frac{1}{3} \times 500 = 166.66\text{mm}$$

$$b = \frac{2}{3} \times 500 = 333.33\text{mm}$$

$\therefore$ Provide b = 250 mm or 0.25 m

Beam size = $b \times D$ = 250 × 550 mm

Step : 2 Calculation of effective span (L)

$$\text{Effective span } (L) = \text{clear span } (l) + \frac{\text{effective depth } (d)}{2}$$

$$\therefore\ L = 3.25 + \frac{0.5}{2} = 3.5\text{m}$$

Step : 3 Load calculation

1. Self weight of beam = $b \times D \times$ density of RCC
   = 0.25 × 0.55 × 25 = 3.437 kN/m
   ≈ 3.44 kN/m
2. Live load (Given) = 15.0 kN/m

   Total load, W = 18.44 kN/m

   $\therefore$ Factored W_u = 1.5 × W = 1.5 × 18.44 = 27.66 kN/m

Step : 4 Calculation of Maximum bending moment

$$M_u = \frac{W_u L^2}{2} = \frac{27.66 \times 3.5^2}{2} = 169.41 \text{ kN.m or } 169.41 \times 10^6 \text{ N.mm}$$

Step : 5 Check for effective depth

$M_{u_{\lim}} = 0.138 f_{ck} bd^2$ ——— For Fe 415 steel

Equating, $M_u = M_{u_{\lim}}$

$$\therefore d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{169.41 \times 10^6}{0.138 \times 20 \times 250}} = 495.50 \text{mm}$$

$d_{req} < d_{provided}$ (500 mm), Design is safe, (ok)

Step : 6 Calculation of main reinforcement

$$M_{u_{\lim}} = 0.138 f_{ck} bd^2 = 0.138 \times 20 \times 250 \times 500^2 = 172.5 \text{ kN.m}$$

$\therefore$ $M_{u_{\lim}} > M_u$ —— Section is under reinforced

Hence singly reinforced section is to be designed

Refer IS:456–2000, G-1.1(b)

$$M_u = 0.87 f_y A_{st} d \left[1 - \left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$$

$$169.41 \times 10^6 = 0.87 \times 415 \times 500 A_{st} \left[1 - \left(\frac{415 A_{st}}{20 \times 250 \times 500}\right)\right]$$

$$29.96 A_{st}^2 - 180525 A_{st} + 169.41 \times 10^6 = 0$$

$\therefore$ $A_{st} = 1162.84 \text{ mm}^2$

- Check for minimum reinforcement - clause 26.5.1.1(a)

$$A_{st_{\min}} = \frac{0.85 bd}{f_y} = \frac{0.85 \times 250 \times 500}{415} = 256.02 \text{ mm}^2$$

$\therefore$ $A_{st} > A_{st_{\min}}$, Hence (ok)

Now Assume 20 mm ϕ bars

$\therefore$ Area of one bar, $a_{st} = \frac{\pi}{4}\phi^2 = \frac{\pi}{4} \times 20^2 = 314.15$ mm²

No. of bars = $\frac{A_{st}}{a_{st}} = \frac{1162.84}{314.15} = 3.70$ say 4

$A_{st_{provided}} = 4 \times 314.15 = 1256.6$ mm^2

∴ Provide 4-20 mm ϕ bars at tension reinforcement (Top) and 2-12 mm ϕ bars at compression reinforcement (Bottom)-[Assume]

Step : 7 Design of shear reinforcement

- Design of shear force

  $V_u = W_u L = 27.66 \times 3.5 = 96.81$ kN

- Nominal shear stress, $\tau_v = \dfrac{V_u}{bd} = \dfrac{96.81 \times 10^3}{250 \times 500} = 0.77 N/mm^2$

- Percentage of tensile steel reinforcement, $p_t = \dfrac{100 A_{st_{prov}}}{bd} = \dfrac{100 \times 1256.6}{250 \times 500}$

  $\therefore p_t = 1\%$

- Calculation of τ_c, Refer IS:456–2000, table No. 19

  using $p_t = 1\%$ and $f_{ck} = 20$ N/mm^2

  $\tau_c = 0.62$ N/mm^2

- Design shear strength, '$\tau_{c_{max}}$', Refer table No.-20

  using $f_{ck} = 20$ N/mm^2

  $\therefore \tau_{c_{max}} = 2.8$ N/mm^2

- Comparisons

(a) $\tau_v > \tau_c$ —— Hence shear reinforcement is required

(i) Calculate shear carried by concrete, $V_{uc} = \tau_c bd = 0.62 \times 250 \times 500 = 77500$ N

$\therefore V_{uc} = 77.5$ kN

(ii) Calculate shear carried by stirrups, $V_{us} = V_u - V_{uc} = 96.81 - 77.5 = 19.31$ kN

spacing of stirrups should be least of the following three

1. $S_v = \dfrac{0.87 f_y A_{sv} d}{V_{us}}$

   Assume $2L$ – 8mm ϕ vertical stirrups

   $\therefore A_{sv} = 2 \times \dfrac{\pi}{4} \times 8^2 = 100.53\,\text{mm}^2$

   $\therefore S_v = \dfrac{0.87 \times 415 \times 100.53 \times 500}{19.31 \times 10^3} = 939.83$

2. $S_v \not> 0.75\, d$, $\therefore S_v = 0.75 \times 500 = 375$ mm
3. $S_v \not> 300$ mm, $\therefore S_v = 300$ mm

   say $S_v = 300$ mm

$\therefore$ Provide 2L–8mm ϕ bars @ 300 mm c/c

(b) $\tau_{c_{max}} > \tau_v$, ——Design is safe (ok)

Step : 8 Check for deflection control

(a) Note down the percentage of steel provided, $p_t = 1\%$

(b) stress in steel, Refer IS:450-2000, Fig 4

$$f_s = 0.58 f_y \left[\frac{A_{st_{req}}}{A_{st_{prov}}}\right] = 0.58 \times 415 \times \frac{1162.84}{1256.6} = 222.74\,\text{N/mm}^2$$

read out the modification factor (k_t)

$\therefore k_t = 1.15$

1. $$\left(\frac{L}{d}\right)_{max} = 7 \times k_t \times k_c \times k_f \qquad k_c = k_f = 1$$

$$\therefore \left(\frac{L}{d}\right)_{max} = 7 \times 1.15 \times 1 \times 1 = 8.05$$

2. $$\left(\frac{L}{d}\right)_{provided} = \frac{3500}{400} = 7$$

$$\therefore \left(\frac{L}{d}\right)_{max} > \left(\frac{L}{d}\right)_{provided}$$

Hence deflection control is satisfied, (ok)

Step : 9 Development length or Anchorage length at supports

Refer IS:456–2000, clause 26.2.1

$$L_d = \frac{\phi\sigma_s}{4\tau_{bd}} \quad \text{where } \sigma_s = 0.87 f_y$$

$$\therefore L_d = \frac{0.87 f_y \phi}{4\tau_{bd}}$$

$\tau_{bd} = 1.2$ N/mm^2 —— from clause 26.2.1.1

For deformed bars increased by 60%

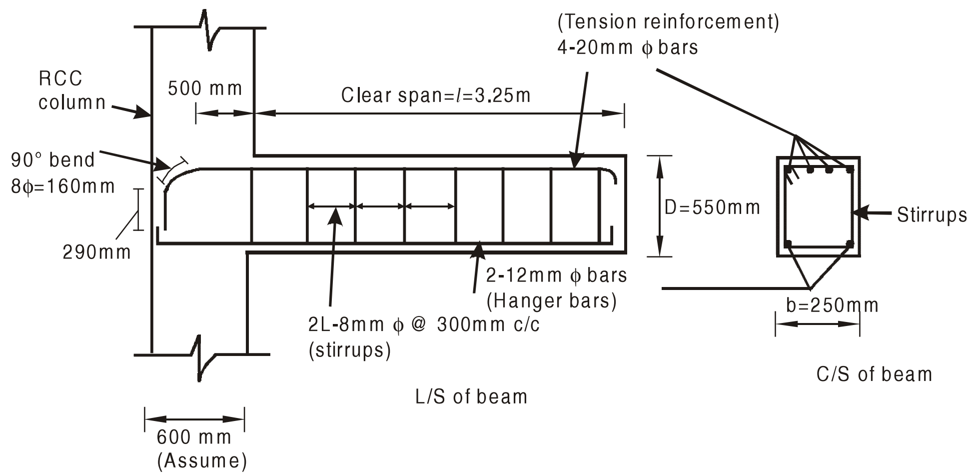
i.e., $\tau_{bd} = 1.2 \times 1.6$

Now, $$L_d = \frac{0.87 \times 415 \times 20}{4 \times 1.2 \times 1.6} = 940.23\,\text{mm say } 950\text{mm}$$

The main tension reinforcement is extended into the column to a length of 500 mm and bent down at 90° and extended upto 8 ϕ and 290 mm

i.e., $L_d = 500 + 8 \times 20 + 290 = 950$ mm

Step : 10 Reinforcement details



3.5 WORKED EXAMPLES ON DESIGN OF DOUBLY REINFORCED RECTANGULAR BEAMS [SIMPLY SUPPORTED AND CANTILEVER BEAMS]

1. **Design a simply supported beam of effective span 8m subjected to super imposed loads of 35 kN/m. The beam dimensions and other data are b = 300mm, D = 700mm, M_{20} grade concrete and Fe 415 steel**

   Solution:

   Given : Effective span, L = 8m or 8000 mm

   Super imposed load = 35 kN/m

   Width of beam, b = 300 mm

   Overall Depth of beam, D = 700 mm

   f_{ck} = 20 N/mm^2

   f_y = 415 N/mm^2

Step : 1 Selection of cross sectional dimensions

Effective depth, $d = D - d'$ Assume d' = 50 mm

$\therefore$ d = 700 – 50 = 650 mm

Step : 2 Load calculation

1. Self weight of beam = 0.3 × 0.7 × 25

   = 5.25 kN/m
2. Super imposed load (Given) = 35 kN/m

   Total load, W = 40.25 kN/m

   $\therefore$ Factored load, $W_u = 1.5 \times W = 1.5 \times 40.25 = 60.38$ kN/m

Step : 3 Calculation of maximum BM

$$M_u = \frac{W_u L^2}{8} = \frac{60.38 \times 8^2}{8} = 483.04 \text{ kN.m} \quad \text{or} \quad 483.04 \times 10^6 \text{ N.mm}$$

$$M_{u_{\lim}} = 0.138 f_{ck} bd^2 = 0.138 \times 20 \times 300 \times 650^2 = 349.83 \text{ kN.m}$$

