Module-4
Limit State Design of Slabs and Stairs

Introduction to one way and two way slabs, Design of cantilever, simply supported and one way continuous slab. Design of two way slabs for different boundary conditions.

Design of dog legged and open well staircases. Importance of bond, anchorage length and lap length.

INTRODUCTION

Slabs are plate element having depth much smaller than its other two dimensions. So slab is a two dimensional element. Slabs form roof or floor of the building. Slabs are designed same as beam with unit width.

A slab may be supported by beams or walls and may be used as the flange of a T-beam or L-beam. A slab may be simply supported or continuous over one or more supports.

- Slabs used in floors and roofs of buildings mostly integrated with the supporting beams, carry the distributed loads primarily by bending.
- It has been mentioned that a part of the integrated slab is considered as flange of T or L - beams because of monolithic construction.
- However, the remaining part of the slab needs design considerations. These slabs are either single span or continuous having different support conditions like fixed, hinged or free along the edges, though normally these slabs are horizontal, inclined slabs are also used in ramps, stair cases and inclined roofs.

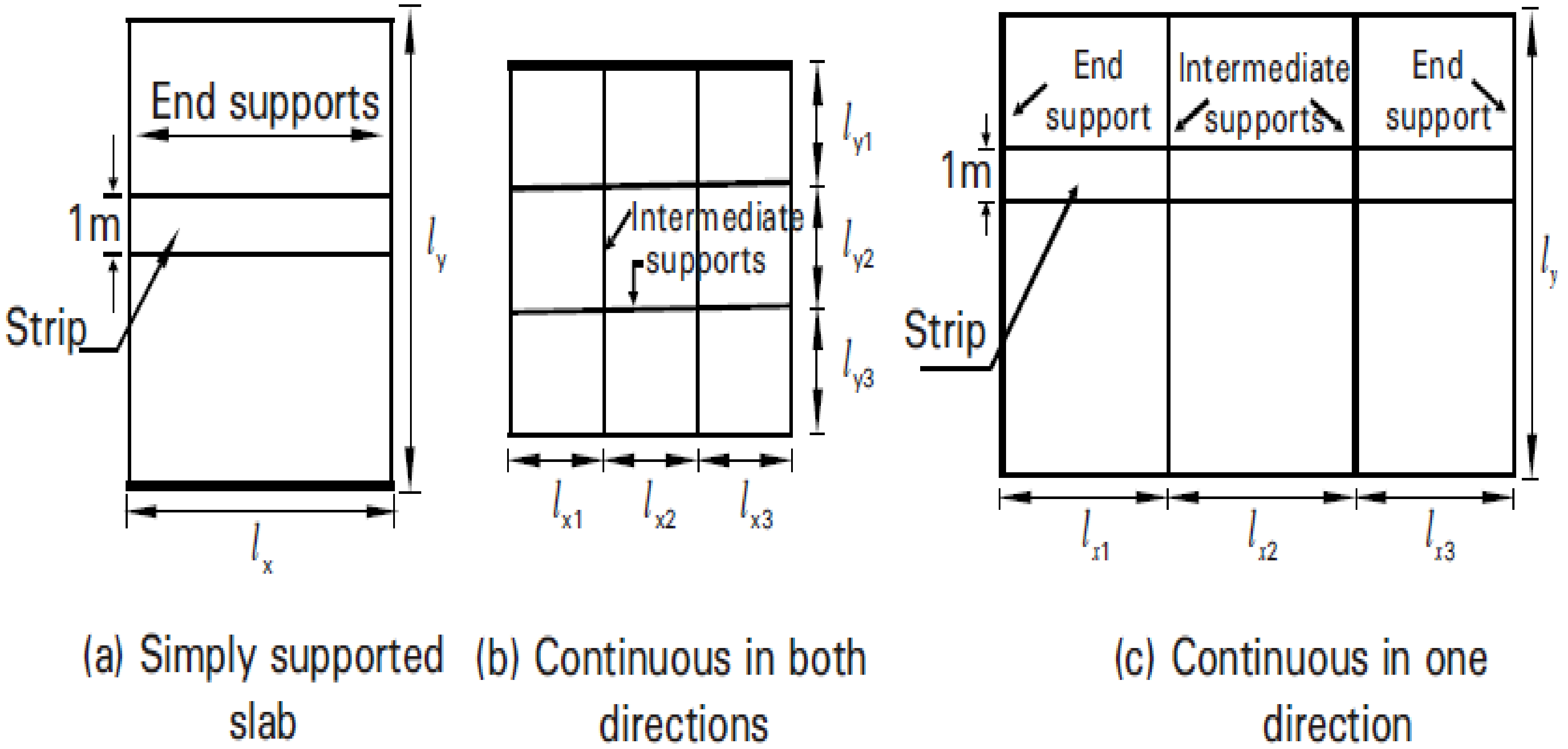


Fig. 1 : Forms of slab

Classification of slabs

1. One way Slabs spanning in one direction.
2. Two way slabs spanning in both directions.
3. Circular slabs.
4. Flat slabs.
5. Grid floor and ribbed slabs.

The difference between a beam and slab can be made as follows:

1. Slabs are analyzes and designed as having a unit width, ie., 1m wide strips.
2. Compression reinforcement is used only exceptional cases in a slab.
3. Shear stresses are usually very low and shear reinforcement is never provided in slabs.
4. Slabs are much thinner than beams.

ONE WAY SLABS

One way slabs are those slabs in which the longer span to shorter span $\frac{l_y}{l_x}$ ratio is greater than 2. This type of slab is also called as slab spanning in one direction as the bending takes place only along the shorter span. Therefore, the main reinforcement is provided along the shorter span. The one way slab is analysed by assuming it to be a beam of 1 m width.

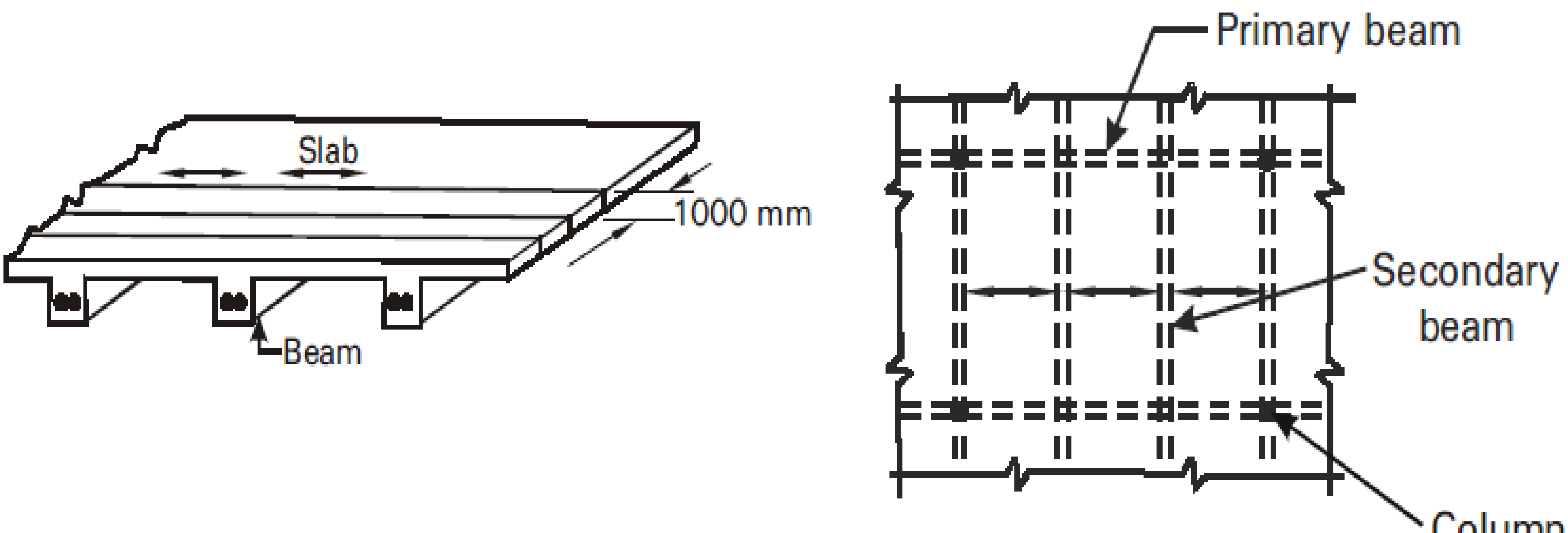


Fig. 2 : One way slabs

TWO WAY SLABS

When slabs are supported on four sides, two-way spanning action occurs. Such slabs may be simply supported or continuous on any or all sides.

If the longer span to shorter span ratio $\frac{l_y}{l_x}$ is less than 2, is called two way slab. Two way slab is also called as slab spanning in two directions because bending takes place in both directions.

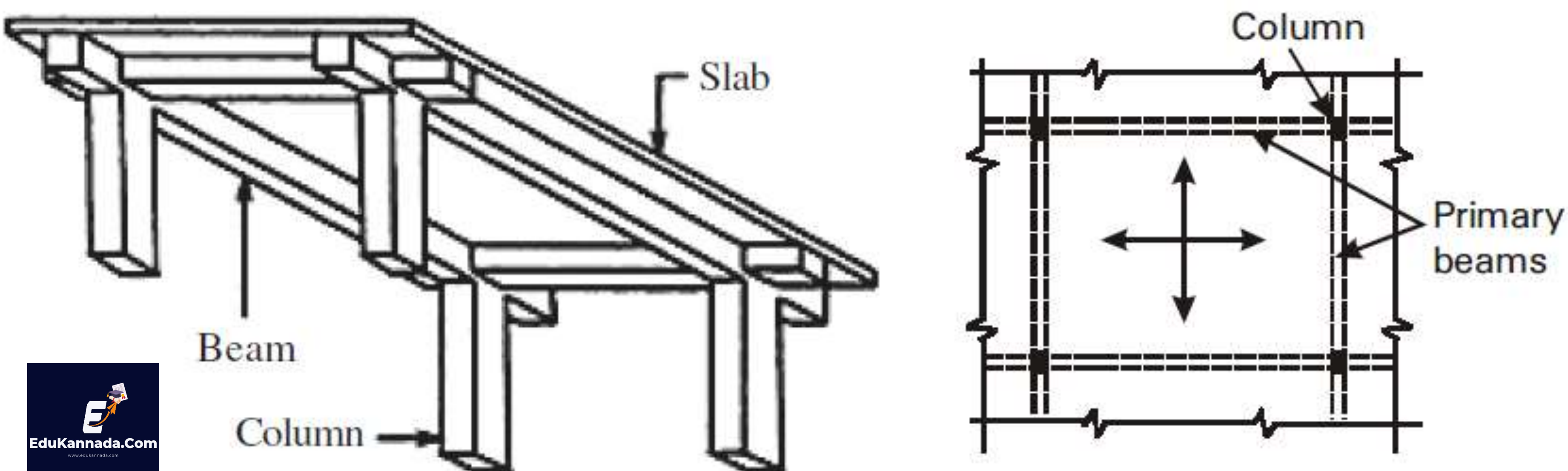


Fig. 3 : Two way slab supported on beams

Types of Two way slab

i) Restrained slabs

When a two way slab is loaded, corners get lifted up. These corners can be prevented from lifting by providing fixidity at the supports by beams or walls. Such type of slabs in which the corners are prevented from lifting are called restrained slabs. In these slabs special torsion reinforcement is provided at the edges to prevent cracking of corners. These are also called as slabs with corners held down.

(IS 456:2000, Annex D-1.0 to 1.11 & Table 26 provides details about restrained slabs and bending moment coefficients for rectangular panels supported with provision for torsion at corners)

Different types of end conditions in case of restrained slabs are:

Case - 1 : Interior Panels i.e., all the four sides are continuous.

Case - 2 : One Short Edge discontinuous i.e., other three sides are continuous.

Case - 3 : One Long Edge discontinuous i.e., other three adjacent sides are continuous.

Case - 4 : Two Adjacent Edges discontinuous i.e., other two adjacent sides are continuous.

Case - 5 : Two Short Edges discontinuous i.e., other two long edges are continuous.

Case - 6 : Two Long Edges discontinuous i.e., other two short edges are continuous.

Case - 7 : Three Edges discontinuous i.e., one long edge continuous.

Case - 8 : Three Edges discontinuous i.e., one short edge continuous.

Case - 9 : Four Edges discontinuous.

Practical illustration of support conditions

There are various cases of support condition encountered in practice and some of them are shown in Fig. (a) and the illustration how these boundary condition are encountered in floor systems are shown in Fig. (b).

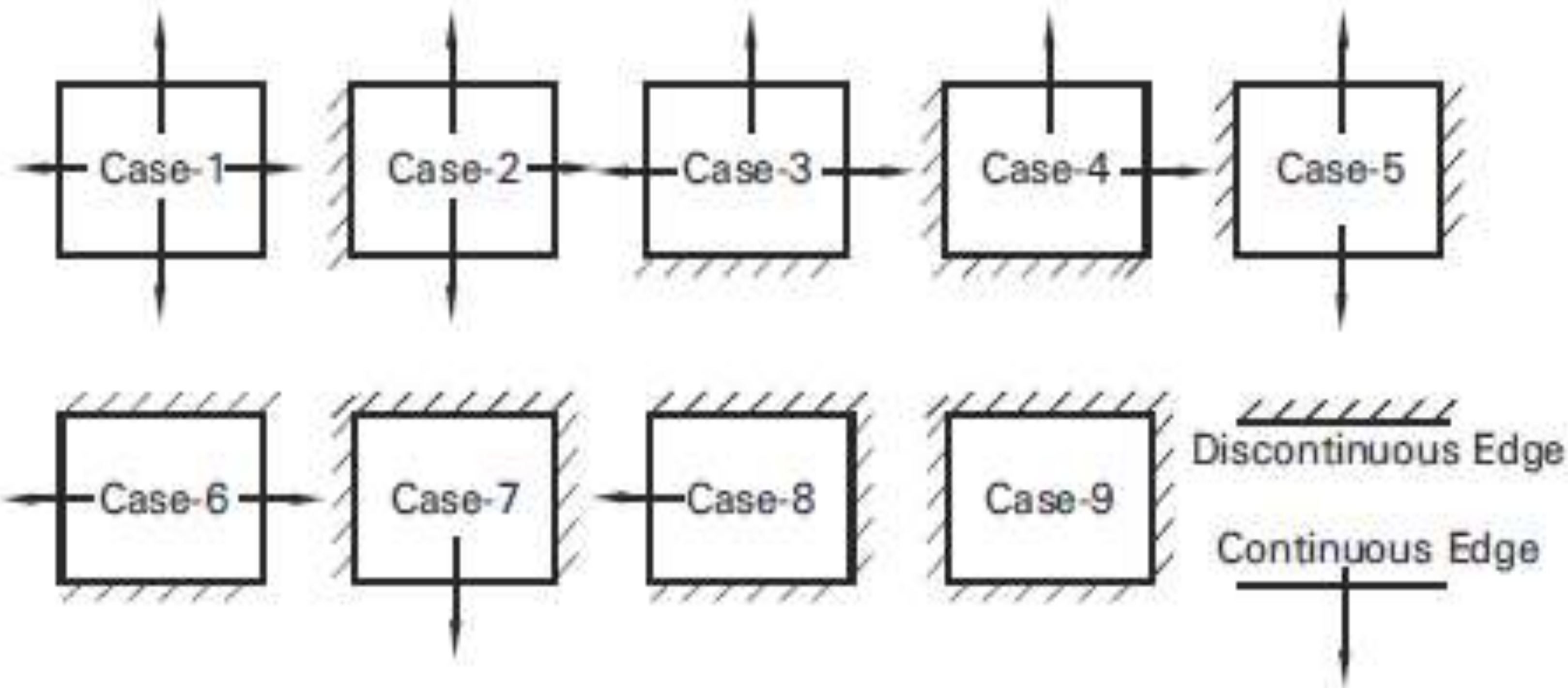


Fig. (a) : Support Conditions

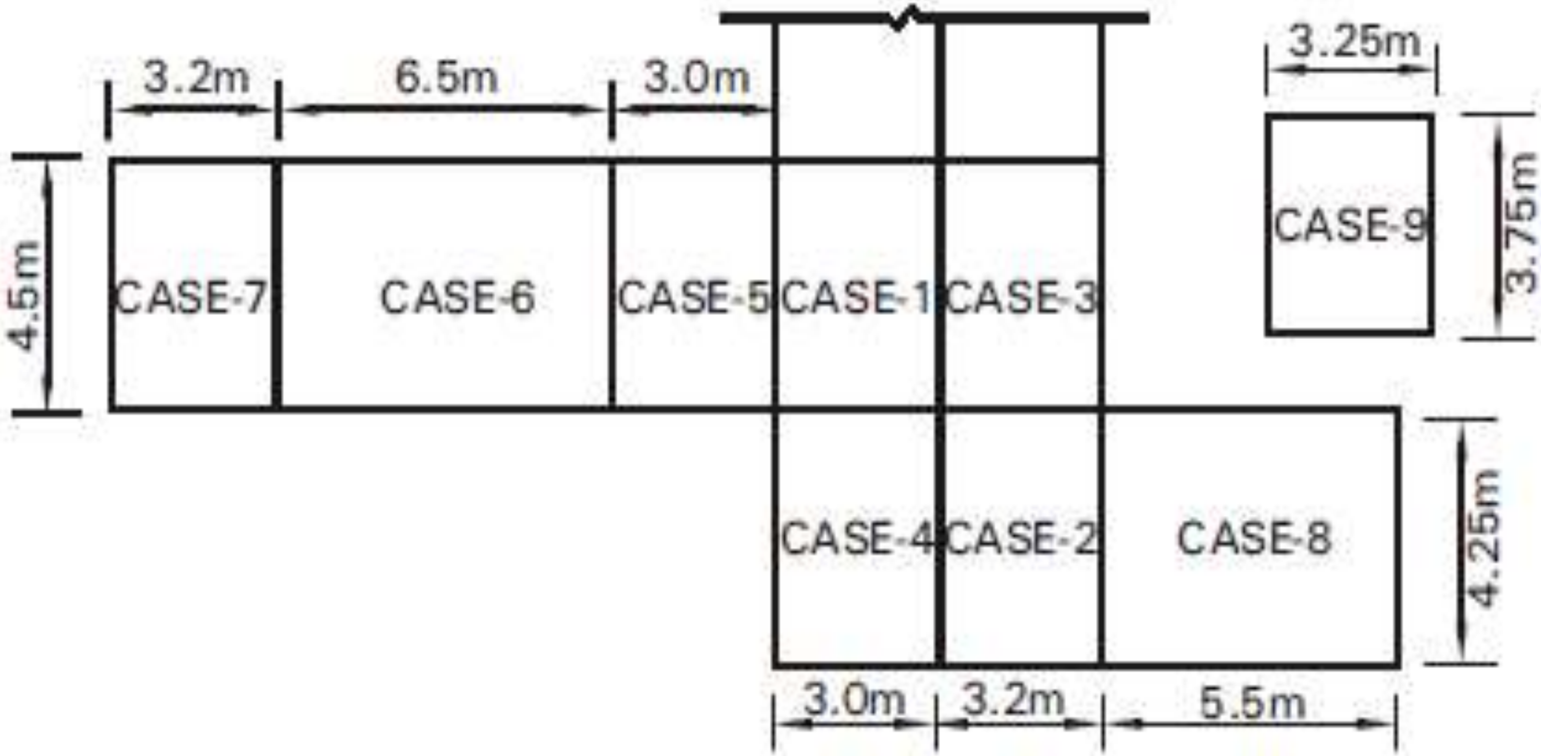


Fig. (b): Different types of support conditions for rectangular two way restrained slab panels

ii) Unrestrained slabs

The slabs in which corners are not prevented from lifting are called as unrestrained slabs or slabs with corners not held down.

(IS 456:2000, Annex D-2 & Table 27 provides details about restrained slabs and bending moment coefficients for slabs spanning in two directions at right angles, simply supported on four sides)

* DIFFERENCE BETWEEN ONE WAY AND TWO WAY SLABS

Sl. No.	One Way Slab	Two Way Slab
1.	$\frac{l_y}{l_x} > 2$	$\frac{l_y}{l_x} \leq 2$
2.	The bending takes place in one direction only i.e. shorter span.	The bending takes place in both the directions.
3.	Depth required is more.	Depth required is less.
4.	Main steel reinforcement is provided along shorter span.	Main steel reinforcement is provided along both the spans.
5.	Less economical as thickness is more and the amount of steel is also more.	More economical as the thickness of slab is less and the amount of steel required is less.

DESIGN OF ONE WAY SLAB (Simply supported and cantilever slab)

Data: Room size or span, type of concrete & steel, Live load and floor finishes.

Design: Depth of slab, Area of main & distribution reinforcement and check for deflection control.

Design Steps:

1. Check slab is one way or Two way if both spans given.

2. Assume overall depth of slab (100mm to 150mm) or $d = \frac{\text{span}}{20}$ to $\frac{\text{span}}{25}$ for simply supported slab and $d = \frac{\text{span}}{7}$ to $\frac{\text{span}}{10}$ for cantilever slab,

   d = effective depth of slab.

   (Note : The depth of slab can be assumed on the basis of control of deflection. Using balanced percentage of steel the $\left(l/d\right)$ ratio are modified. To start with, span to depth ratios are approximated as following for initial depth trial calculations :

   i) For simply supported slabs - 20 to 25

   ii) For cantilever slabs - 7 to 10

   Overall depth, $D = d + \text{clear cover} + \frac{\text{Diameter of bar}}{2}$ or $D = d + d'$

   d' = effective cover

3. Effective span (L) : Least of the following two

   i) L = clear span (l) + bearing

   ii) L = clear span (l) + effective depth (d)

   } For simply supported and continuous slab

L = clear span (l) + half the effective depth(d) } For cantilever slab

ie., $L = l + d/2$

4. Load calculation

   i) Self weight of slab = $D \times 1 \times$ Density of RCC (ρ) = ________ kN/m

   ii) Live load (Given) $\times 1$ = ________ kN/m

   iii) Floor finish (If given) $\times 1$ = ________ kN/m

   Total load, W = ________ kN/m

   Factored or ultimate load, $W_u = 1.5 \times W$

5. Bending moment calculation

$$M_u = \frac{W_u L^2}{8} \text{ for simply supported slab}$$

$$M_u = \frac{W_u L^2}{2} \text{ for Cantilever slab (Eg: Sunshade, Chajja, balacony cantilever slab)}$$

6. Check for effective depth

   Equating M_u = Limiting moment of resistance

$$M_{u\,lim} = 0.148 f_{ck} b d^2 \text{ - Fe250 (Mild steel)}$$

$$M_{u\,lim} = 0.138 f_{ck} b d^2 \text{ - Fe415 (HYSD bars)}$$

$$M_{u\,lim} = 0.133 f_{ck} b d^2 \text{ - Fe500 (HYSD bars)}$$

$d_{required} = \sqrt{\frac{M_u}{0.138 \times f_{ck} b}}$ For Fe415 **Note : b = 1000 mm (1m)**

If $d_{required} < d$ (assumed depth)

Design is safe

If $d_{required} > d$ (assumed depth)

Design is not safe and Revise the depth

7. Design of Main reinforcement (Along Shorter span) or Area of Main reinforcement

Now, refer IS:456-2000, Annex - G, G-1.1(b)

$$M_u = 0.87 f_y A_{st} d \left[1 - \left(\frac{A_{st} f_y}{f_{ck} b d}\right)\right]$$

Solve 'A_{st}' using above expression (Tension reinforcement)

Assume suitable diameter of bars (eg: ϕ = 8mm, 10mm, 12mm)

$$\text{Area of one bar, } a_{st} = \frac{\pi \times \phi^2}{4}$$

Spacing of reinforcement,

$$S = \frac{1000 a_{st}}{A_{st}}$$

(Note: spacing should be less than 3d or 300mm)

8. Distribution steel,

A_{st} = 0.12% area of concrete for HYSD bars (Fe415 and Fe500)

$$A_{st} = \frac{0.12}{100} \times b \times D$$

A_{st} = 0.15% area of concrete for mild steel (Fe250)

$$A_{st} = \frac{0.15}{100} \times b \times D$$

Assume suitable diameter of bars (eg: ϕ = 8mm, 10mm,12mm)

$$\text{Area of one bar, } a_{st} = \frac{\pi \times \phi^2}{4}$$

Spacing of reinforcement,

$$S = \frac{1000 a_{st}}{A_{st}}$$

(Note: spacing should be less than 5d or 450mm)

9. Design for shear stress or check for shear stress

1. $V_u = \frac{W_u L}{2}$ - For simply supported slab
2. $V_u = W_u L$ - For cantilever slab

* Nominal shear stress, $\tau_v = \frac{V_u}{bd}$

* Percentage of steel, $p_t = \frac{100 A_{st}}{bd}$

  Use $A_{st_{provided}}$ value for calculation of % of steel.

* Design shear strength of concrete, τ_c

  Now refer IS:456-2000, Table 19, using p_t and f_{ck}

  Note down the value of 'τ_c'

* Permissible shear stress, $k\tau_c$

  'k' is depends on slab depth, Refer IS:456-2000, clause 40.2.1.1

  If $k\tau_c > \tau_v$ - Design is safe

  if $k\tau_c < \tau_v$ - Design is not safe, Revise the depth

10. Check for deflection control,

1. $\left(\frac{L}{d}\right)_{max} = 20\, k_t \times k_c \times k_f$ - For simply supported slab

$\left(\frac{l}{d}\right)_{max} = 7\, k_t \times k_c \times k_f$ - For cantilever slab

Now refer IS : 456 - 2000, Fig 4

Calculate, $f_s = 0.58\, f_y \left[\frac{A_{st_{req}}}{A_{st_{prov}}}\right]$

$k_t = k_c$ = modification factor for tension and compression

k_f = Reducation factor for flange section

Now using p_t and f_s, Note down the value of Modification factor

2. $\left(\frac{L}{d}\right)_{actual}$

Now, $\left(\frac{L}{d}\right)_{max} > \left(\frac{L}{d}\right)_{actual}$ - Design is safe against deflection

$\left(\frac{L}{d}\right)_{max} < \left(\frac{L}{d}\right)_{actual}$ - Design is not safe. Hence revise the depth

DESIGN OF ONE WAY SLAB

WORKED EXAMPLES ON DESIGN OF ONE WAY SLAB

1. **Design a simply supported slab on masonry walls to the following requirements. Draw plan and section showing reinforcement details.**
   Clear span = 2.5m.
   Live Load = 3000N/m^2
   Use M-20 concrete &Fe-415 steel.

   Solution: Given : $l = 2.5$m, $LL = 3$ kN/m^2, $f_{ck} = 20$ N/mm^2 and $f_y = 415$ N/mm^2

   Assuming bearing = 200mm (eg: 150, 200, 230, 250 and 300 mm)

   Assuming effective cover, $d' = 20$mm

Step : 1 Thickness of slab (*D*) :

$$\text{Effective depth, } d = \frac{\text{span}}{25} = \frac{2500}{25} = 100mm$$

Overall depth of slab, $D = d + d' = 100 + 20 = 120$mm

Step : 2 Effective span: Least of the following two

1) L = Clear span + bearing = 2.5 + 0.2 = 2.7m
2) L = Clear span + effective depth = 2.5 + 0.10 = 2.6m

Therefore effective span, L = 2.6m.