$M_u > M_{u_{\lim}}$ —— section is over reinforced

∴ The beam is designed as doubly reinforced beam section

Note : Depth required is not considered for doubly reinforced beams

Step : 4 Calculation of main reinforcement

(1) Area of compression reinforcement (A_{sc})

Refer IS:456–2000, G-1.2

$$M_u - M_{u_{\lim}} = f_{sc} A_{sc} (d - d')$$

$$\therefore \quad A_{sc} = \frac{M_u - M_{u_{\lim}}}{f_{sc}(d - d')}$$

Now $\varepsilon_{sc} = \dfrac{0.0035\left(x_{u_{\max}} - d'\right)}{x_{u_{\max}}}$ —— from IS:456–2000, G-1.2

$$x_{u_{\max}} = x_{u_{\lim}}$$

$\dfrac{x_{u_{\max}}}{d} = 0.48$ for Fe 415 steel

$$\therefore \; x_{u_{\max}} = 0.48 \times d = 0.48 \times 650 = 312 \text{ mm}$$

Now, $\quad \varepsilon_{sc} = \dfrac{0.0035(312 - 50)}{312} = 0.00293 \approx 0.0030$

Refer IS:456–2000, Fig 23A,

$$f_{sc} = 0.85 f_y = 0.85 \times 415 = 352.75 \text{ N/mm}^2$$

$$A_{sc} = \frac{\left(483.04 \times 10^6 - 349.83 \times 10^6\right)}{352.75(650 - 50)} = 629.38 \text{ mm}^2$$

Assume 16 mm ϕ bars

Area of one bar, $a_{sc} = \dfrac{\pi}{4} \times 16^2 = 201.06 \text{mm}^2$

No. of bars = $\frac{A_{sc}}{a_{sc}} = \frac{629.38}{201.06} = 3.13$, say 4

$A_{sc_{provided}} = 4 \times 201.06 = 804.24\,\text{mm}^2$

Provide 4 – 16mm ϕ bars at compression reinforcement (Top)

(2) Area of tension reinforcement (A_{st})

Refer IS:456 – 2000, G-1.1(a)

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd}$$

Here $x_u = x_{u_{\lim}}$ and $A_{st} = A_{st_1}$

$$\therefore\ A_{st_1} = \frac{0.36 f_{ck} b x_{u_{\lim}}}{0.87 f_y} = \frac{0.36 \times 20 \times 300 \times 312}{0.87 \times 415} = 1866.55\,\text{mm}^2$$

Now Refer, G-1.2

$$A_{st_2} = \frac{A_{sc} f_{sc}}{0.87 f_y} = \frac{629.38 \times 352.75}{0.87 \times 415} = 614.91\,\text{mm}^2$$

$\therefore$ Total tension reinforcement, $A_{st} = A_{st_1} + A_{st_2}$

$= 1866.55 + 614.91$

$A_{st} = 2481.46\ \text{mm}^2$

Assume 25 mm ϕ bars

Area of one bar, $a_{st} = \frac{\pi}{4} \times 25^2 = 490.87\,\text{mm}^2$

No. of bars = $\frac{A_{st}}{a_{st}} = \frac{2481.46}{490.87} = 5.05$ say 5

$A_{st_{provided}} = 5 \times 490.87 = 2454.35\,\text{mm}^2$

$\therefore$ Provide 5 – 25 mm ϕ bars at tension reinforcement

[out of 5 bar, 3 bars in first layer and 2 bars in second layer]

Note : Check for minimum reinforcement is not required in doubly reinforced beams.

Step 5 : Design of shear reinforcement

- Design of shear force, $V_u = \frac{W_u L}{2} = \frac{60.38 \times 8}{2} = 241.52$ kN
- Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{241.52 \times 10^3}{300 \times 650} = 1.23\,\text{N/mm}^2$

- Percentage of tension steel reinforcement, $p_t = \frac{100A_{st_{prov}}}{bd} = \frac{100 \times 2454.35}{300 \times 650}$

$\therefore p_t = 1.25\ \%$

- Calculation of 'τ_c', Refer IS:456–2000, Table No.19 for $p_t = 1.25\%$ and $f_{ck} = 20$ N/mm^2
  $\tau_c = 0.67$ N/mm^2
- Design shear strength $\tau_{c_{max}}$, Refer table No.20
  using $f_{ck} = 20$ N/mm^2, $\tau_{c_{max}} = 2.8$N/mm^2
- Comparisons

(a) $\tau_v > \tau_c$ —— Hence shear reinforcement is required

(i) Calculate shear carried by concrete

$V_{uc} = \tau_c bd = 0.67 \times 300 \times 650 = 130.65$ kN

(ii) Calculate shear carried by stirrups

$V_{us} = V_u - V_{uc} = 241.52 - 130.65 = 110.87$ kN

spacing of stirrups should be least of the following three

1. $S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$ For 2L-8mm ϕ bars, $A_{sv} = 100.53$ mm^2

$$= \frac{0.87 \times 415 \times 100.53 \times 650}{110.87 \times 10^3} = 212.79 \text{ mm}$$

2. $S_v \not> 0.75\,d$, $\therefore S_v = 0.75 \times 650 = 487.5$ mm
3. $S_v \not> 300$ mm $\therefore S_v = 300$ mm, say $S_v = 200$ mm

$\therefore$ Provide 2L-8mm ϕ bars @ 200 mm c/c

(b) $\tau_{c_{max}} > \tau_v$, Design is safe, (ok)

Step : 6 Check for deflection control

(a) Note down the percentage of steel provided

$p_t = 1.25\ \%$ —— Tension steel percentage

$$p_c = \frac{100A_{sc_{prov}}}{bd} = \frac{100 \times 804.24}{300 \times 650} = 0.41\% \text{ —— Compression steel percentage}$$

(b) Stress in steel, refer IS:456-2000, Fig 4

$$f_s = 0.58 f_y \left[\frac{A_{st_{req}}}{A_{st_{prov}}}\right] = 0.58 \times 415 \times \frac{2481.46}{2454.35} = 243.35 \text{ N/mm}^2$$

readout the modification factor (k_t), using p_t and f_s

$\therefore k_t = 0.95$

Similarly, readout the modification factor (k_c), using p_c (Fig 5, IS:456-2000)

$\therefore k_c = 1.13$

1. $\left(\frac{L}{d}\right)_{max.} = 20 \times k_t \times k_c \times k_f$

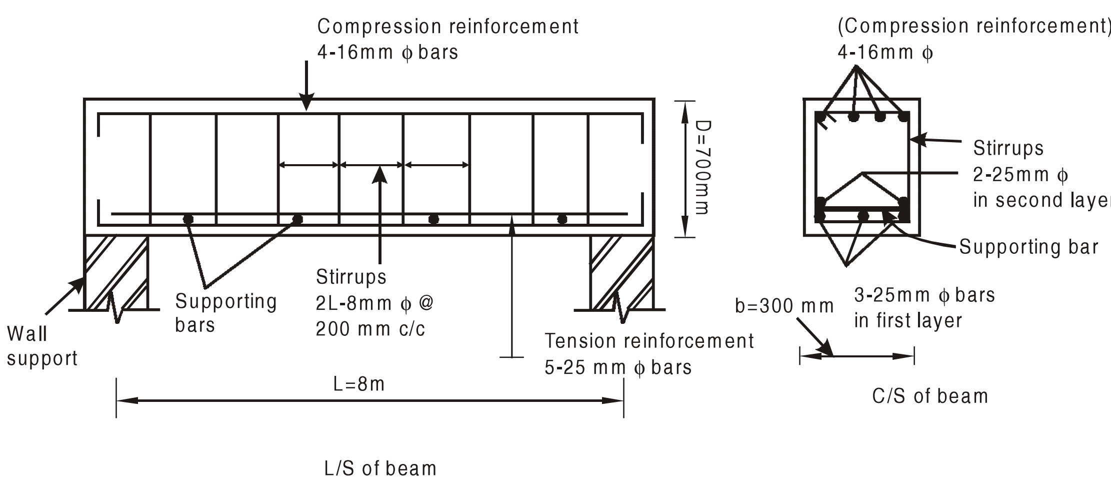
$k_f = 1$ ($\because$ No flanged section)

$\therefore \left(\frac{L}{d}\right)_{max.} = 20 \times 0.95 \times 1.13 \times 1 = 21.47$

2. $\left(\frac{L}{d}\right)_{provided} = \frac{8000}{650} = 12.30$

$\therefore \left(\frac{L}{d}\right)_{max.} > \left(\frac{L}{d}\right)_{provided}$, Hence deflection control is satisfied, (ok)

Step : 7 Reinforcement details



2. **Design a RC beam supported on two walls 500 m thick, spaced at a clear distance of 6m. The beam carries a super-imposed load of 30 kN/m. Use M20 grade concrete and Fe 415 steel.**

Solution :

Given : clear span, l = 6m or 6000 mm, wall thick = 500 mm

super imposed load = 30 kN/m

$f_{ck} = 20$ N/mm^2

$f_y = 415$ N/mm^2

Step : 1 Selection of cross - sectional dimensions

(a) Effective depth of beam (d)

$$d = \frac{\text{clear span}}{14} = \frac{l}{14} = \frac{6000}{14} = 428.57 \text{ mm}$$

say, $d = 450$ mm

(b) Overall depth of beam (D)

$D = d + d'$ (Assume $d' = 50$ mm)

$D = 450 + 50 = 500$ mm or 0.5 m

(c) Width of beam (b)

$$b = \frac{1}{3} \times d \quad \text{to} \quad \frac{2}{3} \times d$$

$$\therefore\ b = \frac{2}{3} \times 450 = 300 \text{ mm} \quad \text{or} \quad 0.3\text{m}$$

$\therefore$ Size of beam = $b \times D = 300 \times 500$ mm

Step : 2 Calculation of effective span (L)