Step : 3 Load calculation

1) Self weight of slab $= 0.12 \times 25 = 3.00 kN/m^2$

2) Live load (Given) $= 3.00 kN/m^2$

3) 20 mm Thick Floor finish $= 0.02 \times 24 \approx 0.50 kN/m^2$
(Assume density 24 kN/m³)

$0.48 \approx 0.50$

Total load, $W = 6.50 kN/m^2$

Factored load per m (W_u) $= 1.5 \times 6.50 \times 1 = 9.75 kN/m$

$1kN.m = 10^6 N.mm$

b = 1m or 1000 mm

Step : 4 Bending Moment Calculation

$$M_u = \frac{W_u L^2}{8} = \frac{9.75 \times 2.6^2}{8} = 8.23 kN-m.$$ or 8.23 × 10⁶ N-mm

Cont…

Step : 5 Check for effective depth

Equating $M_u = M_{u\,\lim} = 0.138\, f_{ck}\, bd^2$

$$d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{8.23\times10^6}{0.138\times20\times1000}} = 54.60mm < 100 \text{ mm (OK)},$$

Design is safe

Step : 6 Area of main reinforcement or Design of main reinforcement

$$M_u = 0.87\, f_y\, A_{st}\, d\left[1-\left(\frac{A_{st} f_y}{f_{ck} bd}\right)\right]$$

$$8.23\times10^6 = 0.87\times415\times100\, A_{st}\left[1-\left(\frac{415\, A_{st}}{20\times1000\times100}\right)\right]$$

$$8.23\times10^6 = 36105\, A_{st} - 749\, A_{st}^2$$

$$\Rightarrow 7.49\, A_{st}^2 - 36105\, A_{st} + 8.23\times10^6 = 0$$

By solving above expression using Quadratic equation, we get,

$$A_{st} = 239.8 \text{ mm}^2 \approx 240 \text{ mm}^2 \left(A_{st_{req}}\right)$$

Using 8mm diameter bars

Cont…

Area of one bar, $a_{st} = \frac{\pi \times 8^2}{4} = 50.26\ mm^2$ $\quad | 3d = 3 \times 100 = 300\text{mm}$

$$\text{Spacing, } S = \frac{1000 a_{st}}{A_{st}} = \frac{1000 \times 50.26}{240} = 209.41\ mm \text{ say } 200 \text{ mm}$$

Provide $8mm\ \phi @ 200\ mm\ c/c.$ ($<$ 300 mm or 3d) $\left[A_{st_{prov}} = 251.3 \text{ mm}^2\right]$

Step : 7 Distribution reinforcement

$$A_{st,\min} = 0.12\% \times b \times D = \frac{0.12}{100} \times 1000 \times 120 = 144\text{mm}^2$$

using 8mm diameters bars

$$\text{Spacing, } S = \frac{1000 a_{st}}{A_{st}} = \frac{1000 \times 50.26}{144} = 349.02\text{mm} \qquad | 5d = 5 \times 100 = 500\text{mm}$$

$\therefore$ Provide 8mm ϕ bars @ 300mm c/c $(< 450\text{mm or } 5d)$

Step 8 : Design for shear stress or check for shear stress

➢ $V_u = \frac{W_u L}{2} = \frac{9.75 \times 2.6}{2} = 12.67 \text{ kN}$

- Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{12.67 \times 10^3}{1000 \times 100} = 0.126$ N/mm²
- Percentage of steel, $p_t = \frac{100 A_{st}}{bd} = \frac{100 \times 251.3}{1000 \times 100} = 0.25\%$
- Design shear strength of concrete, τ_c

  Now, refer IS : 456-2000, Table 19, using $p_t = 0.25\%$ and $f_{ck} = 20$ N/mm²

$$\therefore \tau_c = 0.36 \text{ N/mm}^2$$

- Permissible shear stresses, $k\tau_c$

  $k = 1.30$ for slab depth = 120 mm, (Refer clause 40.2.1.1)

  $\therefore k\tau_c = 1.3 \times 0.36 = 0.468 \text{ N/mm}^2 > \tau_v$

  Hence the shear stresses are within safe permissible limits

Step 9 : Check for deflection control

1. $\left(\frac{L}{d}\right)_{max} = 20\ k_t \times k_c \times k_f$ | For simply supported = 20

   Now, refer IS : 456-2000, Fig. 4

$$p_t = 0.25\%$$

$$f_s = 0.58 f_y \left[\frac{A_{st_{\text{required}}}}{A_{st_{\text{provided}}}}\right] = 0.58 \times 415 \times \frac{240}{251.3} = 229.8 \approx 230 \text{ N/mm}^2$$

Now using, f_s and p_t, calculate modification factor (k_t)

$$\therefore k_t = 1.6, \text{ (from IS : 456-2000, Fig. 4)}$$

Here, $k_c = 1$ and $k_f = 1$ | $\because$ no compression and flange section

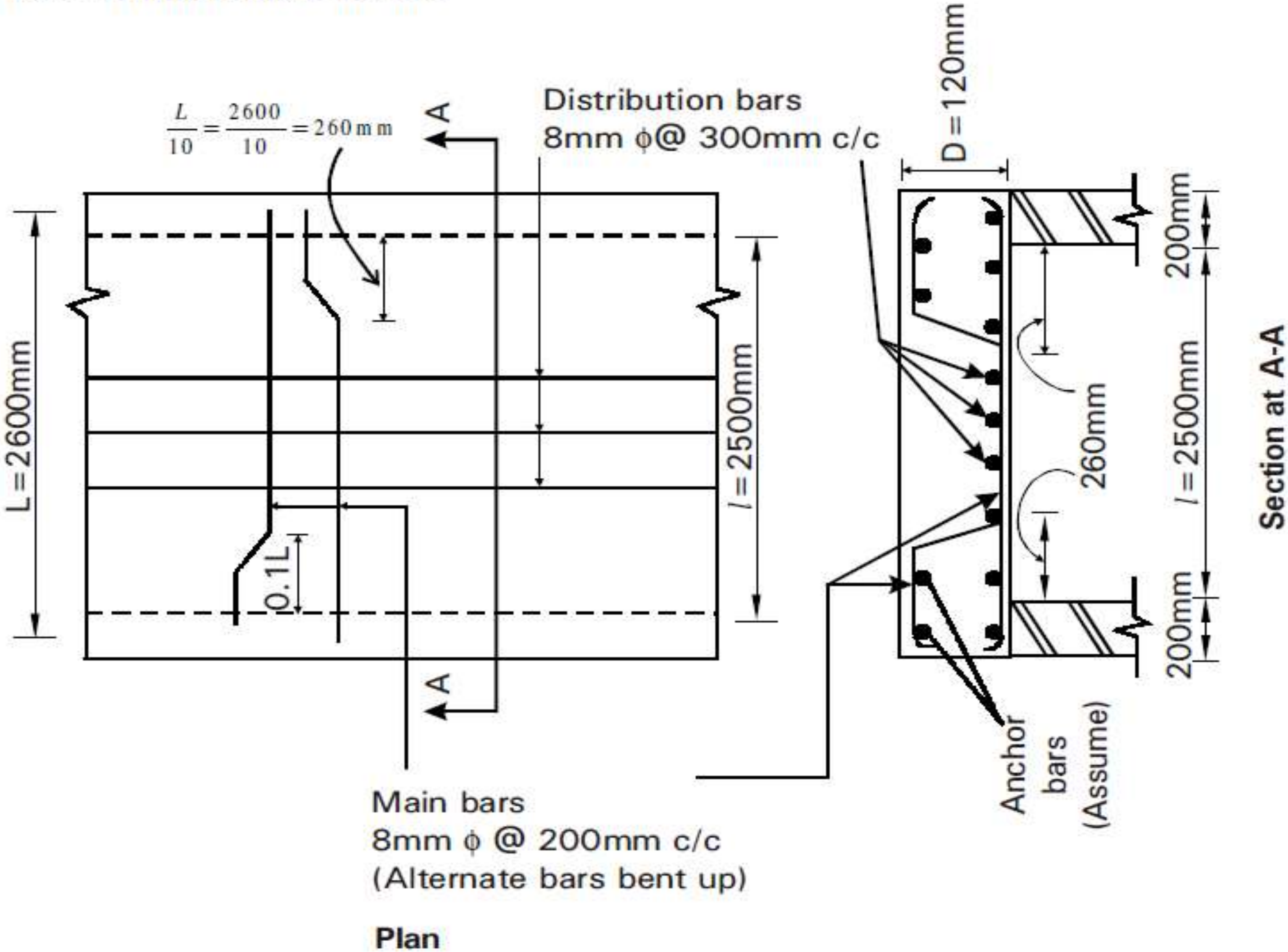
$$\therefore \left(\frac{L}{d}\right)_{\max} = 20 \times 1.6 \times 1 \times 1 = 32$$

2. $\left(\frac{L}{d}\right)_{\text{actual}} = \frac{2600}{100} = 26$

$$\therefore \left(\frac{L}{d}\right)_{\max} > \left(\frac{L}{d}\right)_{\text{actual}}$$

Hence deflection is within permissible limits

Step 10 : Reinforcement details



2. **A room has clear dimension 7m × 3m. The live load on the slab is 3.5 kN/m² and a floor finishing load of 1.2 kN/m². Using M 20 concrete and Fe415 steel, design the slab. Use Limit state method. Take l/d = 25 and d' = 20mm.**

Solution:

Given : Room dimension = 7 × 3m, $LL = 3.5$ kN/m², $FF = 1.2$ kN/m², $f_{ck} = 20$ N/mm² and $f_y = 415$ N/mm²

Step : 1 Check for longer span to shorter span ratio

$$\frac{l_y}{l_x} = \frac{7}{3} = 2.33 > 2, \text{ Therefore, slab is designed as one way slab}$$

Step : 2 Thickness of slab (D) :

$$\text{Effective depth, } d = \frac{l}{25} = \frac{3000}{25} = 120\,mm$$

Assuming effective cover $d' = 20$mm

Overall depth, $D = d + d' = 120 + 20 = 140$mm

Assuming 230mm bearing or wall thickness

Step : 3 Effective span: Least of the following :

i) $L =$ Clear span + bearing $= 3 + 0.23 = 3.23\,m$

ii) $L =$ Clear span + effective depth $= 3.0 + 0.12 = 3.12\,m$

Therefore effective span $L = 3.12\,m$.

Step : 4 Load calculation

1) Self weight of slab = 0.14 × 25 $= 3.5 kN/m^2$

2) Live load (Given) $= 3.5 kN/m^2$

3) Floor finishing (Given) $= 1.2 kN/m^2$

Total load, $W = 8.2 KN/m^2$

Factored load per m, $(W_u) = 1.5 \times 8.2 \times 1 = 12.3$ kN/m

Step : 5 Bending moment calculations

$$M_u = \frac{W_u L^2}{8} = \frac{12.3 \times 3.12^2}{8} = 14.97 kN-m$$

$$= 14.97 \times 10^6 \ N-mm$$

Step : 6 Check for effective depth

Equating $M_u = M_{ulim} = 0.138 \ f_{ck} \ bd^2$ Consider $b = 1$m or 1000mm

$$d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{14.97 \times 10^6}{0.138 \times 20 \times 1000}} = 73.64mm < 120mm \ (\text{OK})$$

Design is safe.

Step 7 : Area of main reinforcement

$$M_u = 0.87\ f_y A_{st} d \left[1 - \left(\frac{A_{st} f_y}{f_{ck} bd}\right)\right]$$

$$14.97 \times 10^6 = 0.87 \times 415 \times 120\ A_{st}\left[1 - \left(\frac{415 A_{st}}{20 \times 1000 \times 120}\right)\right]$$

$$\Rightarrow 7.49 A_{st}^2 - 43326 A_{st} + 14.97 \times 10^6 = 0$$

$$\therefore A_{st} = 369.06 \approx 370\ \text{mm}^2 \left(A_{st_{req}}\right)$$

Providing 8 *mm* dia bars

$$\text{Area of one bar, } a_{st} = \frac{\pi \times 8^2}{4} = 50.26\ mm^2$$

$3d = 3 \times 120 = 360\text{mm}$

$5d = 5 \times 120 = 600\text{mm}$

$$\text{Spacing, } S = \frac{1000 a_{st}}{A_{st}} = \frac{1000 \times 50.26}{370}$$

$$S = 135.83\ mm. \text{ say } 130\ mm$$

$$A_{st_{provided}} = 386.61\ \text{mm}^2$$

Provide 8 *mm* ϕ @ 130 mm c/c as main reinforcement. (< 300 mm or 3*d*)

Step : 8 Distribution reinforcement

$$A_{st,\min} = 0.12\% \times b \times D = \frac{0.12}{100} \times 1000 \times 140 = 168 mm^2$$

using 8mm diameter bars

$$S = \frac{1000 a_{st}}{A_{st}} = \frac{1000 \times 50.26}{168} = 299.16\ mm$$

$\therefore$ Provide $8mm\ \phi$ @ $290mm\ c/c$ as distribution reinforcement (<450mm or $5d$)

Step 9 : Design for shear stress

- $V_u = \frac{W_u L}{2} = \frac{12.3 \times 3.12}{2} = 19.18 \approx 19.2$ kN
- Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{19.2 \times 10^3}{1000 \times 120} = 0.16$ N/mm²
  - Percentage of steel, $p_t = \frac{100 A_{st}}{bd} = \frac{100 \times 386.61}{1000 \times 120} = 0.33\%$
  - Design shear strength of concrete, τ_c

    Now, refer IS : 456-2000, Table 19, using $p_t = 0.33\%$ and $f_{ck} = 20$ N/mm²

p_t (%)	τ_c (N/mm²)
0.25	0.36
0.33	?
0.50	0.48

By using interpolation, we get

$$\therefore \tau_c = 0.398 \text{ N/mm}^2$$

- Permissible shear stress, $k\tau_c$

  $k = 1.3$ for slab depth = 140 mm, (refer clause 40.2.1.1)

  $\therefore k\tau_c = 1.3 \times 0.398 = 0.51 \text{ N/mm}^2 > \tau_v$

  Hence the shear stresses are within safe permissible limits

Step 10 : Check for deflection control

1. $\left(\dfrac{L}{d}\right)_{max} = 20\, k_t \times k_c \times k_f$

   Now, refer IS : 456-2000, Fig. 4

$$p_t = 0.33\%$$

$$f_s = 0.58\, f_y \left[\frac{A_{st_{required}}}{A_{st_{provided}}}\right] = 0.58 \times 415 \times \frac{370}{386.61} = 230.35 \text{ N/mm}^2$$

   Now using, f_s and p_t, calculate modification factor (k_t)

$$\therefore k_t = 1.4 \quad \text{[from IS : 456-2000, Fig. 4]}$$

$$\text{Here,} \quad k_c = 1 \text{ and } k_f = 1$$

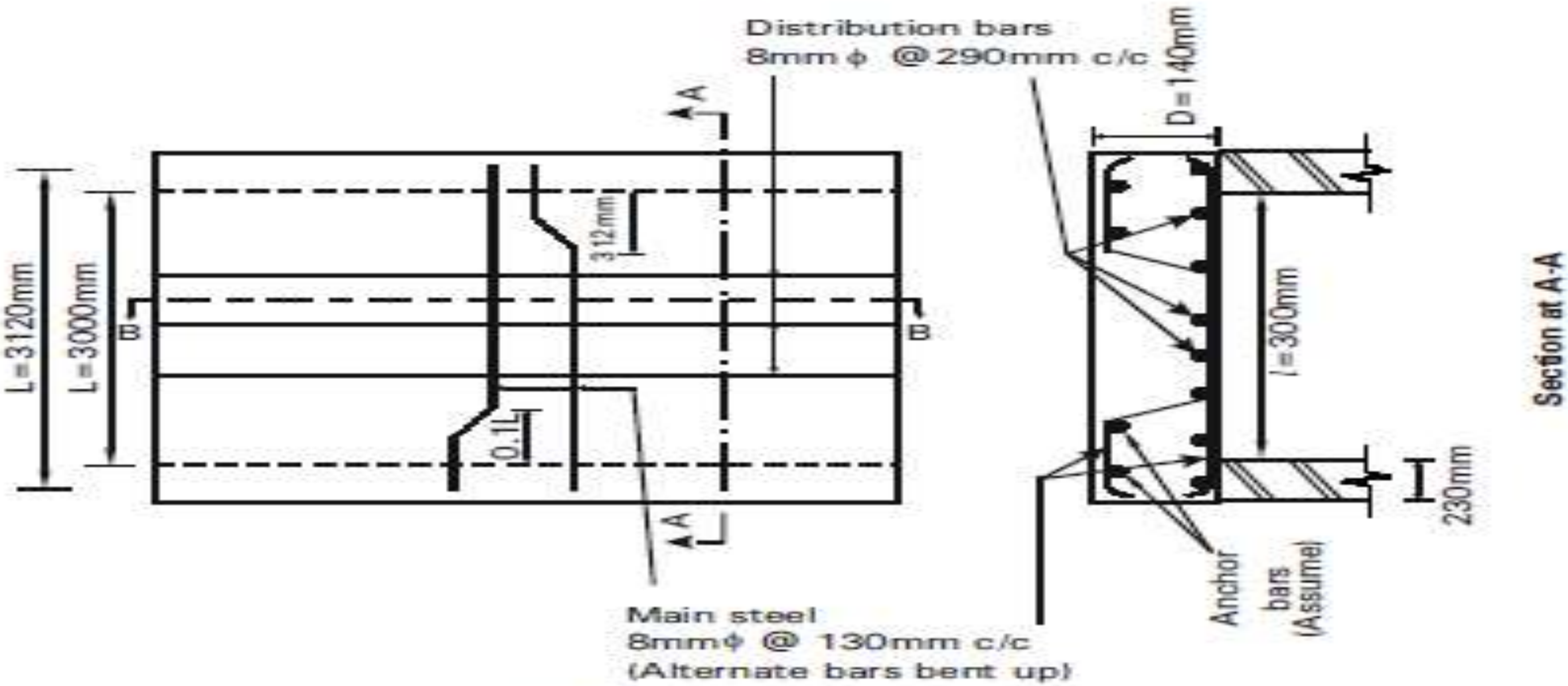
$$\therefore \left(\frac{L}{d}\right)_{max} = 20 \times 1.4 \times 1 \times 1 = 28$$

2. $\left(\dfrac{L}{d}\right)_{actual} = \dfrac{3120}{120} = 26$

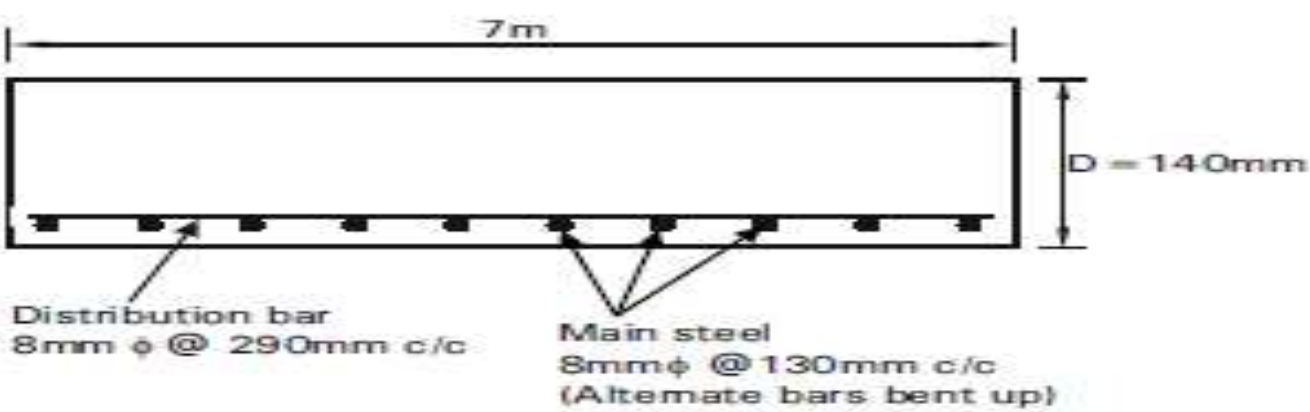
$$\therefore \quad \left(\frac{L}{d}\right)_{max} > \left(\frac{L}{d}\right)_{actual}$$

   Hence deflection is within permissible limits

Step 11 : Reinforcement details



Plan



Section @ B-B

3. **Calculate the safe super imposed load for slab 125 mm thick, which is simply supported over an effective span of 3.2 m. The slab is reinforced with 12 mm dia bars at 120 mm c/c spacing. Take f_{ck} = 20 Mpa, f_y = 415 Mpa, density of concrete = 25 kN/m³ and effective cover = 25 mm.**

Solution:

Given: $D = 125\ mm, L = 3.2\ m, \phi = 12\ mm, S = 120\ mm$

$f_{ck} = 20$ Mpa $= 20\ N/mm^2, f_y = 415$ Mpa $= 415\ N/mm^2$

$\rho = 25\ kN/m^3$, eff. cover $d' = 25\ mm$

effect depth, $d = 125 - 25 = 100$mm

Step : 1 Area of main reinforcement

$$\text{Spacing}, S = \frac{1000\, a_{st}}{A_{st}}$$

$$\text{Area of one bar, } a_{st} = \frac{\pi \times 12^2}{4} = 113.09\ mm^2$$

$$A_{st} = \frac{1000 a_{st}}{S} = \frac{1000 \times 113.09}{120}$$

$$A_{st} = 942.41\ mm^2$$

Step : 2 Depth of neutral axis (x_u) - from IS : 456-2000, G-1.1 (a)

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d} \Rightarrow x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 942.41}{0.36 \times 20 \times 1000} = 47.25\text{mm}$$

Balanced section, $x_{u_{max}} = 0.48d = 0.48 \times 100 = 48$mm

$$x_{u_{max}} > x_u \text{ or } x_u < x_{u_{max}}$$

Step : 3 Ultimate Moment of resistance or Bending Moment Calculation :

$$M_u = 0.87\, f_y\, A_{st}\, d\left[1-\left(\frac{f_y A_{st}}{f_{ck}\, bd}\right)\right]$$

$$= 0.87 \times 415 \times 942.41 \times 100\left[1-\frac{415\times 942.41}{20\times 1000\times 100}\right]$$

$$M_u = 27.37 \times 10^6\ N\text{–}mm = 27.37\ kN\text{–}m$$

Step : 4 Calculation of safe super imposed load :

w.k.t. simply supported beam carry udl, Maximum bending moment is given by

$$M_u = \frac{W_u L^2}{8}$$

$$\therefore \text{Total load, } W_u = \frac{8 M_u}{L^2} = \frac{8\times 27.37}{3.2^2}$$

$$W_u = 21.38 \text{ kN/m}$$

$$\text{Working load, } W = \frac{W_u}{1.5} = \frac{21.38}{1.5}$$

$$W = 14.25 \text{ kN/m}$$

Total load,

$W = W_{DL} + W_{LL}$

W_{LL} = super imposed load

$$\text{Self weight of slab, } (W_{DL}) = D \times 1 \times \rho$$

$$= 0.125 \times 1 \times 25 = 3.125\ kN/m$$

$$\text{Super imposed load} = (W_{LL}) = W - W_{DL}$$

$$= 14.25 - 3.125 = \mathbf{11.125\ kN/m}$$

4. **Design a cantilever balcony slab projecting 1.5 m from wall. Take live load of 2.75 kN/m². Use M20 concrete and Fe 415 steel. Sketch reinforcement details.**

Solution:

Given : $l = 1.5\text{m}$, live load = 2.75kN/m^2, $f_{ck} = 20\ \text{N/mm}^2$ and $f_y = 415\ \text{N/mm}^2$

Step : 1 Thickness of slab (*D*)

$$\text{Effective depth of slab, } d = \frac{l}{10} = \frac{1500}{10}$$

$$= 150\ \text{mm}$$

$$\text{overall depth } D = d + 25 \text{ (assume effective cover = 25 mm)}$$

$$= 150 + 25$$

$$D = 175\ \text{mm}$$

The depth D gradually reduced to 100 mm

(Note : The depth at the free end is minimum and kept $\frac{1}{2}$ to $\frac{1}{3}$ of the depth at fixed end)

Step : 2 Effective Span

$$L = 1.5 + \frac{0.15}{2} = 1.575 \approx 1.58\text{m}$$

(Note : Effective span = projection from free of support + half the effective depth of slab for cantilever) - From IS:456 - 2000, clause 22.2(c)

Step : 3 Load calculation

i) Self weight of slab = Average thickness × 1 × 25

$$= \frac{0.175 + 0.1}{2} \times 1 \times 25 \quad = 3.44\ \text{kN/m}$$

ii) Live load (Given) $= 2.75 \times 1 \quad = 2.75\ \text{kN/m}$

iii) Floor finishing (Assume) = 1.0×1 = 1.0 kN/m

Total load, W = 7.19 kN/m

Factored load, $W_u = 1.5 \times W = 1.5 \times 7.19 = 10.79$ kN/m

Step 4 : Bending Moment Calculation

$$M_u = \frac{W_u L^2}{2} = \frac{10.79 \times 1.58^2}{2} = 13.46 \text{ kN-m}$$

Step 5 : Check for effective depth

$$d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck}\, b}} = \sqrt{\frac{13.46 \times 10^6}{0.138 \times 20 \times 1000}}$$

$$= 69.83 \text{ mm} < 150 \text{ mm (provided)}$$

Design is safe

Step 6 : Area of main reinforcement

$$M_u = 0.87\, f_y\, A_{st} d \left[1 - \left(\frac{A_{st} f_y}{f_{ck} bd}\right)\right]$$

$$13.46 \times 10^6 = 0.87 \times 415 \times 150\, A_{st}\left[1 - \left(\frac{415 A_{st}}{20 \times 1000 \times 150}\right)\right]$$

$$7.49\, A_{st}^2 - 54157.5\, A_{st} + 13.46 \times 10^6 = 0$$

$$\therefore A_{st} = 257.72 \approx 258 \text{ mm}^2$$

$3d = 3 \times 150 = 450$mm

$5d = 5 \times 150 = 750$mm

Spacing of 8 mm dia bars, $S = \dfrac{1000\, a_{st}}{A_{st}}$

Area of one bar, $a_{st} = 50.26 \text{ mm}^2$

$$S = \frac{1000 \times 50.26}{258} = 194.8 \text{ say } 190 \text{ mm c/c}$$

$\therefore$ Provide 8mm ϕ @ 190mm c/c as main reinforcement ($< 3d$ or 300 mm) [$A_{st_{pro}} = 264.52 \text{ mm}^2$]

Step 7 : Distribution reinforcement

$$A_{st} = 0.12\% \times b \times D = \frac{0.12}{100} \times 1000 \times 175$$

$$= 210 \text{ mm}^2$$

Spacing of 8 mm dia bars, $S = \frac{1000 \times 50.26}{210} = 239.33 \text{ mm say } 230 \text{ mm C/C}$

$\therefore$ Provide 8mm ϕ @ 230mm c/c as distribution reinforcement ($< 5d$ or 450 mm)

Step 8 : Design for shear stress or check for shear

- $V_u = W_u L = 10.79 \times 1.58 = 17.04$ kN
- Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{17.04 \times 10^3}{1000 \times 150} = 0.113$ N/mm^2
- Percentage of steel, $p_t = \frac{100 A_{st}}{bd} = \frac{100 \times 264.52}{1000 \times 150} = 0.17\%$
- Design shear strength of concrete, τ_c

  Now, refer IS : 456-2000, Table 19, using $p_t = 0.17\%$ and $f_{ck} = 20$ N/mm^2

  $\therefore \tau_c = 0.398$ N/mm^2 [By using interpolation method]
- Permissible shear stresses, $k\tau_c$

  $k = 1.25$ for slab depth = 175 mm

  $\therefore k\tau_c = 1.25 \times 0.296 = 0.37$ N/mm$^2 > \tau_v$

  Hence the shear stresses are within safe permissible limits

Step 9 : Check for deflection control

1. $\left(\frac{L}{d}\right)_{max} = 7\ k_t \times k_c \times k_f$

Now, refer IS : 456-2000, Fig. 4

$p_t = 0.17\%$

$$f_s = 0.58\ f_y \left[\frac{A_{st_{required}}}{A_{st_{provided}}}\right]$$

$$= 0.58 \times 415 \times \frac{258}{264.52} = 234.76\ \text{N/mm}^2$$

Now using, f_s and p_t, calculate modification factor (k_t)