Should be least of the following two

(i) $L = l$ + support width = 6000 + 500 = 6500 mm

(ii) $L = l + d = 6000 + 450 = 6450$ mm

$\therefore L = 6450$ mm or 6.45m

Step : 3 Load calculation

1. Self weigth of beam = 0.3 × 0.5 × 25
   = 3.75 kN/m
2. Super imposed load (Given) = 30.0 kN/m

Total load, W = 33.75 kN/m

$\therefore$ Factored load, $W_u = 1.5 \times W = 1.5 \times 33.75 = 50.62$ kN/m

Step : 4 Calculation of Maximum BM

$$M_u = \frac{W_u L^2}{8} = \frac{50.62 \times 6.45^2}{8} = 263.23 \text{ kN.m} \quad \text{or} \quad 263.23 \times 10^6 \text{ N.mm}$$

$$M_{u_{\lim}} = 0.138 f_{ck} b d^2 = 0.138 \times 20 \times 300 \times 450^2 = 167.67 \text{kN.m} \quad \text{or} \quad 167.67 \times 10^6 \text{ N.mm}$$

$M_u > M_{u_{\lim}}$ —— section is over reinforced

$\therefore$ The beam is designed as doubly reinforced beam section

Step : 5 Calculation of main reinforcement

(1) Area of compression reinforcement (A_{sc})

$$A_{sc} = \frac{M_u - M_{u_{\lim}}}{f_{sc}(d - d')}$$

Now $\varepsilon_{sc} = \dfrac{0.0035\left(x_{u_{\max}} - d'\right)}{x_{u_{\max}}}$ $\qquad x_{u_{\lim}} = x_{u_{\max}}$

$$\frac{x_{u_{\max}}}{d} = 0.48 \Rightarrow x_{u_{\max}} = 0.48 \times d = 0.48 \times 450 = 216\,\text{mm}$$

$$\therefore\ \varepsilon_{sc} = \frac{0.0035(216 - 50)}{216} = 0.00268$$

Refer IS:456-2000, Fig 23A

$$f_{sc} = 0.83 f_y = 0.83 \times 415 = 344.45\,\text{N/mm}^2$$

$$\therefore\ A_{sc} = \frac{\left(263.23 \times 10^6 - 167.67 \times 10^6\right)}{344.45(450 - 50)} = 693.56\,\text{mm}^2$$

Assume 20 mm ϕ bars

Area of one bar, $a_{sc} = \dfrac{\pi}{4} \times 20^2 = 314.15\,\text{mm}^2$

$\therefore$ No. of bars $= \dfrac{A_{sc}}{a_{sc}} = \dfrac{693.56}{314.15} = 2.20$, say 3

$$A_{sc_{prov}} = 3 \times 314.15 = 942.45\,\text{mm}^2$$

Provide 3 – 20 mm ϕ bars at compression reinforcement (Top)

(2) Area of tension reinforcement (A_{st})

$$A_{st_1} = \frac{0.36 f_{ck} b x_{u_{\lim}}}{0.87 f_y} = \frac{0.36 \times 20 \times 300 \times 216}{0.87 \times 415} = 1292.23\,\text{mm}^2$$

Now refer, G-1.2

$$A_{st_2} = \frac{A_{sc} f_{sc}}{0.87 f_y} = \frac{693.56 \times 344.45}{0.87 \times 415} = 661.87\,\text{mm}^2$$

$\therefore$ Total tension reinforcement, $A_{st} = A_{st_1} + A_{st_2} = 1292.23 + 661.67$

$$A_{st} = 1953.90\ \text{mm}^2$$

Assume 25 mm ϕ bars

Area of one bar, $a_{st} = \dfrac{\pi}{4} \times 25^2 = 490.87\,\text{mm}^2$

No.of bars $= \frac{A_{st}}{a_{st}} = \frac{1953.90}{490.87} = 3.98$, say 4

$A_{st_{prov}} = 4 \times 490.87 = 1963.48\,\text{mm}^2$

$\therefore$ Provide 4 - 25 mm ϕ bars at tension reinforcement

Step : 6 Design of shear reinforcement

- $V_u = \frac{W_u L}{2} = \frac{50.62 \times 6.45}{2} = 163.25\,\text{kN}$
- $\tau_v = \frac{V_u}{bd} = \frac{163.25 \times 10^3}{300 \times 450} = 1.20\,\text{N/mm}^2$
- $p_t = \frac{100 A_{st_{prov}}}{bd} = \frac{100 \times 1963.48}{300 \times 450} = 1.45\%$
- Calculate 'τ_c', Refer table 19, for p_t = 1.45% and f_{ck} = 20 N/mm^2

  $\therefore$ τ_c = 0.71 N/mm^2 (By interpolation method)
- $\tau_{c_{max}} = 2.8$ N/mm^2 for f_{ck} = 20 N/mm^2 (refer table 20)
- Comparisons

(a) $\tau_v > \tau_c$ — Hence shear reinforcement is required

(i) $V_{uc} = \tau_c bd = 0.71 \times 300 \times 450 = 95.85$ kN

(ii) $V_{us} = V_u - V_{uc} = 163.25 - 95.85 = 67.4$ kN

Spacing of stirrups should be least of the following three

1. $S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$, A_{sv} = 100.53 mm^2 for 2L–8mm ϕ bars

$= \frac{0.87 \times 415 \times 100.53 \times 450}{67.4 \times 10^3} = 242.33\,\text{mm}$

2. $S_v \ngtr 0.75\,d$, $\therefore S_v = 0.75 \times 450 = 337.5$ mm
3. $S_v \ngtr 300\,d$, $\therefore S_v$ = 300mm

say S_v = 225mm

$\therefore$ Provide 2L-8mm ϕ bars @ 225 mm c/c

(b) $\tau_{c_{max}} > \tau_v$, Design is safe, (ok)

Step 7 : Check for deflection control

(a) Note down the percentage of steel

$p_t = 1.45\%$ —— Tension steel percentage

$$p_c = \frac{100 A_{sc_{prov}}}{bd} = \frac{100 \times 942.45}{300 \times 450} = 0.69\%$$

(b) Stress in steel

$$f_s = 0.58 f_y \left[\frac{A_{st_{req}}}{A_{st_{prov}}} \right] = 0.58 \times 415 \times \frac{1953.90}{1963.48} = 239.52 \text{ N/mm}^2$$

read out the modification factor (k_t), using p_t and f_s

$\therefore k_t = 0.9$

Similarly readout the modification factor (k_c), using p_c (Fig. 5 of IS : 456-2000)

1. $\left(\frac{L}{d}\right)_{\text{max.}} = 20 \times k_t \times k_c \times k_f$ $\therefore k_c = 1.19,\ k_f = 1$

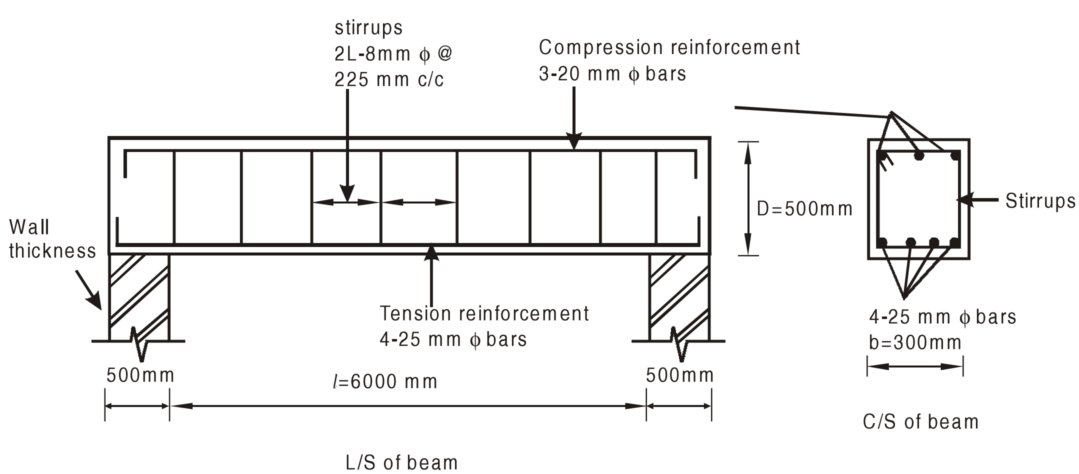
$= 20 \times 0.9 \times 1.19 = 21.42$

2. $\left(\frac{L}{d}\right)_{\text{max.}} = \frac{6450}{450} = 14.33$

$\therefore \left(\frac{L}{d}\right)_{\text{max.}} > \left(\frac{L}{d}\right)_{provided}$

Hence deflection control is satisfied (ok)

Step 8 : Reinforcement details



3. A cantilever beam of 4m, Effective Span carries a service live load of 40 kN/m. The width of the beam is 230 mm. Design the beam for flexure and shear. Sketch the details of reinforcement, use M20 and Fe 415 steel.

Solution :

Given : Effective span, $L = 4$m,

Service load = 40 kN/m

width of beam, $b = 230$ mm

$f_{ck} = 20$ N/mm^2 and $f_y = 415$ N/mm^2

Step : 1 Selection of cross - sectional dimensions

(a) Effective depth of beam (d)

$$d = \frac{\text{Span length}}{7} = \frac{L}{7} = \frac{4000}{7} = 571.42 \text{ mm, say } 575 \text{ mm}$$

(b) Overall depth of beam (D)

$D = d + d^1 = 575 + 50 = 625$mm (Assume $d^1 = 50$ mm)

Step : 2 Load Calculation

Self weight of beam = $b \times D \times$ RCC density

= $0.23 \times 0.625 \times 25$

= 3.59 kN/m

Live load (Given) = 40.0 kN/m

Total Load, W = 43.59 kN/m

Factored load, W_u = $1.5 \times W = 1.5 \times 43.59 = 65.38$ kN/m

Step : 3 Calculation of Maximum Bending Moment

$$M_u = \frac{W_u L^2}{2} = \frac{65.38 \times 4^2}{2} = 523.04 \text{ kN.m or } 523.04 \times 10^6 \text{ N.mm}$$

$$M_{u_{\lim}} = 0.138 f_{ck} bd^2 = 0.138 \times 20 \times 230 \times 575^2 = 209.88 \text{ kN.m}$$

$M_u > M_{u_{\lim}}$ — section is over reinforced

∴ The beam is designed as doubly reinforced beam section

Step : 4 Calculation of Main reinforcement

(1) Area of compression reinforcement (A_{sc})