$\therefore k_t = 1.75$

Here, $k_c = 1$ and $k_f = 1$

$\therefore \left(\frac{L}{d}\right)_{max} = 7\ k_t \times k_c \times k_f = 7 \times 1.75 \times 1 \times 1 = 12.25$

2. $\left(\frac{L}{d}\right)_{actual} = \frac{1580}{150} = 10.53$

$\therefore \quad \left(\frac{L}{d}\right)_{max} > \left(\frac{L}{d}\right)_{actual}$

Hence deflection is within permissible limits

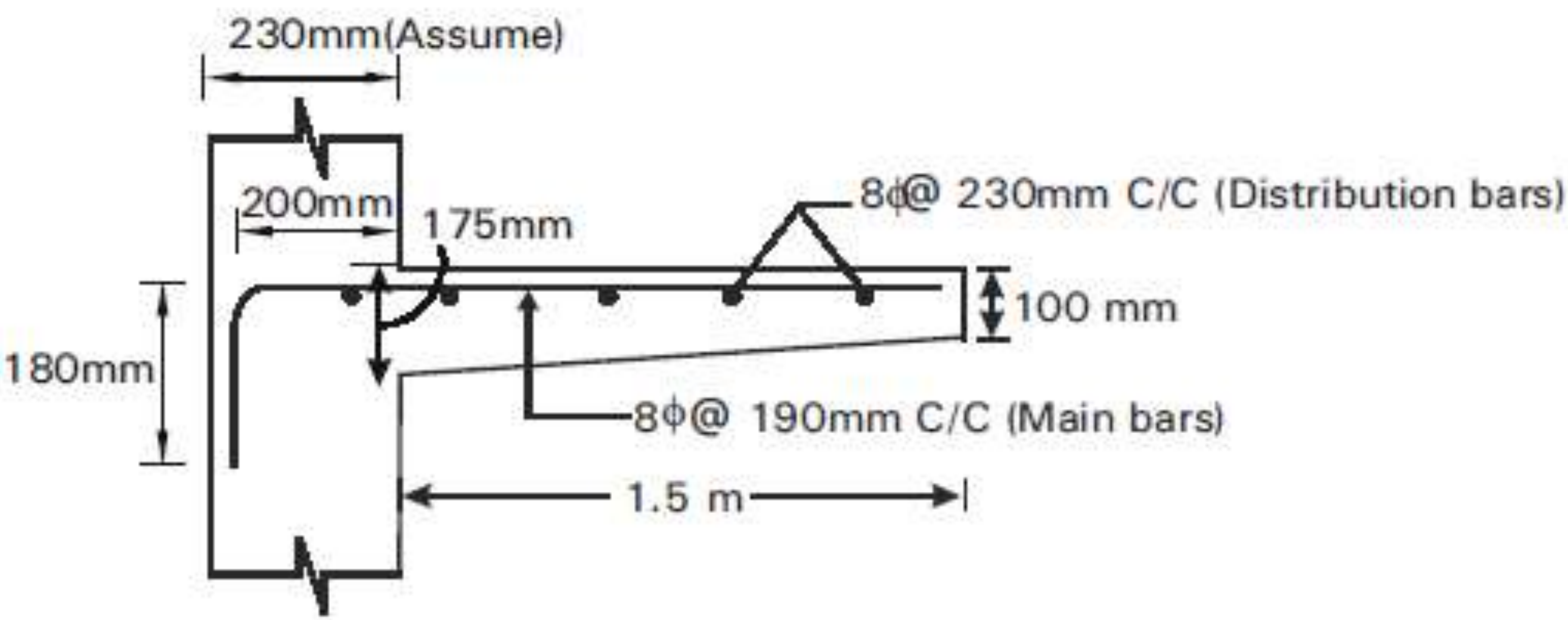
Step 10 : Development length at the support

From IS : 456-2000, clause 26.2.1, $L_d = \dfrac{\phi\sigma_s}{4\tau_{bd}} = \dfrac{0.87 f_y \phi}{4\tau_{bd}}$

$\tau_{bd} = 1.2$ for M20 glade (clause 26.2.1.1) and for HYSD bars these values shall be increased by 60%

$$\therefore L_d = \frac{0.87 \times 415 \times 8}{4 \times 1.6 \times 1.2} = 376.09\text{mm} \approx 380\text{mm}$$

Therefore extending the bars into the support after giving a 90° bend for a distance of 380mm (200 + 180) as shown in figure.



DESIGN OF ONE WAY CONTINUOUS SLAB

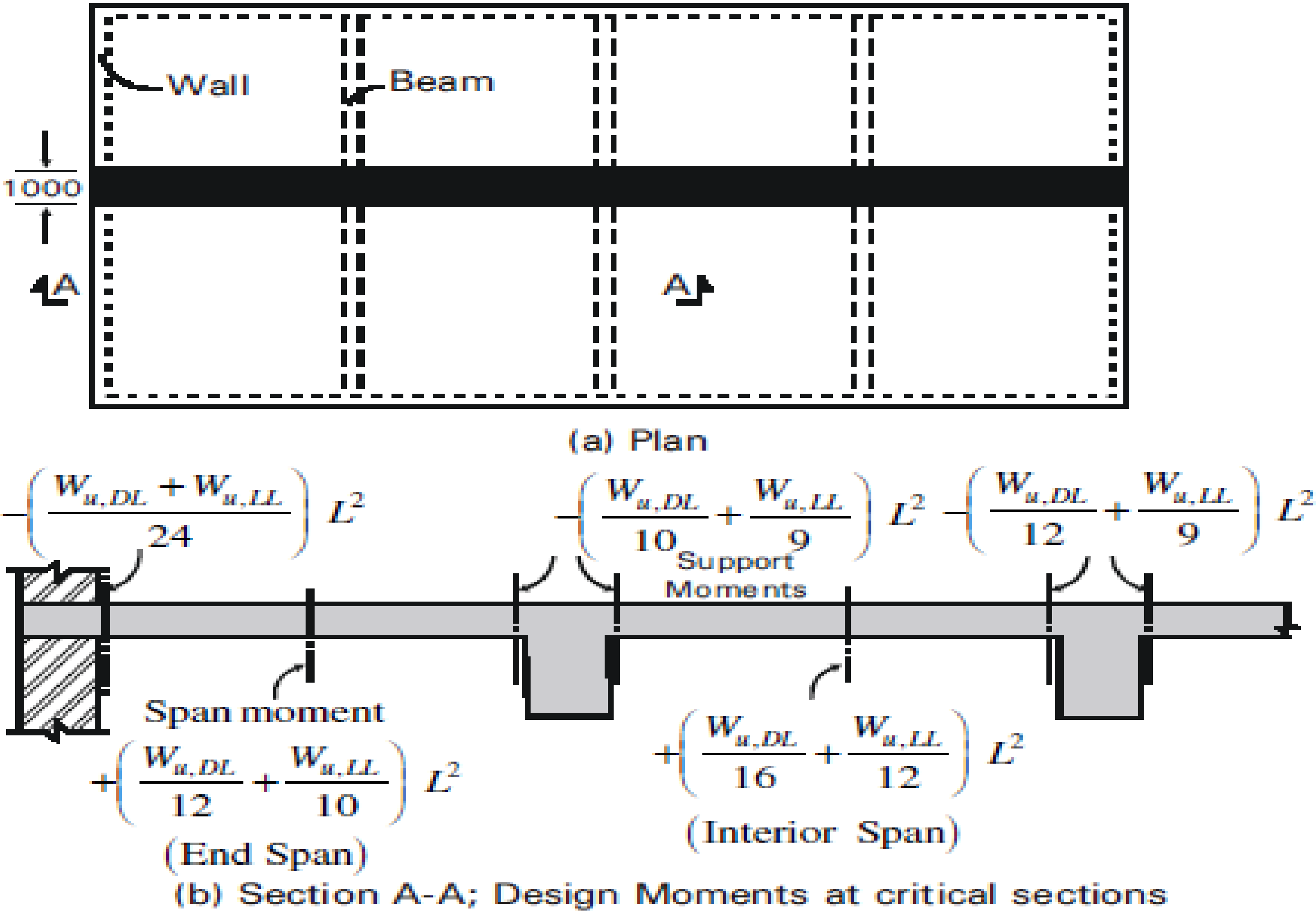


Fig. 4 : Plan and section of one way continuous slab

When a slab is continuous over several spans, negative bending moment is induced over intermediate supports and hence reinforcement has to be provided at the top of the slab portion over the intermediate supports. The design of such one way continuous slab can be done by moment coefficient method given in IS:456-2000. These coefficients give the moment at the critical sections.

Table . Bending Moment Coefficients

Loading	Span moments		Support Moments	
	Near middle of End span	At middle of Interior span	At support next to the end support	At other interior supports
Dead Load and imposed load (fixed)	$+\frac{1}{12}$	$+\frac{1}{16}$	$-\frac{1}{10}$	$-\frac{1}{12}$
Imposed load (not fixed)	$+\frac{1}{10}$	$+\frac{1}{12}$	$\frac{-1}{9}$	$\frac{-1}{9}$

For obtaining the bending moment, the coefficients shall be multiplied by the total design load ($W_u L$) and effective span(L).

DESIGN OF ONE WAY CONTINUOUS SLAB

Data : Span, type of concrete and steel, Live load and Floor Finish

Design : Depth of slab, Area of main and distribution reinforcement and check for deflection control

Design steps

1. Thickness of slab (D)

$$d = \frac{\text{Span}}{26 \text{ to } 35}$$

then calculate $D = d + d^1$ Assume d^1

2. Effective span (L) : Same as Design of one way slab
3. Load Calculation

(a) Dead load (W_{DL})

(i) Self weight of slab = $D \times 1 \times$ RCC density	= ______	kN/m
(ii) Floor Finish	= ______	kN/m
Total Dead Load (W_{DL})	= ______	kN/m
∴ Factored Dead Load, $W_{u_{DL}} = 1.5 \times W_{DL}$	= ______	kN/m
(b) Live load (W_{LL})	= ______	kN/m
∴ Factored Live Load, $W_{u_{LL}} = 1.5 \times W_{LL}$	= ______	kN/m

4. Bending Moment Calculation

Refer IS : 456-2000, Table 12 (BM coefficients)

Maximum Negative BM (at support) - $M_{u_{-ve}}$
Maximum Positive BM (at span) - $M_{u_{+ve}}$
} refer Fig

5. Check for depth

Consider maximum values out of the two values of moments $\left(M_{u\_ve} \text{ and } M_{u+ve}\right)$

$$d_{required} = \sqrt{\frac{M_u}{0.138 f_{ck} b}} \quad \text{—— for Fe 415}$$

if $d_{required} < d_{provided}$ Design is safe (ok)

if $d_{required} > d_{provided}$ Design is not safe

Hence, revise the depth

6. Determination of area of steel

1. Maximum Negative BM $\left(M_{u\_ve}\right)$

Calculate 'A_{st}' using $M_u = 0.87\, f_y A_{st} d \left[1 - \left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$

Spacing, $S = \frac{1000 a_{st}}{A_{st}}$

2. Maximum Positive BM $\left(M_{u+ve}\right)$

Calculate A_{st} and Spacing _ 'S'

7. Distribution of Steel — same as Design of one way slab

8. Check for shear — As per IS : 456-2000, Table 13 (shear force coefficients) calculate V_u using shear coefficients

- Nominal shear stress, $\tau_v = \frac{V_u}{bd}$
- Percentage of steel, $p_t = \frac{100A_{st}}{bd}$ Use A_{st} at support reinforcement,

Remaining procedure is same as Design of one way slab

9. Check for deflection control

1. $\left(\frac{L}{d}\right)_{max}$ 2. $\left(\frac{L}{d}\right)_{actual}$

Use A_{st} at mid span reinforcement and calculate p_t as per A_{st},

Remaining procedure is same as Design of one way slab

WORKED EXAMPLES ON DESIGN OF ONE WAY CONTINUOUS SLAB

1. **Design a simply supported one way continuous three span slab of clear span 4m each. The live load on the slab is 3 kN/m² and a floor finish (imposed dead load) of 1.5 kN/m². Use M20 grade concrete and Fe 500 grade steel.**

Solution :

Given $l = 4$m, $LL = 3$ kN/m², $FF = 1.5$ kN/m², $f_{ck} = 20$ N/mm² and $f_y = 500$ N/mm²

Assume 230 mm wall thickness

Step 1 : Thickness of slab (D)

$$\text{Effective depth, } d = \frac{l}{30} = \frac{4000}{30} = 133.33 \text{ mm, say } 140 \text{ mm}$$

$$\text{Assume } d^1 = 20 \text{ mm}$$

$$\therefore \text{Overall depth, } D = d + d^1 = 140 + 20 = 160 \text{ mm}$$

Step 2 : Effective Span (L) : least of the following two

(i) L = Clear span + Bearing = 4 + 0.23 = 4.23 m

(ii) L = Clear span + Effective depth = 4 + 0.14 = 4.14 m

say $L = 4.15$ m

Step 3 : Load calculation

(a) Dead Load (W_{DL})

(i) Self Weight of slab $= D \times 1 \times \text{density of RCC}$

$= 0.16 \times 1 \times 25 \quad = 4 \text{ kN/m}$

(ii) Floor Finish $= 1.5 \times 1 \quad = 1.5 \text{ kN/m}$

Total dead load, W_{DL} $= 5.5 \text{ kN/m}$

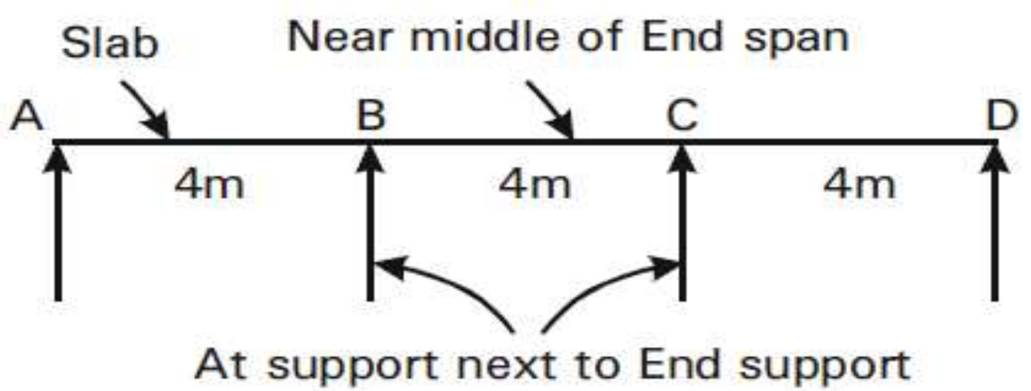
$\therefore$ Factored dead load, $\left(W_{u_{DL}}\right) = 1.5 \times 5.5 \quad = 8.25 \text{ kN/m}$

(b) Live load (W_{LL}) $= 3 \times 1 \quad = 3 \text{ kN/m}$

$\therefore$ Factored Live loads, $\left(W_{u_{LL}}\right) = 1.5 \times 3 \quad = 4.5 \text{ kN/m}$

Step 4 : Bending Moment Calculation

Refer IS : 456 -2000, Table 12 (Bending Moment Coefficients)



Cont…

- Maximum Negative BM (At supports)

$$M_u = -1.5\left(\frac{W_{u_{DL}} L^2}{10} + \frac{W_{u_{LL}} L^2}{9}\right)$$

$$= -1.5\left(\frac{8.25 \times 4.15^2}{10} + \frac{4.5 \times 4.15^2}{9}\right) = -1.5\,(14.20 + 8.61)$$

$$M_u = -34.21 \text{ kN.m}$$

$$\text{i.e., } M_{u_{-ve}} = 34.21 \text{ kN.m or } 34.21 \times 10^6 \text{ N.mm}$$

- Maximum Positive BM (At Mid span)

$$M_u = 1.5\left(\frac{W_{u_{DL}} L^2}{12} + \frac{W_{u_{LL}} L^2}{10}\right)$$

$$= 1.5\left(\frac{8.25 \times 4.15^2}{12} + \frac{4.5 \times 4.15^2}{10}\right) = 1.5\,(11.84 + 7.75)$$

$$M_{u_{+ve}} = 29.38 \text{ kN.m or } 29.38 \times 10^6 \text{ N.mm}$$

Step 5 : Check for depth

$$d_{\text{required}} = \sqrt{\frac{M_u}{0.133 f_{ck} b}} \quad \text{—— for Fe 500}$$

Out of the two values of moments $\left(M_{u_{-ve}} \text{ and } M_{u_{+ve}}\right)$, the effective depth will be determined for maximum value

$$\therefore M_{u_{-ve}} = M_u = 34.21 \times 10^6 \text{ N.mm}$$

$$d_{\text{required}} = \sqrt{\frac{34.21 \times 10^6}{0.133 \times 20 \times 1000}} = 113.40 \text{ mm} < 140 \text{ mm (ok)} \qquad | \therefore d_{\text{required}} < d_{\text{provided}}$$

$\therefore$ Design is safe

Step 6 : Determination of area of steel

1. Maximum negative BM - Support Moment

$$M_u = 34.21 \times 10^6 \text{ N.mm}$$

From IS : 456-2000,

$$M_u = 0.87 f_y A_{st} d \left[1 - \left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$$

$$34.21 \times 10^6 = 0.87 \times 500 \times 140\, A_{st}\left[1-\left(\frac{500A_{st}}{20\times1000\times140}\right)\right]$$

$$10.87\, A_{st}^2 - 60900\, A_{st} + 34.21 \times 10^6 = 0$$

$$A_{st} = 633.4\text{mm}^2$$

Assume 10 mm ϕ bars

$$\therefore \text{Spacing, } S = \frac{1000a_{st}}{A_{st}} = \frac{1000\times\frac{\pi}{4}\times10^2}{633.4} = 123.99 \text{ mm}$$

But maximum spacing, 3d = 3 × 140 = 420 mm or 300 mm, whichever is less

$\therefore$ Provide 10 mm ϕ bars @ 120 mm c/c at supports $\left(A_{st_{provided}} = 655\text{mm}^2\right)$

2. Maximum positive BM - Span Moment

$$M_u = 29.38 \times 10^6 \text{ N.mm} = 0.87\, f_y A_{st} d\left[1-\left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$$

$$29.38 \times 10^6 = 0.87 \times 500 \times 140\, A_{st}\left[1-\left(\frac{500A_{st}}{20\times1000\times140}\right)\right]$$

$$10.87\, A_{st}^2 - 60900\, A_{st} + 29.38 \times 10^6 = 0$$

$$A_{st} = 533.16\text{ mm}^2$$

Assume 10 mm ϕ bars

$$\therefore \text{Spacing}, S = \frac{1000 a_{st}}{A_{st}} = \frac{1000 \times \frac{\pi}{4} \times 10^2}{533.16} = 147.31 \text{ mm}$$

∴ Provide 10 mm ϕ bars @ 140 mm c/c at mid span (< 3d or 300 mm) $\left[A_{st_{\text{provided}}} = 561 \text{mm}^2\right]$

3. Distribution of steel

$$A_{st} = 0.12\% \text{ of gross cross sectional area}$$

$$A_{st} = \frac{0.12}{100} \times b \times D = \frac{0.12}{100} \times 1000 \times 160 = 192 \text{ mm}^2$$

Assume 8mm ϕ bars

$$\therefore \text{Spacing}, S = \frac{1000 a_{st}}{A_{st}} = \frac{1000 \times \frac{\pi}{4} \times 8^2}{192} = 261.8 \text{ mm}$$

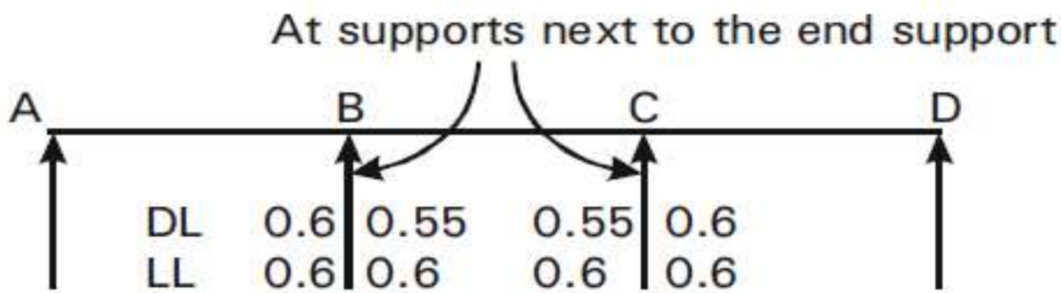
But maximum spacing, 5d = 5 × 140 = 700 mm or 450 mm, whichever is less

∴ Provide 8mm ϕ bars @ 250 mm c/c along longer span

Step 7 : Check for shear **Cont...**

Refer IS : 456 - 2000

Table - 13, shear force coefficients



$$\text{Dead Load, } DL = 0.6 \text{ (Take maximum value)}$$

$$\text{Live Load, } LL = 0.6 \text{ (Take maximum value)}$$

$$V_u = \left(0.6\,W_{u_{DL}} + 0.6\,W_{u_{LL}}\right) \times L$$

$$= 0.6\,(8.25 + 4.5) \times 4.15$$

- Shear Force, $V_u = 31.75$ kN
- Nominal Shear Stress, $\tau_v = \dfrac{V_u}{bd} = \dfrac{31.75 \times 10^3}{1000 \times 140} = 0.22 \text{ N/mm}^2$
- Percentage of steel, $p_t = \dfrac{100A_{st}}{bd} = \dfrac{100 \times 655}{1000 \times 140} = 0.46\%$
- Design shear strength of concrete, τ_c

  Now refer, IS : 456-2000, Table 19, using $p_t = 0.46\%$ and $f_{ck} = 20 \text{N/mm}^2$

$$\tau_c = 0.46 \text{ N/mm}^2 \quad \text{(By using interpolation method)}$$

➤ Permissible shear stress, $k\tau_c$

Slab depth	k
150	1.30
160	?
175	1.25

$$\therefore k = 1.28 \text{ (By using interpolation method)}$$

$$\therefore k\tau_c = 1.28 \times 0.46 = 0.58 \text{ N/mm}^2 > \tau_v$$

Hence the shear stresses are within safe permissible limits (ok)

Step 8 : Check for deflection control

1. $\left(\frac{L}{d}\right)_{max} = 26\ k_t \times k_c \times k_f$ $\qquad \left|\ \frac{L}{d} = 26\right.$ – for continuous slab

Now, refer IS : 456-2000, Fig. 4

$$p_t = \frac{100 A_{st}}{bd} = \frac{100 \times 561}{1000 \times 140} = 0.4\%$$

$\left.\begin{array}{l} A_{st_{required}} = 533.16 \text{mm}^2 \\ A_{st_{provided}} = 561 \text{mm}^2 \end{array}\right\}$ Span reinforcement

$$f_s = 0.58 f_y \left[\frac{A_{st_{required}}}{A_{st_{provided}}}\right] = 0.58 \times 500 \times \frac{533.16}{561}$$

$$\therefore f_s = 275.60 \text{ N/mm}^2$$

Now using, f_s and p_t, calculate modification factor (k_t)

$$\therefore k_t = 1.2$$

Here $k_c = 1$ and $k_f = 1$ (No flange section and no compression steel)

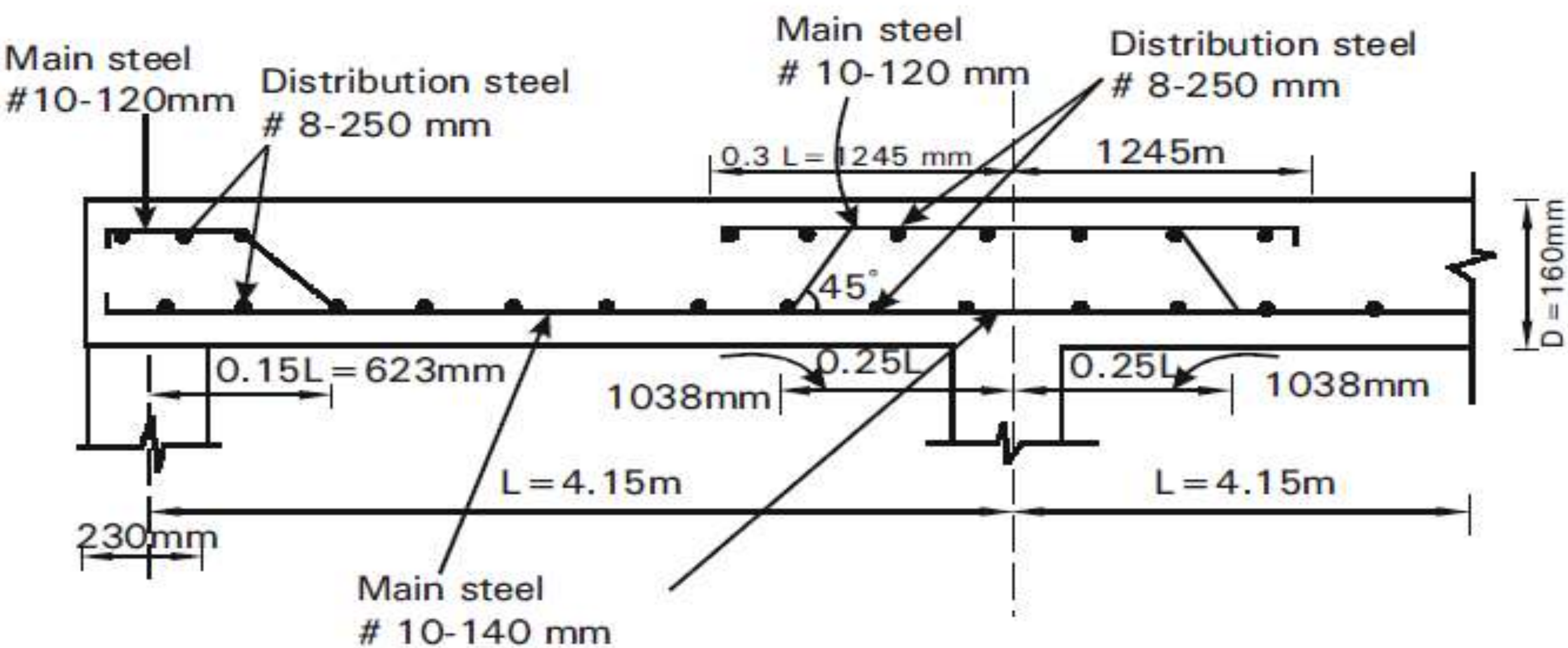
$$\therefore \left(\frac{L}{d}\right)_{max} = 26 \times 1.2 \times 1 \times 1 = 31.2$$

2. $$\left(\frac{L}{d}\right)_{actual} = \frac{4150}{140} = 29.64$$

$$\therefore \left(\frac{L}{d}\right)_{max} > \left(\frac{L}{d}\right)_{actual}$$

Hence, the slab is safe against deflection control

Step 9 : Reinforcement details



DESIGN OF TWO WAY SLAB

Given Data: Room size, Grade of concrete & steel, Live load and floor finishes.

Design: Depth of slab, Area of main reinforcement and check for deflection control.

Steps:

1. Check slab is One way or Two way. (If $\frac{l_y}{l_x} \leq 2$, two way slab)

   Calculate $\frac{l_y}{l_x}$ ratio and check one way or two way slab.

2. For simply supported two way slabs, the shorter span to depth ratio, generally assumed as 35. For HYSD bars of grade Fe415, the value should be multiplied by 0.8 (i.e., 35×0.8).

   [Refer IS : 456 - 2000, clause 24.1(2)]

   Assume effective depth of slab (100mm to 150mm) or $d = \frac{l}{25 \text{ to } 35}$

   Overall depth, D = d + d' (Assume d')

3. Effective span (L_x): Least of the following two

   i) L_x = clear span + bearing.

   ii) L_x = clear span + effective depth (d).