Refer IS : 456 - 2000, G-1.2

$$M_u - M_{u_{\lim}} = f_{sc} A_{sc} (d - d^1)$$

$$\therefore \quad A_{sc} = \frac{M_u - M_{u_{\lim}}}{f_{sc}(d - d^1)}$$

$$\text{Now, } \varepsilon_{sc} = \frac{0.0035\left(x_{u_{max}} - d'\right)}{x_{u_{max}}} \quad \text{— from G-1.2}$$

$$\frac{x_{u_{max}}}{d} = 0.48 \text{ for Fe 415 steel} \qquad x_{u_{lim}} \text{ or } x_{u_{max}}$$

$$\therefore\ x_{u_{max}} = 0.48\,d = 0.48 \times 575 = 276 \text{ mm}$$

$$\Rightarrow\ \varepsilon_{sc} = \frac{0.0035(276-50)}{276} = 0.00286$$

Refer IS : 456 - 2000, Fig 23 A

$$\therefore\ f_{sc} = 0.84 f_y = 0.84 \times 415 = 348.6 \text{ N/mm}^2$$

$$A_{sc} = \frac{\left(523.04\times10^6 - 209.88\times10^6\right)}{348.6\times(575-50)} = 1711.11 \text{ mm}^2$$

Assume 25 mm ϕ bars

$$\therefore\ \text{Area of one bar, } a_{sc} = \frac{\pi}{4}\times 25^2 = 490.87 \text{ mm}^2$$

$$\therefore\ \text{No. of bars} = \frac{A_{sc}}{a_{sc}} = \frac{1711.11}{490.87} = 3.48, \text{ say } 4$$

$$A_{sc_{provided}} = 4 \times 490.87 = 1963.48 \text{ mm}^2$$

Provide 4-25mm ϕ bars at compression reinforcement (Bottom)

(2) Area of tension reinforcement (A_{st})

Refer IS : 456 - 2000, G-1.1 (a)

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd} \qquad \text{Here } x_u = x_{u_{lim}},\ A_{st} = A_{st_1},$$

$$\therefore\ A_{st_1} = \frac{0.36 f_{ck} b x_{u_{lim}}}{0.87 f_y} = \frac{0.36\times20\times230\times276}{0.87\times415} = 1265.90 \text{ mm}^2$$

Now refer, G - 1.2

$$A_{st_2} = \frac{A_{sc} f_{sc}}{0.87 f_y} = \frac{1711.11\times348.6}{0.87\times415} = 1652.10 \text{ mm}^2$$

$\therefore$ Total tension reinforcement, $A_{st} = A_{st_1} + A_{st_2} = 1265.90 + 1652.10$

$$A_{st} = 2918 \text{ mm}^2$$

Assume 25 mm ϕ bars

$$\text{No. of bars} = \frac{A_{st}}{a_{st}} = \frac{2918}{490.87} = 5.94, \text{ say } 6$$

$$A_{st_{provided}} = 2945.22 \text{ mm}^2$$

Provide 6 – 25 mm ϕ bars at Tension reinforcement (Top)

[out of 6 bars, 3 bars in first layer and another 3 bars in second layer]

Step : 5 Design of shear reinforcement

- Design of shear force, $V_u = W_u L = 65.38 \times 4 = 261.52$ kN
- Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{261.52 \times 10^3}{230 \times 575} = 1.97$ N/mm^2
- Percentage of tension steel reinforcement, $p_t = \frac{100 A_{st_{prov}}}{bd} = \frac{100 \times 2945.22}{230 \times 575}$

$$\therefore p_t = 2.22 \%$$

- Calculation of τ_c, Refer Table No.19 of IS : 456 - 2000, for $p_t = 2.22\%$ and $f_{ck} = 20$ N/mm^2

$$\therefore \tau_c = 0.80 \text{ N/mm}^2 \quad \text{(By interpolation)}$$

- Design shear strength, $\tau_{c_{max}} = 2.8$N/mm^2 [From Table No. 20]
- Comparisons

(a) $\tau_v > \tau_c$ —— Hence shear reinforcement is required

(i) Calculate shear carried by concrete

$V_{uc} = \tau_c bd = 0.80 \times 230 \times 575 = 105.8$ kN

(ii) Calculate shear carried by stirrups

$V_{us} = V_u - V_{uc} = 261.52 - 105.8 = 155.72$ kN

spacing of stirrups should be least of the following three

1. $S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$, $A_{sv} = 100.53$ mm^2 for 2L-8mm ϕ vertical stirrups

$$S_v = \frac{0.87 \times 415 \times 100.53 \times 575}{155.72 \times 10^3} = 134.02 \text{ mm}$$

2. $S_v \ngtr 0.75\, d$, $\therefore S_v = 0.75 \times 575 = 431.25$ mm
3. $S_v \ngtr 300$ mm $\therefore S_v = 300$ mm

say $S_v = 125$ mm

$\therefore$ Provide 2L-8mm ϕ bars @ 125 mm c/c

(b) $\tau_{c_{max}} > \tau_v$ $\therefore$ Design is safe, (ok)

Step : 6 Check for deflection control

(a) Note down the percentage of steel provided

$p_t = 2.22\%$ —— Tension steel percentage

$$p_c = \frac{100 A_{sc_{prov}}}{bd} = \frac{100 \times 1963.48}{230 \times 575} = 1.48\% \text{ —— Compression steel percentage}$$

(b) Stress in steel, Refer IS : 456-2000, Fig 4

$$f_s = 0.58 f_y \left[\frac{A_{st_{req}}}{A_{st_{prov}}}\right] = 0.58 \times 415 \times \frac{2918}{2945.22} = 238.47 \text{ N/mm}^2$$

read out the modification factor (k_t), using p_t and f_s

$$\therefore k_t = 0.8$$

Similarly, read out the modification factor (k_c) using p_c (From Fig. 5)

$$\therefore k_c = 1.33$$

1. $$\left(\frac{L}{d}\right)_{max} = 7 \times k_t \times k_c \times k_f \qquad k_f = 1$$

$$= 7 \times 0.8 \times 1.33 \times 1$$

$$= 7.45$$

2. $$\left(\frac{L}{d}\right)_{provided} = \frac{4000}{575} = 6.95$$

$$\therefore \left(\frac{L}{d}\right)_{max} > \left(\frac{L}{d}\right)_{provided}$$

Hence deflection control is satisfied, (ok)

Step : 7 Development length or Anchorage length at support

Refer IS : 456 - 2000, Clause 26.2.1

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}$$

$$\tau_{bd} = 1.2 \text{ N/mm}^2 \text{ —— From clause 26.2.1.1}$$

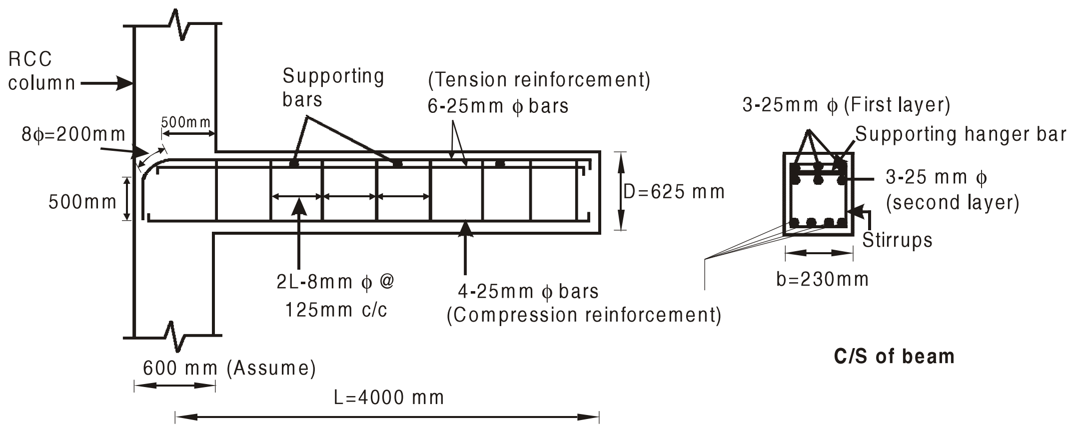
For deformed bar (Fe 415) increased by 60%

$$\text{i.e., } \tau_{bd} = 1.2 \times 1.6$$

$$\therefore L_d = \frac{0.87 \times 415 \times 25}{4 \times 1.2 \times 1.6} = 1175.29 \text{ mm, say 1200 mm}$$

The Main tension reinforcement is extended into the column to a length of 500mm and bent down at 90° and extended upto 8 ϕ and 500mm

$\therefore L_d = 500 + 8\,\phi + 500 = 500 + 8 \times 25 + 500 = 1200$mm



C/S of beam

L/S of beam

3.6 DESIGN STEPS — FLANGED BEAMS (T-BEAMS) - SIMPLY SUPPORTED BEAM

Refer IS : 456 - 2000, clause 23.1

1. **Selection of cross sectional dimensions**

(a) Effective depth of beam (d) } same as rectangular beam
(b) Overall depth of beam (D) }

(c) Width of web (b_w)

Nominal width of T-beams varies from 150 to 400 mm (Generally 300 mm)

(d) Flange thickness (D_f)

The flange thickness is generally the same as the thickness of the slab between the ribs or webs. The slab thickness depends upon the spacing of ribs, type of loading and $\frac{\text{Span}}{\text{depth ratio}}$

Generally the thickness of the slab varies from a minimum of 100mm to a maximum of 250 mm

(e) Effective width of flange (b_f)

The effective width of flange should, in no case be greater than the breadth or width of the web (b_w) plus half the sum of the clear distance to the adjacent beams on either side.

Refer IS : 456 - 2000, clause 23.1.2 (a)

For T-beams, $b_f = \left(\frac{l_0}{6}\right) + b_w + 6D_f$

Where l_0 = distance between points of Zero moments in the beam

Effective span (l_0) = should be least of the following two

(i) l_0 = clear span + bearing

(ii) l_0 = clear span + effective depth

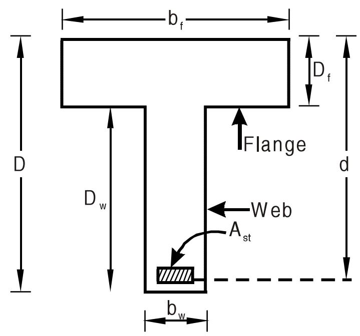


Fig : 3.4 T-beam (C/S)

2. Load Calculation

1. Self weight of beam = $(b_f \times D_f + b_w \times D_w) \times$ RCC density

= ______ kN/m

2. Live load (Given) on slab= ______ kN/m

Total load, W = ______ kN/m

Factored load, W_u = $1.5 \times W$ = ______ kN/m

3. Calculation of maximum bending moment

$$M_u = \frac{W_u l_0^2}{8}$$

4. Calculation of moment capacity of flange

Assume Neutral axis lies within the flange, $x_u \leq D_f$

$$M_{uf} = 0.36 f_{ck} b_f D_f (d - 0.42 D_f)$$

Since $M_u < M_{uf}$, $D_f > x_u$

Neutral axis lies within the flange. In this case, the beam can be treated as a normal rectangular beam of width $b_f = b$ and depth d

Hence singly reinforced section is to be designed

5. Calculation of Main reinforcement

a. If $M_u < M_{uf}$ – section is to be designed as singly reinforced.

b. If $M_u > M_{uf}$ – section is to be designed as doubly reinforced.