4. Load calculation

i) Self weight of slab = $D \times 1 \times$ Density of RCC (ρ) = __________ kN/m

ii) Live load (given) × 1 = __________ kN/m

iii)Floor finish (If given) × 1 = __________ kN/m

Total load W = __________ kN/m

Factored or ultimate load, $W_u = 1.5 \times W$

5. Bending Moment calculation

For calculated $\frac{l_y}{l_x}$ referring IS : 456-2000, Table No. 26 or 27, depending on end conditions read out the values of α_x and α_y, then calculate

$$M_{ux} = \alpha_x W_u L_x^2$$

$$M_{uy} = \alpha_y W_u L_x^2$$

6. Check for depth

Equating $M_{ux} = M_u$

$$M_u = 0.148 f_{ck} bd^2 \text{ – Fe250 (Mild steel)}$$

$$M_u = 0.138 f_{ck} bd^2 \text{ – Fe415 (HYSD bars)}$$

$$M_u = 0.133 f_{ck} bd^2 \text{ – Fe500 (HYSD bars)}$$

$$d_{\_required} = \sqrt{\frac{M_{ux}}{0.138 \times f_{ck} b}} \quad \text{For Fe 415}$$

If $d_{\_required} < d_{\text{provided,}}$ Design is safe

If $d_{\_required} > d_{\text{provided}}$, Design is not safe and revise the depth.

7. Main reinforcement along Shorter span (A_{stx})

From IS : 456 - 2000, G-1.1(b)

$$M_u = 0.87 f_y A_{st} d\left[1-\left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right],$$ calculate 'A_{st_x}'

Assume suitable diameter of bars (eg: $\phi = 8$mm, 10mm, 12mm)

$$\text{Area of one bar, } a_{st} = \frac{\pi \times \phi^2}{4}$$

$$\text{Spacing of reinforcement, } S = \frac{1000 a_{st}}{A_{stx}}$$

8. Main reinforcement along Longer span (A_{sty})

$$M_u = 0.87 f_y A_{st} d\left[1-\left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right],$$ calculate 'A_{st_y}'

Assume suitable diameter of bars (eg: $\phi = 8$mm, 10mm,12mm)

$$\text{Area of one bar, } a_{st} = \frac{\pi \times \phi^2}{4}$$

$$\text{Spacing of reinforcement, } S = \frac{1000 a_{st}}{A_{sty}}$$

9. Torsional steel or corner steel reinforcement

If corners are held down, then calculate Torsional steel

$\therefore$ Area of torsional steel at each of the corners in 4 layers

$$\text{Area of torsional steel} = \frac{3}{4} \times \text{mid span steel}$$

$$\text{or} \quad \frac{3}{4} \times \text{main steel along shorter span}$$

$$\text{Spacing, S} = \frac{1000 a_{st}}{A_{st_t}}$$

| A_{st_t} - torsional steel area

10. Check for shear stresses or Design for shear stresses

Considering the short span and unit width of slab

- $V_u = \frac{W_u L_x}{2}$ — For simply supported slab
- Nominal shear stress, $\tau_v = \frac{V_u}{bd}$
- Percentage of steel, $p_t = \frac{100 A_{st}}{bd}$
- Design shear strength of concrete, τ_c

Now refer IS : 456-2000, Table 19, using p_t and f_{ck}

Note down the value of 'τ_c'

- Permissible shear stress, $k\tau_c$

  'k' is depends on slab depth, Refer IS : 456-2000, clause 40.2.1.1

if $k\tau_c > \tau_v$ — Design is safe

if $k\tau_c < \tau_v$ — Design is not safe, Revise the depth

11. Check for deflection control,

    Same as Design of one way slab

12. Reinforcement details

DESIGN OF TWO WAY SLAB

WORKED EXAMPLES ON DESIGN OF TWO WAY SLABS

1. **Design a slab over a room of internal dimensions 4m × 5m supported on 230mm thick brick wall. All the edges are simply supported (The corners are free to lift). Use live load 2kN/m², Floor finish 1kN/m². Take M20 concrete and Fe415 Steel. Sketch the reinforcement details.**

Solution:

Given : $l_x = 4$m, $l_y = 5$m, brick wall thick = 230 mm, $LL = 2$ kN/m², $FF = 1$ kN/m²

$F_{ck} = 20$ N/mm² and $f_y = 415$ N/mm²

Step : 1 Check for longer to shorter span ratio

$$\frac{l_y}{l_x} = \frac{5}{4} = 1.25 < 2$$

∴ slab is designed as a two way slab

Step : 2 Thickness of slab (D)

$$\text{Effective depth, } d = \frac{l_x}{28} = \frac{4000}{28} = 142.85 \text{ mm, say } d = 140 \text{ mm}$$

Assume $d' = 20$mm

∴ Overall depth, $D = d + d' = 140 + 20 = 160$ mm

Step : 3 Effective span (L_x): Least of the following two

i) L_x = clear span (l) + bearing = 4 + 0.23 = 4.23 m

ii) L_x = clear span (l) + effective depth (d) = 4 + 0.14 = 4.14 m

$L_x = 4.14$ m.

Cont...

Step : 4 Load calculation

i) Self weight of slab = $0.16 \times 1 \times 25$ $= 4\ kN/m$

ii) Live load (Given) = 2×1 $= 2.0\ kN/m$

iii) Floor finish (Given) = 1×1 $= 1.0\ kN/m$

Total load, $W = 7\ kN/m$

Factored or ultimate load, $W_u = 1.5 \times 7 = 10.5\ kN/m$

Step : 5 Bending Moment calculation

Refer IS : 456 : 2000, Annex - D, D-1.1

$$M_{ux} = \alpha_x W_u Lx^2$$

$$M_{uy} = \alpha_y W_u Lx^2$$

By referring, Table No.27, read out the values of α_x & α_y corresponding value of

$\frac{l_y}{l_x} = \frac{5}{4} = 1.25.$

$\frac{l_y}{l_x}$	α_x	α_y
1.20	0.084	0.059
1.30	0.093	0.055

$$\text{For } 1.25,\ \alpha_x = 0.084 + \frac{0.009}{0.1} \times 0.05$$
$$= 0.0885$$
$$\alpha_y = 0.059 - \frac{0.004}{0.1} \times 0.05$$
$$= 0.057$$

$M_{ux} = 0.0885 \times 10.5 \times 4.14^2 = 15.93\ kN\text{-}m = 15.93 \times 10^6\ N\text{-}mm$

$M_{uy} = 0.057 \times 10.5 \times 4.14^2 = 10.25\ kN\text{-}m = 10.25 \times 10^6\ N\text{-}mm$

Step : 6 Check for depth

Equating $M_{ux} = M_{u\lim} = 0.138\ f_{ck}\ bd^2$

$$d_{required} = \sqrt{\frac{M_{ux}}{0.138 \times f_{ck} b}} = \sqrt{\frac{15.93 \times 10^6}{0.138 \times 20 \times 1000}} = 75.97\,mm$$

$d_{req} < d_{provided}$, Design is safe.

Step : 7 Main reinforcement along Shorter span (A_{stx})

$$M_u = 0.87 f_y A_{st} d \left[1 - \left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$$

$$15.93 \times 10^6 = 0.87 \times 415 \times 140 A_{st_x} \left[1 - \left(\frac{415 A_{st_x}}{20 \times 1000 \times 140}\right)\right]$$

$$7.49 A_{st_x}^2 - 50547 A_{st_x} + 15.93 \times 10^6 = 0$$

$$\therefore A_{st_x} = 331.42\text{mm}^2$$

Providing of 10mm dia bars

$$\text{Spacing, } S = \frac{1000a_{st}}{A_{st_x}} = \frac{78.53}{331.42} \times 1000 = 236.97mm, say\ 225mm$$

Provide 10mm dia bars @225mm c/c along shorter span $\left[A_{st_{prov}} = 349.02mm^2\right](< 3d \text{ or } 300\text{mm})$

Step : 8 Main reinforcement along longer span (A_{st_y})

$$M_u = 0.87 f_y A_{st} d\left[1 - \left(\frac{f_y A_{st_y}}{f_{ck} bd}\right)\right]$$

$$10.25 \times 10^6 = 0.87 \times 415 \times 140 A_{st_y}\left[1 - \left(\frac{415 A_{st_y}}{20 \times 1000 \times 140}\right)\right]$$

$$7.49 A_{st_y}^2 - 50547 A_{st_y} + 10.25 \times 10^6 = 0$$

$$A_{st_y} = 209.27mm$$

Provide of 8*mm* dia bars

$$\text{Spacing, S} = \frac{1000a_{st}}{A_{sty}} = \frac{\frac{\pi(8)^2}{4}}{209.27} \times 1000 = 240.19mm.$$

Cont…

$\therefore$ Provide 8mm dia bars @ 225mm c/c along longer span $\left[A_{st_{prov}} = 223.40mm^2\right]$

(<3d or 300 mm)

* Check for minimum reinforcement $A_{st,\min} = \frac{0.12}{100} \times 1000 \times 160 = 192mm^2 < A_{st_x}$ and A_{st_y} (OK)

Step 9 : Check for shear stresses

Consider the shorter span

- $V_u = \frac{W_u L_x}{2} = \frac{10.5 \times 4.14}{2}$ 21.73 kN
- Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{21.73 \times 10^3}{1000 \times 140} = 0.15$ N/mm²
- Percentage of steel, $p_t = \frac{100A_{st}}{bd} = \frac{100 \times 349.02}{1000 \times 140} = 0.25\%$
- Design shear strength of concrete, τ_c

  Now refer IS : 456-2000, Table 19, using $p_t = 0.25\%$ and $f_{ck} = 20$ N/mm², $\tau_c = 0.36$ N/mm²
- Permissible shear stress, $k\tau_c$

  Refer IS : 456-2000, Clause 40.2.1.1

  $k = 1.28$ for slab depth = 160 mm, using interpolation method

  $\therefore k\tau_c = 1.28 \times 0.36 = 0.46$ N/mm² $> \tau_v$, Design is safe

  Hence the shear stress are within safe permissible limits

Step 10 : Check for deflection control

1. $\left(\frac{L}{d}\right)_{max} = 20\ k_t \times k_c \times k_f$

Now, refer IS : 456-2000, Fig. 4

$$p_t = 0.25\%$$

$$f_s = 0.58\ f_y \left[\frac{A_{st_{required}}}{A_{st_{provided}}}\right] = 0.58 \times 415 \times \frac{331.42}{349.02} = 228.56\ \text{N/mm}^2$$

Readout modification factor (k_t) from curve (From Fig. 4 of IS : 456-2000) for f_s = 228.56 N/mm² and p_t = 0.25%

$$\therefore k_t = 1.6, \quad \because k_c = 1 \text{ and } k_f = 1$$

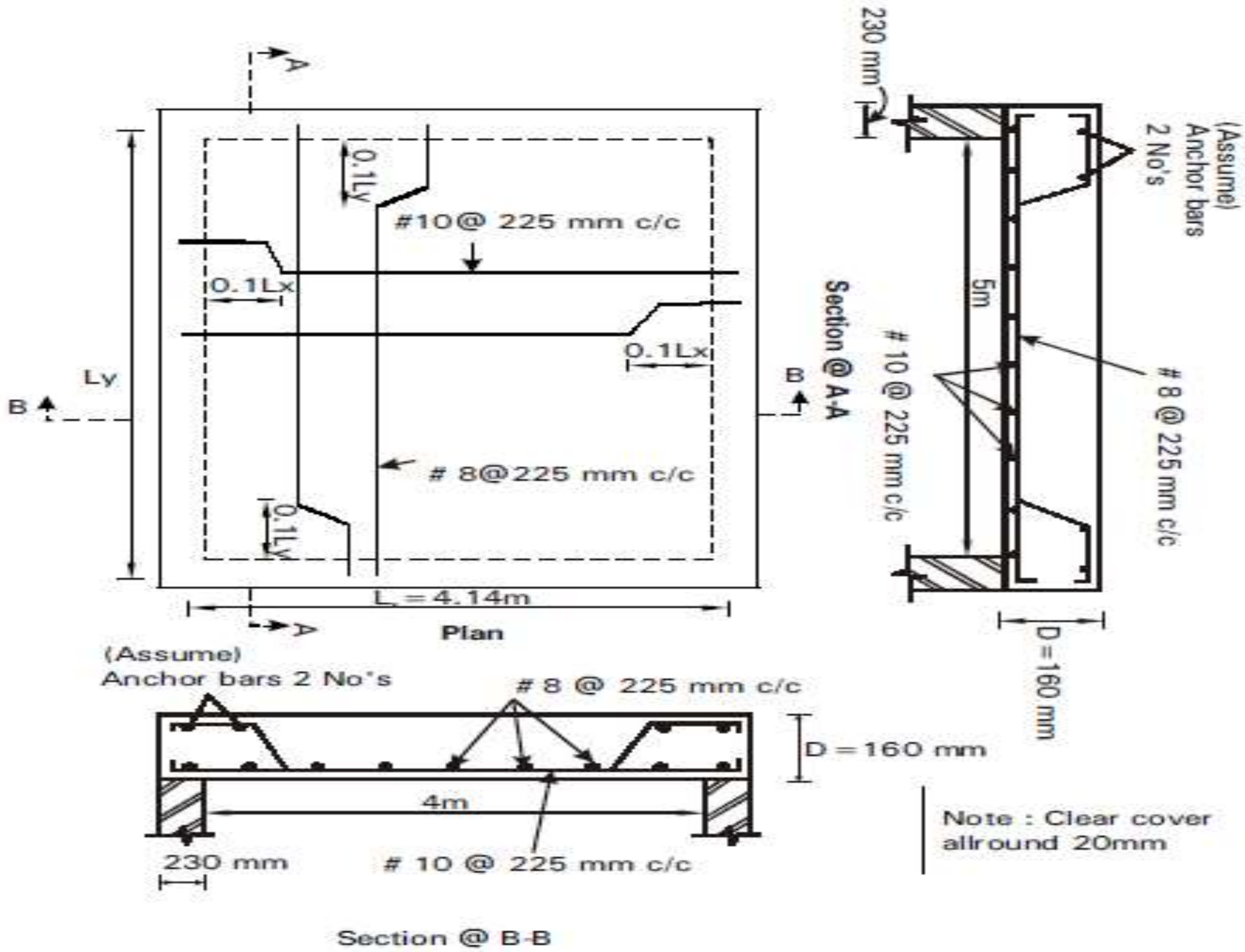
$$\left(\frac{L}{d}\right)_{max} = 20\ k_t \times k_c \times k_f = 20 \times 1.6 \times 1 \times 1 = 32$$