Procedure is same as rectangular beam procedure for calculation of A_{st}, Number of bars and $A_{st_{provided}}$

6. Design of Shear reinforcement

Here use b_w instead of b

$$\text{ex : } \tau_v = \frac{V_u}{b_w d}$$

Procedure is same as rectangular beam procedure

7. Check for deflection control

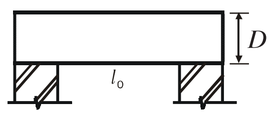
1. $\left(\frac{L}{d}\right)_{\text{max.}} = 20 \times k_t \times k_c \times k_f$

Here K_f is calculated using IS : 456 - 2000, Fig. 6

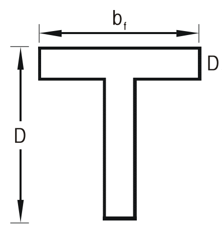
Remaining procedure is same

8. Reinforcement details

1. Longitudinal section



2. Cross section



3.7 WORKED EXAMPLES ON DESIGN OF T - BEAMS (SIMPLY SUPPORTED BEAM)

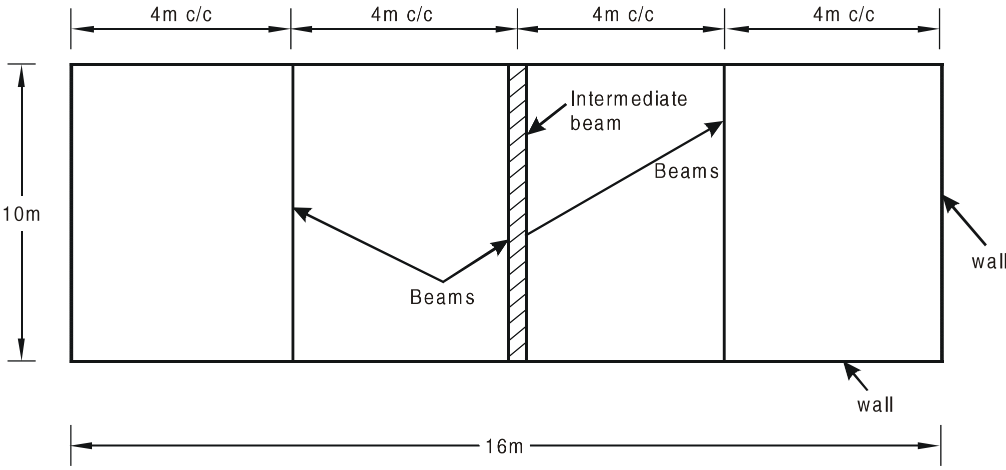
1. A floor of a hall measures 16m × 10m to the faces of the supporting walls. The floor consists of three beams spaced at 4m c/c and the slab thickness is 150 mm. The floor carries a live lead of 4 kN/m². Design the intermediate T - beam. Use M20 concrete and *Fe* 415 steel. The support width may be assumed as 300mm.

Solution:

Given : l = 10m, support width = 300mm, LL = 4kN/m^2

Slab depth = D_f = 150mm, Beam spaced = 4m c/c, f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2

Hall measures = 16m × 6m



Step : 1 Selection of cross-sectional dimensions

(a) Effective depth of beam (d)

$$d = \frac{\text{span length}}{15} = \frac{10000}{15} = 666.67\text{, Say 700mm}$$

(b) Overall depth of beam (D)

$$D = d + d' = 700 + 50 = 750\text{mm (Assume } d' = 50\text{mm)}$$

(c) Width of web (b_w) : Assume b_w = 300mm

(d) Flange thickness (D_f) : D_f = 150mm (Given)

(e) Effective width of flange (b_f)

Effective span (l_0) : should be least of the following two

(i) l_0= Clear span + bearing = 10000 + 300 = 10,300mm or 10.3m

(ii) l_0 = clear span + Effective depth (d) = 1000 + 700 = 10,700mm or 10.7m

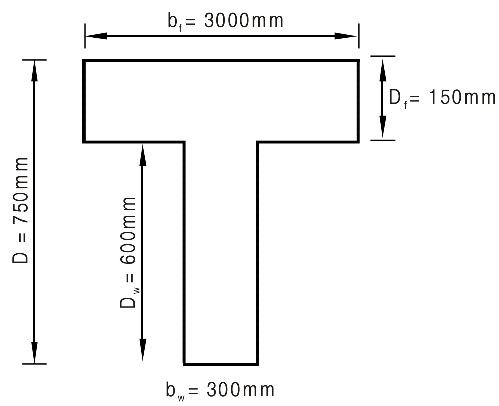
Say l_0 = 10.3 m

Refer IS : 456-2000, Clause 23.1.2 (a)

For T - beam,

$$b_f = \left(\frac{l_0}{6}\right) + b_w + 6D_f = \left(\frac{10300}{6}\right) + 300 + 6 \times 150 = 2916.66\text{ mm}$$

$\therefore$ Say b_f = 3000mm or 3m



Step : 2 Load Calculation

1. Self weight of beam $= [(b_f \times D_f) + (b_w \times D_w)] \times$RCC density
   $= (3 \times 0.15 + 0.3 \times 0.6) \times 25$
   $= 15.75$ kN/m
2. Live load (Given) $= 4 \times 1 = 4$ kN/m

   Total load, $W = 19.75$ kN/m

   $\therefore$ Factored load, $W_u = 1.5 \times W = 1.5 \times 19.75$

   $W_u = 29.62$ kN/m

Live load on slab = $4\ \text{kN}/\text{m}^2$
Consider
Per meter width
$\therefore$ LL $= 4 \times 1 = 4\text{kN}/\text{m}^2$

Step : 3 Calculation of Maximum BM

$$M_u = \frac{W_u l_0^2}{8} = \frac{29.62 \times 10.3^2}{8} = 392.79 \text{ kN.m}$$

Step : 4 Calculation of moment capacity of flange

$$M_{uf} = 0.36 f_{ck} b_f x_u (d - 0.42\, x_u)$$

Assuming N – A (Neutral Axis) lies with in the flange

$$\therefore x_u \le D_f$$

Consider $x_u = D_f$

Here $D_f = 150$mm

$$\therefore M_{uf} = 0.36 f_{ck} b_f D_f (d - 0.42\, D_f)$$

$$= 0.36 \times 20 \times 3000 \times 150\,(700 - 0.42 \times 150)$$

$$M_{uf} = 2063.88 \text{ kN.m}$$

Since $M_{uf} > M_u$

$$\Rightarrow D_f > x_u$$

Hence Neutral axis lies with in the flange. In this case the beam can be treated as a normal rectangular beam of width $b_f = b$ and depth d

Hence singly reinforced section is to be designed

Step : 5 Calculation of Main reinforcement

$$d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck} b_f}} = \sqrt{\frac{392.79\times10^6}{0.138\times20\times3000}} = 217.80\text{mm} < d_{provided}\ (700\text{mm})$$

Hence design is safe

$$M_u = 0.87 f_y A_{st}\, d \left[1-\left(\frac{f_y A_{st}}{f_{ck} b_f d}\right)\right]$$

$$392.79 \times 10^6 = 0.87 \times 415 \times 700\, A_{st} \left[1-\left(\frac{415 A_{st}}{20\times3000\times700}\right)\right]$$

$$\therefore\ 2.49A_{st}^2 - 252735\ A_{st} + 392.79 \times 10^6 = 0$$

$$\therefore\ A_{st} = 1578.71\ \text{mm}^2$$

Assume 25mm ϕ bars

$$\therefore\ \text{No. of bars} = \frac{A_{st}}{a_{st}} = \frac{1578.71}{\frac{\pi}{4}\times25^2} = 3.21,\ \text{say } 4$$

$$A_{st_{provided}} = 4\times\frac{\pi}{4}\times25^2 = 1963.49\text{mm}^2$$

∴ Provide 4-25 mm ϕ bars at tension side and 2-12mm ϕ bars at compression side (Anchor bars) - [Assume]

Step : 6 Design of Shear reinforcement

- Design of shear force, $V_u = \frac{W_u l_o}{2} = \frac{29.62\times10.3}{2} = 152.54$ kN
- Nominal shear stress, $\tau_v = \frac{V_u}{b_w d} = \frac{152.54\times10^3}{300\times700} = 0.72$ N/mm^2
- Percentage of tension steel reinforcement, $p_t = \frac{100 A_{st_{provided}}}{b_w d} = \frac{100\times1963.49}{300\times700} = 0.93\%$
- Calculation of τ_c, Refer table 19 of IS : 456-2000
  for $p_t = 0.93\%$ and $f_{ck} = 20$ N/mm^2
  ∴ $\tau_c = 0.60$ N/mm^2 (By interpolation)

- Design shear strength, $\tau_{c_{max}}$ = 2.8 N/mm^2 (From table 20)
- Comparisons

  $\tau_v > \tau_c$ – Hence shear reinforcement is required

(i) Calculate shear carried by concrete

$V_{uc} = \tau_c b_w d = 0.6 \times 300 \times 700 = 126$ kN

(ii) Calculate shear carried by stirrups

$V_{us} = V_u - V_{uc} = 152.54 - 126 = 26.54$ kN

Spacing of stirrups should be least of the following three

1. $$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

$$A_{sv} = 100.53 \text{ mm}^2 \text{ for 2L – 8mm } \phi \text{ vertical stirrups}$$

$$\therefore S_v = \frac{0.87 \times 415 \times 100.53 \times 700}{26.54 \times 10^3} = 957.32\text{mm}$$

2. $S_v \not> 0.75d$, $\therefore S_v = 0.75 \times 700 = 525$mm

3. $S_v \not> 300$mm, $\therefore S_v = 300$mm

Say, $S_v = 300$mm

Hence provide 2L – 8mm ϕ bars @ 300 mm c/c

Step : 7 Check for deflection control

(a) Note down the percentage of steel provided

$$p_t = 0.93\%$$

(b) Stress in steel, Refer IS: 456-2000, Fig. 4

$$f_s = 0.58 f_y \left[\frac{A_{st_{req}}}{A_{st_{pro}}}\right] = 0.58 \times 415 \times \frac{1578.71}{1963.49} = 193.53 \text{ N/mm}^2$$

read out the modification factor (k_t) using p_t and f_s

$$\therefore k_t = 1.22$$

Now, ratio $\frac{b_w}{b_f} = \frac{300}{3000} = 0.1$

read out the reduction factor (k_f) using $\frac{b_w}{b_f}$ referring IS : 456–2000, Fig. 6.

$$\therefore k_f = 0.8$$

(1) $$\left(\frac{L}{d}\right)_{max.} = 20 \times k_t \times k_c \times k_f$$ $k_c = 1$ $\because$ No compression reinforcement

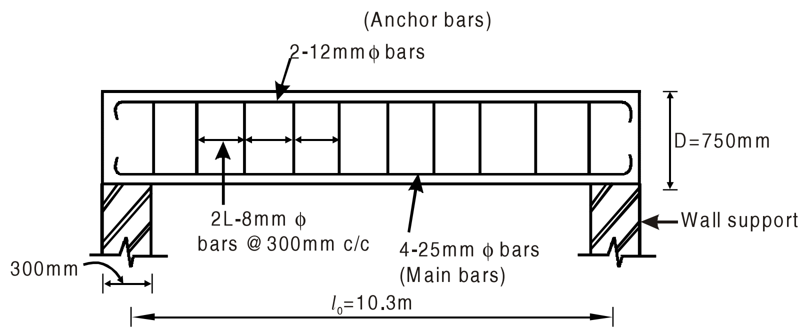
$$= 20 \times 1.22 \times 1 \times 0.8 = 19.52$$

(2) $\left(\frac{L}{d}\right)_{provided} = \frac{10300}{700} = 14.71$

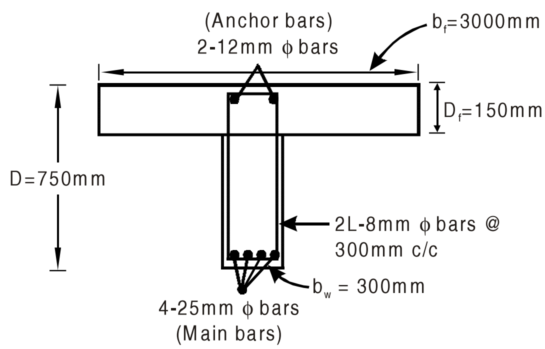
$\therefore \left(\frac{L}{d}\right)_{max.} > \left(\frac{L}{d}\right)_{provided}$

Hence deflection control is satisfied

Step : 8 Reinforcement details



L/S of T-beam



C/S of T-beam

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2. Design the T-beam as per IS:456-2000. The beam is subjected to an ultimate moment of 400kN-m. Use M20 concrete and Fe 415 steel. Following are the parameters which are used for design.

b_f = 800 mm, b_w = 200 mm, D_f = 100mm, d = 400 mm

Solution:

Given : M_u = 400 kN-m, f_{ck} = 20 N/mm^2, f_y = 415 N/mm^2, b_f = 800 mm, b_w = 200 mm, D_f = 100mm and d = 400mm

Step : 1 Calculate limiting depth of neutral axis

$$\frac{x_{u\,max}}{d} = 0.48 \quad \Rightarrow x_{umax} = 0.48 \times 400 = 192\text{mm}$$

Step : 2 Calculate moment capacity of flange

$$M_{uf} = 0.36 f_{ck} b_f x_u \ (d – 0.42 x_u)$$

Assuming N – A (Neutral Axis) lies with in the flange

$$\therefore x_u \le D_f$$

$$\text{Here } D_f = 100\text{mm}$$

$$\Rightarrow M_{uf} = 0.36 f_{ck} b_f D_f (d - 0.42\, D_f)$$

$$= 0.36 \times 20 \times 800 \times 100\ (400 - 0.42 \times 100)$$

$$M_{uf} = 206.20 \text{ kN.m}$$

$$\text{Since } M_{uf} < M_u$$

$$\Rightarrow D_f \not> x_u$$

$$\text{i.e., } D_f < x_u \quad \text{(Hence Assumption is wrong)}$$

$\therefore$ N-A lies outside the flange.

Step : 3 Calculate limiting moment of resistance

$$M_u = 0.36 \frac{x_{u,\max}}{d}\left(1 - 0.42 \frac{x_{u,\max}}{d}\right) f_{ck} b_w d^2 + 0.45 f_{ck}\left(b_f - b_w\right) D_f \left(d - \frac{D_f}{2}\right)$$

$$= 0.36 \times 0.48\ (1 - 0.42 \times 0.48) \times 20 \times 200 \times 400^2 + 0.45 \times 20\ (800 - 200)\ 100\ (400 - \frac{100}{2})$$

$$M_{u_{\lim}} = 277.29 \times 10^6 \text{ N-mm} \quad \text{or} \quad 277.29 \text{ kN-m}$$

Step : 4 Calculation of Area of steel reinforcement

(1) Area of compression steel (A_{sc})

$$\varepsilon_{sc} = \frac{0.0035\left(x_{u\max} - d'\right)}{x_{u\max}} \qquad \text{| Assume } d' = 50\text{mm}$$

$$= \frac{0.0035(192 - 50)}{192} = 2.58 \times 10^{-3}$$

Now using ε_{sc}, note down the value of f_{sc} from IS:456-2000, Fig. 23A

$$f_{sc} = 0.84 f_y = 0.84 \times 415 = 348.6 \text{ N/mm}^2$$

$$\text{Now, } A_{sc} = \frac{M_u - M_{u_{\lim}}}{f_{sc}\left(d - d'\right)} = \frac{400 \times 10^6 - 277.29 \times 10^6}{348.6\,(400 - 50)} = 1005.73 \text{ mm}^2$$

Now, Assume 20mm dia bars

$$\therefore \text{No. of bars} = \frac{A_{sc}}{a_{sc}} = \frac{1005.73}{\frac{\pi}{4} \times 20^2} = 3.20 \text{ say } 4$$

Provide 4-20mm ϕ bar @ compression side (Top)

(2) Area of tension steel (A_{st})

Area of steel corresponding to balanced section

$$M_{u_{\lim}} = 0.87 f_y A_{st_1} (d - 0.42\ x_{u_{\lim}})$$

$$277.29 \times 10^6 = 0.87 \times 415\ A_{st_1} (400 - 0.42 \times 192)$$

$$\therefore\ A_{st_1} = 2404.84 \text{ mm}^2$$

$$A_{st_2} = \frac{f_{sc} A_{sc}}{0.87 f_y} = \frac{348.6 \times 1005.73}{0.87 \times 415} = 971.04 \text{mm}^2$$

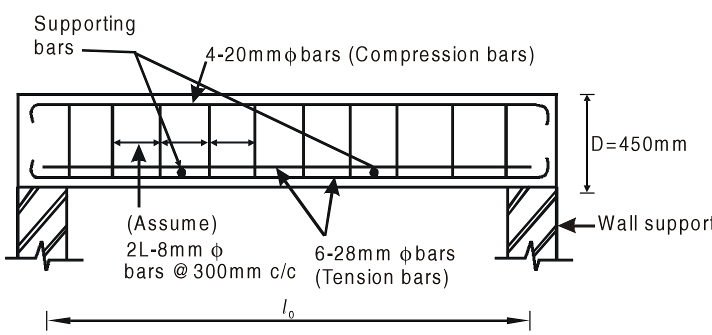
$$\therefore \text{ Total } A_{st} = A_{st_1} + A_{st_2} = 2404.84 + 971.04 = 3375.88 \text{ mm}$$

Assume 28mm ϕ bars

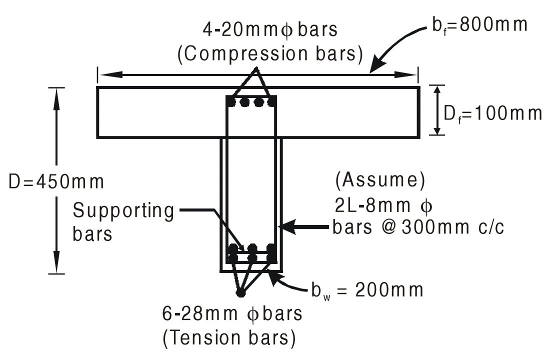
$$\therefore \text{ No. of bars } = \frac{A_{st}}{a_{st}} = \frac{3375.88}{\frac{\pi}{4} \times 28^2} = 5.48 \text{ say } 6$$

$\therefore$ Provide 6-28mm ϕ bars @ Tension side (Bottom)

Step : 5 Reinforcement details



L/S of T-beam



C/S of T-beam

3.8 DESIGN STEPS - FLANGED BEAMS (L - BEAMS) - SIMPLY SUPPORTED BEAM

Refer IS : 456 - 2000, clause 23.1

1. Selection of cross-sectional dimensions

(a) Effective depth of beam (d)
(b) Overall depth of beam (D)
(c) Width of web (b_w)
(d) Flange thickness (D_f)

} Procedure is same as T - beams

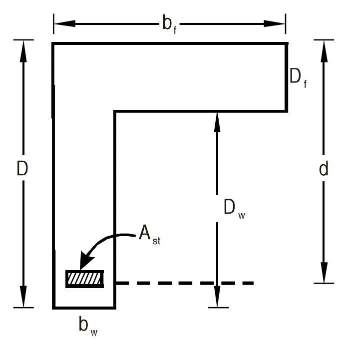
(e) Effective width of flange (b_f), Refer clause 23.1.2 (b)

$$\text{For L- beams, } b_f = \left(\frac{l_0}{12}\right) + b_w + 3D_f$$

Effective Span (l_0) = should be least of the following two

(i) l_0 = clear span + bearing

(ii) l_0 = clear span + Effective depth



2. Load calculation is same as T - beams

3. Calculation of maximum bending moment =

(i) At support section, $M_u = \dfrac{W_u l_0^2}{12}$

(ii) At centre of span section, $M_u = \dfrac{W_u l_0^2}{24}$

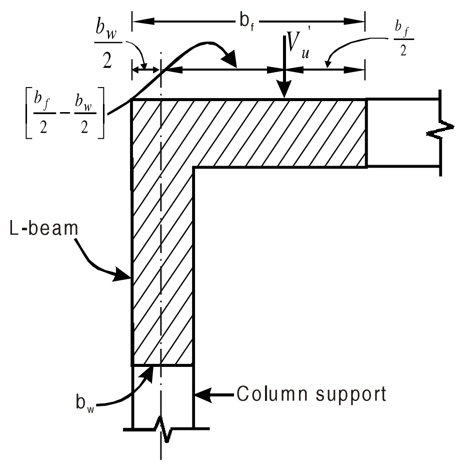
4. Calculation of Torsional moment at support section

Torsional moment is produced due to dead load of slab and live load on it

Load = Factored load – [Self weight of web × Rcc density] = ———— kN/m

∴ Total ultimate load on slab = Load × Effective span (l_o) = ———— kN

$$\text{Total ultimate shear force, } V_u' = \frac{\text{Total ultimate load on slab}}{2}$$



$\therefore$ Ultimate Torsional moment, $T_u = V_u' \times \left[\frac{b_f}{2} - \frac{b_w}{2}\right]$

5. Calculation of Equivalent Bending moment and shear force

Refer IS : 456-2000, Clause 41.4.2

- Equivalent Bending moment

$$M_{e_1} = M_u + M_t$$

Where, M_u = BM at the cross section

$$M_t = T_u \left(\frac{1+\frac{D}{b}}{1.7}\right)$$

Find M_{e_1}

- Equivalent Shear Force

Refer clause 41.3.1

$$V_e = V_u + 1.6\left(\frac{T_u}{b}\right)$$

Where V_u = Shear force at support section

i.e., $\therefore V_u = \frac{W_u l_0}{2}$

6. Calculation of Main longitudinal reinforcement

$$M_{u_{\lim}} = 0.138\, f_{ck} b_w d^{\,2}$$

If $M_{e_1} < M_{u_{\lim}}$ – section is under reinforced

(a) At support section

$$\therefore\ M_{e_1} = 0.87 f_y A_{st} d\left[1-\left(\frac{A_{st} f_y}{f_{ck} bd}\right)\right]$$

Find A_{st}, No. of bars and $A_{st_{provided}}$

(b) At centre of span section

using 'M_u', Find A_{st}, No. of bars and $A_{st_{provided}}$

7. Calculation of side face reinforcement

Refer clause 26.5.1.7

Where the depth of the web in a beam exceeds 450 mm side face reinforcement shall be provided along the two faces. The total area of such reinforcement shall be not less than 0.1 percent of the web area.

i.e. $A_{st} = \frac{0.1}{100} \times b_w \times D$

Find No. of bars

8. Design of shear reinforcement or Transverse reinforcement

Refer clause 41.3.1

- Equivalent Nominal shear stress, $\tau_{ve} = \frac{V_e}{b_w d}$
- Percentage of steel, $p_t = \frac{100 A_{st_{proivided}}}{b_w d}$
- Calculation of τ_c

  read out the value of τ_c using p_t and f_{ck} from table 19 of IS : 456-2000
- Design shear strength, $\tau_{c_{\max}}$ using f_{ck}, table 20 of IS : 456-2000
- Comparisons

(a) If $\tau_{ve} < \tau_c$ - Shear reinforcement is not required. Hence provide minimum shear reinforcement as per clause 26.5.1.6

(b) If $\tau_{ve} > \tau_e$ - shear reinforcement is required

$\therefore$ Both longitudinal and transverse reinforcement shall be provided in accordance with clause 41.4

(c) If $\tau_{ve} < \tau_{c_{\max}}$ - Design is safe, (OK)

(d) If $\tau_{ve} > \tau_{c_{\max}}$ - Design is not safe, Hence revise the section

- If Shear reinforcement is required, then refer clause 41.4.3

  Spacing of stirrups shall not exceeds the least of

1. $A_{sv} = \dfrac{T_u S_v}{b_1 d_1 (0.87 f_y)} + \dfrac{V_u S_v}{2.5 d_1 (0.87 f_y)}$, solve S_v
2. $A_{sv} = \dfrac{(\tau_{ve} - \tau_c) b . S_v}{0.87 f_y}$, solve S_v
3. $S_v = x_1$ = shorter dimension of the stirrup
4. $S_v = \dfrac{x_1 + y_1}{4}$
5. $S_v \ngtr 0.75\, d$
6. $S_v \ngtr 300$mm

Where, y_1 = Longer dimension of the stirrup

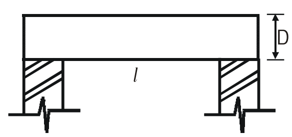
b_1 = c/c distance between corner bars in the direction of width of the beam

d_1 = c/c distance between corner bars in the direction of the depth of the beam

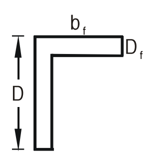
9. Check for deflection control: Same as T - beams

10. Reinforcement details

1. Longitudinal section



2. Cross - section



3.9 WORKED EXAMPLES ON DESIGN OF L - BEAMS, ANALYSIS AND DESIGN FOR COMBINED BENDING AND TORSION

1. A reinforced concrete beam of rectangular section having a width of 230mm and overall depth of 450mm is reinforced with 4-25mm ϕ bars distributed at each of the corners at an effective cover of 50mm in the direction of depth and side covers of 25mm in the direction of width. 8mm ϕ, 2 - Legged torsional strength of the section adopting Fe - 500 HYSD bars for the following cases

(a) Torsional strength, if V_u = 0

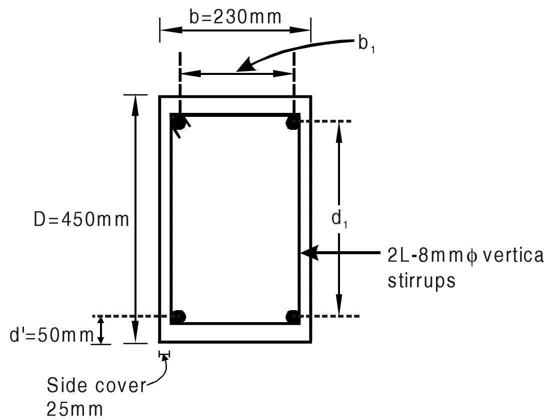
(b) Torsional strength, if V_u = 150 kN

(c) Torsional strength, if V_u = 200 kN

Solution :

Given Data

b = 230mm, D = 450mm, d^1 = 50 mm, S_v = 125mm, f_y = 500 N/mm^2



$$b_1 = b - 2\left(\text{side cover} + \text{stirrup dia} + \frac{25\text{mm}}{2}\phi\right)$$

$$= 230 - 2\left(25 + 8 + \frac{25}{2}\right) = 139\text{mm}$$

$d_1 = D - 2 \times$ Effective cover $= 450 - 2 \times 50 = 350$mm

Case (a) : Calculate Torsional strength, if $V_u = 0$

Refer IS : 456-2000, Clause 41.4.3

$$A_{sv} = \frac{T_u S_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u S_v}{2.5 d_1 (0.87 f_y)}$$

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

$$S_v = 125\text{mm c/c}$$

$$V_u = 0, f_y = 500 \text{ N/mm}^2$$

$$\therefore T_u = \frac{A_{sv} b_1 d_1 (0.87 f_y)}{S_v} = \frac{100.53 \times 139 \times 350 \times 0.87 \times 500}{125}$$

$$\therefore T_u = 17.02 \text{ kN.m or } 17.02 \times 10^6 \text{ N.mm}$$

Case (b) : Calculate Torsional strength, if V_u =150 kN

$$100.53 = \frac{T_u \times 125}{139 \times 350 \times 0.87 \times 500} + \frac{150 \times 10^3 \times 125}{2.5 \times 350 \times 0.87 \times 500}$$

$$100.53 = 5.90 \times 10^{-6} T_u + 49.26$$

$$\therefore T_u = 8.68 \text{ kN.m or } 8.68 \times 10^6 \text{ N.mm}$$

Case (c) : Calculate Torsional strength, if V_u = 200kN

$$100.53 = \frac{125\,T_u}{139 \times 350 \times 0.87 \times 500} + \frac{200 \times 10^3 \times 125}{2.5 \times 350 \times 0.87 \times 500}$$

$$100.53 = 5.90 \times 10^{-6} + 65.68$$

$$\therefore T_u = 5.90 \times 10^6 \text{ N.mm or } 5.90 \text{ kN.m}$$

Comment: As Shear force increase with decrease in Torsional strength

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2. **A RC beam of rectangular section with a width of 300mm and overall depth 600mm is reinforced with 4 bars of 25mm diameter on the tension side at an Effective depth of 550mm. The section is subjected to a unfactored *BM* of 150 kN.m. If f_{ck} = 25N/mm² and f_y = 415 N/mm², Calculate the ultimate torsional resistance that can be allowed on the section.**

Solution:

Given Data :

b = 300mm, D = 600mm, d = 550 mm, M = 150 kN.m

f_{ck} = 25N/mm² and f_y = 415N/mm²

$$A_{st} = 4 \times \frac{\pi}{4} \times 25^2 = 1963.49 \text{ mm}^2$$

$$M_u = 1.5 \times M = 1.5 \times 150 = 225 \text{ kN.m}$$

Step : 1 Depth of Neutral axis, G - 1.1 (a)

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd} = \frac{0.87 \times 415 \times 1963.49}{0.36 \times 25 \times 300 \times 550} = 0.47$$

$$\therefore x_u = 0.47 \times d = 0.47 \times 550 = 262.56 \text{ mm}$$

Step : 2 Limiting values of the depth of neutral axis

Clause 38.1,

$$\frac{x_{u_{\max}}}{d} = 0.48 \text{ —— for Fe 415 steel}$$

$$\therefore x_{u_{\max}} = 0.48 \times 550 = 264 \text{ mm}$$

$$x_{u_{\max}} > x_u$$

Hence section is under reinforced

Step : 3 Calculation of Equivalent ultimate moment capacity of section

$$M_{e_1} = 0.87 f_y A_{st} d\left[1 - \left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$$

$$= 0.87 \times 415 \times 1963.49 \times 550 \left[1 - \left(\frac{415 \times 1963.49}{25 \times 300 \times 550}\right)\right]$$

$$M_{e_1} = 312.88 \text{ kN.m}$$

Step : 4 Calculation of ultimate torsional resistance

Clause 41.4.2

$$M_{e_1} = M_u + M_t$$

$$M_t = T_u \left[\frac{1 + \left(\frac{D}{b}\right)}{1.7}\right]$$

$$312.88 = 225 + T_u \left[\frac{1 + \left(\frac{600}{300}\right)}{1.7}\right]$$

$$312.88 = 225 + 1.76\, T_u$$

$$T_u = 49.93 \text{ kN.m}$$

---

3. **A rectangular beam 230 × 550 mm deep is subjected to a sagging ultimate BM of 40kN.m, ultimate SF of 30 kN and ultimate Torsional moment of 11.5 kN.m at a given section. Design the reinforcement if M20 grade concrete and Fe 415 steel are used. Sketch the details.**

Solution:

Given, M_u = 40kN.m, V_u = 30 kN, T_u = 11.5 kN.m

f_{ck} = 20N/mm^2 , f_y = 415N/mm^2, b = 230mm, D = 550mm

Assume d^1 = 50 mm $\therefore$ $d = D - d^1$ = 550 – 50 = 500 mm

Step : 1 Calculation of Equivalent BM and limiting moment

Refer IS: 456-2000, Clause 41.4.2

$$M_{e_1} = M_u + M_t$$

$$= 40 + T_u \left[\frac{1 + \left(\frac{D}{b}\right)}{1.7}\right] = 40 + 11.5 \left[\frac{1 + \left(\frac{550}{230}\right)}{1.7}\right]$$

$$\therefore M_{e_1} = 62.94 \text{ kN.m or } 62.94 \times 10^6 \text{ N.mm}$$

$$M_{u_{\lim}} = 0.138 f_{ck} bd^2 = 0.138 \times 20 \times 230 \times 500^2 = 158.7 \text{ kN.m}$$

Since $M_{u_{\lim}} > M_u$ - Section is under reinforced

Hence the beam is designed like singly reinforced section

Since $M_u > M_t$, there is no need of compression reinforcement due to twisting moments

Step : 2 Calculation of Main longitudinal reinforcement

Now Refer IS : 456-2000, G 1.1 (b)

$$\therefore M_{e_1} = 0.87 f_y A_{st} d \left[1 - \left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$$

$$62.94 \times 10^6 = 0.87 \times 415 \times 500 A_{st} \left[1 - \left(\frac{415 A_{st}}{20 \times 230 \times 500}\right)\right]$$

$$32.57 A_{st}^2 - 180525 A_{st} + 62.94 \times 10^6 = 0$$

$$\therefore A_{st} = 373.86 \text{ mm}^2$$

Assume 12mm ϕ bars

$$\text{Area of one bar, } a_{st} = \frac{\pi}{4} \times 12^2 = 113.09 \text{ mm}^2$$

$$\text{No. of bars} = \frac{A_{st}}{a_{st}} = \frac{373.86}{113.09} = 3.30, \text{ say } 4$$

$$A_{st_{provided}} = 4 \times 113.09 = 452.36 \text{ mm}^2$$

Provide 4 – 12 mm ϕ bars at Tension face or reinforcement and

Provide 2 – 12 mm ϕ bars at Compression reinforcement (Hanger bars)

Step : 3 Calculation of side face reinforcement

As the depth of the section is more than 450mm, side face reinforcement of 0.10% of the web section is to be provided

$$\therefore A_{st} = \frac{0.1}{100} \times b \times D = \frac{0.1}{100} \times 230 \times 550 = 126.5 \text{ mm}^2$$

Assume 10 mm ϕ bars

$$\text{Area of one bar, } a_{st} = \frac{\pi}{4} \times 10^2 = 78.53 \text{ mm}^2$$

$$\text{No. of bars} = \frac{A_{st}}{a_{st}} = \frac{126.5}{78.53} = 1.61, \text{ say } 2$$

∴ Provide 2-10 mm ϕ bars on each face

Step : 4 Calculation of Equivalent shear force

Clause 41.3

Equivalent Shear, $V_e = V_u + 1.6\left(\frac{T_u}{b}\right) = 30 + 1.6 \times \left(\frac{11.5}{0.23}\right) = 110$ kN

Step : 5 Design of shear reinforcement or transverse reinforcement

Refer clause 41.3.1

- Equivalent nominal shear stess, $\tau_{ve} = \frac{V_e}{bd} = \frac{110 \times 10^3}{230 \times 500} = 0.95$ N/mm^2
- Percentage of steel, $p_t = \frac{100 A_{st_{prov}}}{bd} = \frac{100 \times 452.36}{230 \times 500} = 0.39\%$
- Calculation of τ_c
  using $p_t = 0.39\%$ and $f_{ck} = 20$ N/mm^2 from table 19 of IS: 456 -2000,
  $\tau_c = 0.42$ N/mm^2
- Design shear strength, $\tau_{c_{max}} = 2.8$ N/mm^2 from table 20
- Comparisons

(a) $\tau_{ve} > \tau_e$ – Shear reinforcement is required

Refer IS: 456-200, clause 41.4.3

Spacing of stirrups shall not exceed the least of

1. $$A_{sv} = \frac{T_u S_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u S_v}{2.5 d_1 (0.87 f_y)}$$

Assume 2L – vertical 8 mm ϕ stirrups

$$\therefore A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

Assume 25 mm clear cover allround

Where, b_1 = c/c distance between corner bars in the direction of the width of the beam

$$b_1 = b - 2 \times \text{clear cover} - 2 \times \text{stirrup } \phi - 2 \times \frac{\text{longitudinal diameter}}{2}$$

$$\therefore b_1 = 230 - 2\left[25 + 8 + \frac{12}{2}\right] = 152\text{mm}$$

d_1 = c/c distance between corner bars in the direction of the depth of the beam

$$d_1 = D - 2\left[25 + 8 + \frac{12}{2}\right] = 550 - 2[39] = 472\text{mm}$$

$$100.53 = \frac{11.5\times10^6 S_v}{152\times472\times0.87\times415} + \frac{30\times10^3 S_v}{2.5\times472\times0.87\times415}$$

$$100.53 = 0.44\, S_v + 0.07\, S_v$$

$$\therefore S_v = 197.11\text{mm}$$

2. $$A_{sv} = \frac{(\tau_{ve} - \tau_c)bS_v}{0.87 f_y}$$

$$100.53 = \frac{(0.95-0.42)\times230 S_v}{0.87\times415}$$

$$\therefore S_v = 297.75 \text{ mm}$$

3. $$S_v = x_1 = \text{shorter dimension of stirrup}$$

$$x_1 = b - 2\left[\text{clear cover } + \frac{\text{stirrup diameter}}{2}\right]$$

$$= 230 - 2\left[25 + \frac{8}{2}\right] = 172\text{mm}$$

$$\therefore S_v = 172 \text{ mm}$$

4. $$S_v = \frac{x_1 + y_1}{4}$$

$$y_1 = \text{longer dimension of stirrup}$$

$$y_1 = D - 2\left[\text{clear cover } + \frac{\text{stirrup diameter}}{2}\right]$$

$$= 550 - 2\left[25 + \frac{8}{2}\right] = 492 \text{ mm}$$

$$\therefore S_v = \frac{172+492}{4} = 166 \text{ mm}$$

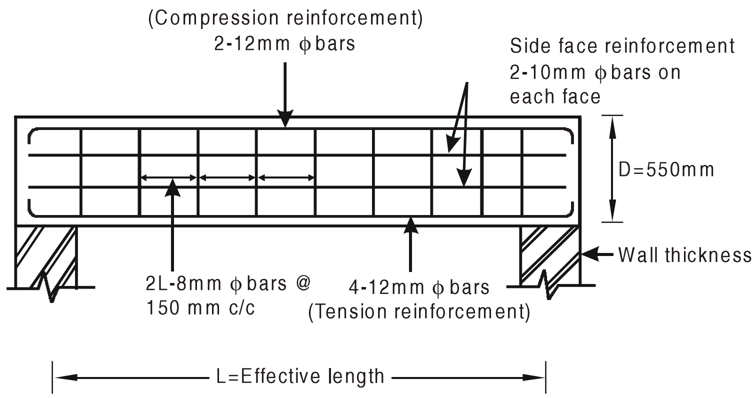
5. $S_v \not> 0.75\, d$, $\therefore S_v = 0.75 \times 500 = 375$mm

6. $S_v \not> 300$ mm $\therefore S_v = 300$mm

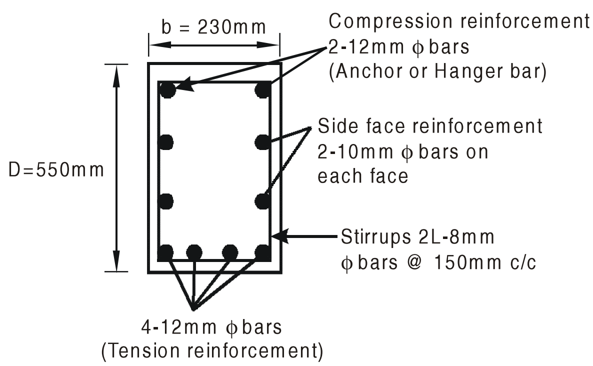
Say $S_v = 150$ mm

Provide $2L$ - 8 mm ϕ bars @ 150 mm c/c

Step : 6 Reinforcement details



L/S of beam



C/S of beam

4. Design a *L* - beam for an office floor to suit the following date
Clear span (l) = 7m, thickness of flange (D_f) = 150mm, live load = 4 kN/m²
Spacing of beams = 3m, f_{ck} = 20 N/mm², f_y = 415 N/mm² and
Width of column = 300 mm

Solution

Given: $l = 7$m, $D_f = 150$mm, $LL = 4$ kN/m^2, $f_{ck} = 20$N/mm^2, $f_y = 415$ N/mm^2,
Spacing of beams = 3m and width of column = 300mm

Step :1 Section of Cross-sectional dimensions

(a) Effective depth of beam (d)

$$d = \frac{\text{Clear span}}{12 \text{ to } 15} \text{ for span range 5 to 10m}$$