2. $\left(\frac{L}{d}\right)_{actual} = \frac{4140}{140} = 29.57$

$$\therefore \left(\frac{L}{d}\right)_{max} > \left(\frac{L}{d}\right)_{actual}$$

Hence safe, deflection is within permissible limits

Step 11 : Reinforcement details



2. **Design a slab over a room of internal (clear) dimension 4m × 5m supported on 230 mm thick brick wall. All the edges are simply supported (The corners of slab are held down) or (Four edges discontinuous). Use Live load 3 kN/m², Floor finish 1 kN/m². Take M20 concrete and Fe 415 steel. Sketch the reinforcement details.**

Solution :

Given : $l_x = 4\text{m}, l_y = 5\text{m}$, wall thickness = 230 mm

$LL = 3\text{ kN/m}^2, FF = 1\text{ kN/m}^2, f_{ck} = 20\text{ N/mm}^2$ and $f_y = 415\text{ N/mm}^2$

Step 1 : Check for longer to shorter span ratio

$$\frac{l_y}{l_x} = \frac{5}{4} = 1.25 < 2$$

∴ Slab is designed as a two way slab

Step 2 : Thickness of slab (D)

$$\text{Effective depth, } d = \frac{l_x}{28} = \frac{4000}{28} = 142.85\text{ mm, say } d = 140\text{ mm}$$

$$\text{Assume } d^1 = 20\text{ mm}$$

$$\therefore \text{Overall depth, } D = d + d^1 = 140 + 20 = 160\text{ mm}$$

Step 3 : Effective Span (L_x) : least of the following two

(i) L_x = clear span (l_x) + bearing = 4 + 0.23 = 4.23 m

(ii) L_x = clear span (l_x) + effective depth = 4 + 0.14 = 4.14 m

$$\therefore L_x = 4.14\text{ m}$$

Step 4 : Load Calculation

(i)	Self weight of slab	= $0.16 \times 1 \times 25$	=	4 kN/m
(ii)	Live load (Given)	= 3×1	=	3 kN/m
(iii)	Floor Finish (Given)	= 1×1	=	1 kN/m
		Total load, W	=	8 kN/m

$\therefore$ Factored or ultimate load, $W_u = 1.5 \times 8 = 12$ kN/m

Step 5 : Bending Moment Calculation

Refer IS : 456-2000, Annex–D, D – 1.1

$$M_{ux} = \alpha_x W_u L_x^2$$

$$M_{uy} = \alpha_y W_u L_x^2$$

By referring Table No. 26, read out the values of α_x and α_y corresponding value of

$$\frac{L_y}{L_x} = \frac{5}{4} = 1.25$$

Refer case No. 9 — Four Edges Discontinuous

$\frac{L_y}{L_x}$	α_x
1.2	0.072
1.25	?
1.3	0.079

$\therefore \alpha_x = 0.0755$ (by interpolation)

$\alpha_y = 0.056$

$\therefore M_{ux} = 0.0755 \times 12 \times 4.14^2 = 15.52$ kN.m or 15.52×10^6 N.mm

$M_{uy} = 0.056 \times 12 \times 4.14^2 = 11.51$ kN.m or 11.51×10^6 N.mm

Step 6 : Check for depth

$$d_{\text{required}} = \sqrt{\frac{M_{ux}}{0.138 f_{ck} b}} = \sqrt{\frac{15.52\times10^6}{0.138\times20\times1000}} = 74.98 \text{ mm} < d_{\text{provided}}$$

Hence (ok), Design is safe

Step 7 : Main reinforcement along shorter span $\left(A_{st_x}\right)$

$$M_u = 0.87\, f_y A_{st} d \left[1-\left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$$

$$15.52\times10^6 = 0.87\times415\times140\, A_{st_x}\left[1-\left(\frac{415 A_{st_x}}{20\times1000\times140}\right)\right]$$

$$7.49\, A_{st_x}^2 - 50547\, A_{st_x} + 15.52\times10^6 = 0$$

$$\therefore A_{st_x} = 322.45 \text{ mm}^2$$

Providing 10 mm dia bars

$$\text{Spacing}, S = \frac{1000 a_{st}}{A_{st_x}} = \frac{1000\times78.53}{322.45} = 243.54 \text{mm}$$

$\therefore$ Provide 10 mm dia bars @ 240 mm c/c along shorter span (<3d or 300mm)

$$\left[A_{st_{\text{provided}}} = 327.20 \text{mm}^2\right]$$

Step 8 : Main reinforcement along longer span $\left(A_{st_y}\right)$

$$M_u = 0.87\, f_y A_{st} d \left[1-\left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$$

$$11.51 \times 10^6 = 0.87 \times 415 \times 140\, A_{st_y}\left[1-\left(\frac{415 A_{st_y}}{20 \times 1000 \times 140}\right)\right]$$

$$7.49\, A_{st_y}^2 - 50547\, A_{st_y} + 11.51 \times 10^6 = 0$$

$$\therefore A_{st_y} = 235.95 \text{ mm}^2$$

Providing 8 mm dia bars

$$\text{Spacing}, S = \frac{1000 a_{st}}{A_{st_y}} = \frac{1000 \times 50.26}{235.95} = 213.03 \text{ mm}$$

$\therefore$ Provide 8mm dia bars @ 200 mm c/c along longer span (< 3d or 300 mm)

$\left[A_{st_{provided}} = 251.3 \text{mm}^2\right]$

* Check for minimum reinforcement,

$$A_{st_{min}} = \frac{0.12}{100} \times b \times D = \frac{0.12}{100} \times 100 \times 160 = 192 \text{mm}^2 < A_{st_x} \text{ and } A_{st_y} \text{ (ok)}$$

Step 9 : Torsional steel or corner steel reinforcement

Area of torsional steel at each of the corners in 4 layers

Area of torsional steel $= \frac{3}{4} \times$ mid span steel or main steel along shorter span

$$= \frac{3}{4} \times 327.20$$

$$A_{st_t} = 245.4 \text{ mm}^2$$

Providing 8mm dia bars

$$\text{Spacing}, S = \frac{1000 a_{st}}{A_{st_t}} = \frac{1000 \times 50.26}{245.4} = 204.80 \text{ mm}$$

Length over which torsion steel is provided $= \frac{1}{5} \times$ short span $= \frac{1}{5} \times 4140 = 828 \text{ mm} \approx 830 \text{ mm}$

$\therefore$ Provide 8mm dia bars @ 200 mm c/c for a length of 830 mm at all four corners in 4 layers

Step 10 : Check for Shear Stresses

Consider the shorter span

- $V_u = \frac{W_u L_x}{2} = \frac{12 \times 4.14}{2} = 24.84 \text{ kN}$

- Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{24.84 \times 10^3}{1000 \times 140} = 0.17 \text{ N/mm}^2$

- Percentage of steel, $p_t = \frac{100 A_{st_x}}{bd} = \frac{100 \times 327.20}{1000 \times 140} = 0.23\%$
- Design shear strength of concrete, τ_c

  Now, refer IS : 456-2000, Table 19, using $p_t = 0.23\%$ and $f_{ck} = 20$ N/mm^2

  $\tau_c = 0.344$ N/mm^2 (By interpolation method)
- Permissible shear stresses, $k\tau_c$

  Refer IS : 456-2000, clause 40.2.1.1

  $k = 1.28$ for slab depth = 160 mm, (By Interpolation method)

  $\therefore k\tau_c = 1.28 \times 0.344 = 0.44$ N/mm^2 $> \tau_v$, Design is safe

  Hence the shear stresses are within safe permissible limits

Step 11 : Check for deflection control

1. $\left(\frac{L}{d}\right)_{max} = 20\ k_t \times k_c \times k_f$

   Now, refer IS : 456-2000, Fig. 4

$$p_t = 0.23\%$$

$$f_s = 0.58\, f_y \left[\frac{A_{st_{required}}}{A_{st_{provided}}}\right] = 0.58 \times 415 \times \frac{322.45}{327.20} = 237.20 \text{ N/mm}^2$$

Readout modification factor (k_t) from curve (From Fig. 4 of IS : 456-2000) for f_s = 237.20 N/mm^2 and p_t = 0.23%

$$\therefore k_t = 1.6, \quad \because k_c = 1 \text{ and } k_f = 1$$

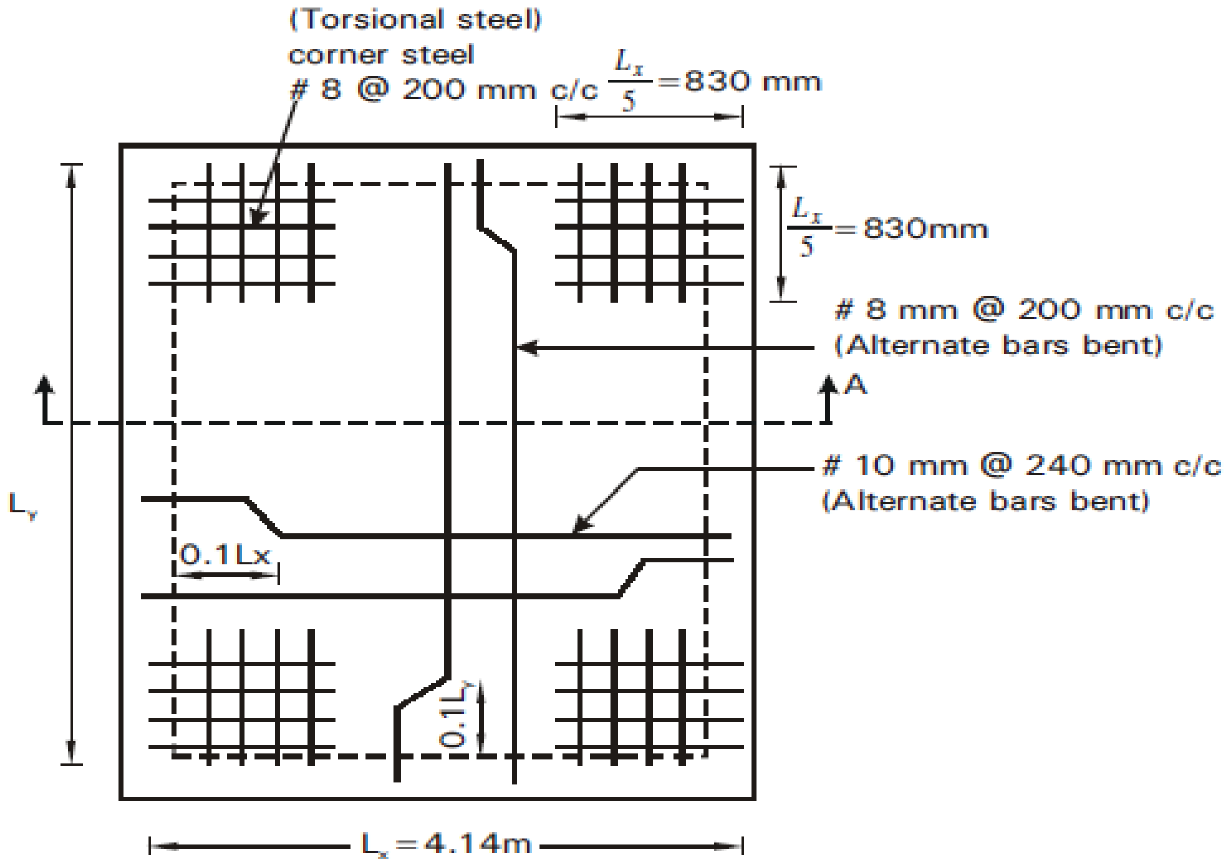
$$\left(\frac{L}{d}\right)_{max} = 20\, k_t \times k_c \times k_f = 20 \times 1.6 \times 1 \times 1 = 32$$

2. $\left(\frac{L}{d}\right)_{actual} = \frac{4140}{140} = 29.57$

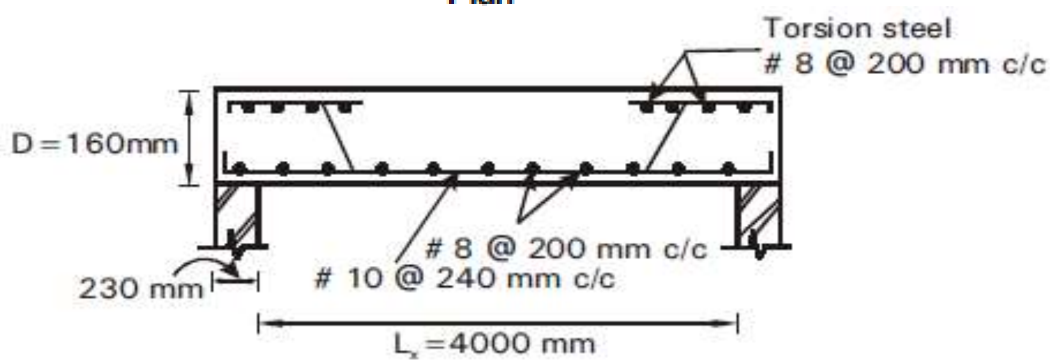
$$\therefore \quad \left(\frac{L}{d}\right)_{max} > \left(\frac{L}{d}\right)_{actual}$$

Hence safe, deflection is within permissible limits

Step 12 : Reinforcement details



Plan



Section @ A-A

DESIGN OF STAIRCASE

Stairs are needed for ascending and descending from floor to floor. A stair consists of a number of steps to move from one level to another. The room/space housing stairs is called stair case.

Staircase flights are designed as slabs spanning between wall supports. Staircase behaves like an ordinary slab. The design of staircase requires proportioning of its different components and determination of reinforcement and its detailing to satisfy both the serviceability and strength requirements. Types of staircase

1. Single flight staircase
2. Single or quarter turn staircase
3. Dog legged staircase
4. Open well staircase
5. Spiral staircase
6. Helocoidal staircase
7. Free standing staircase

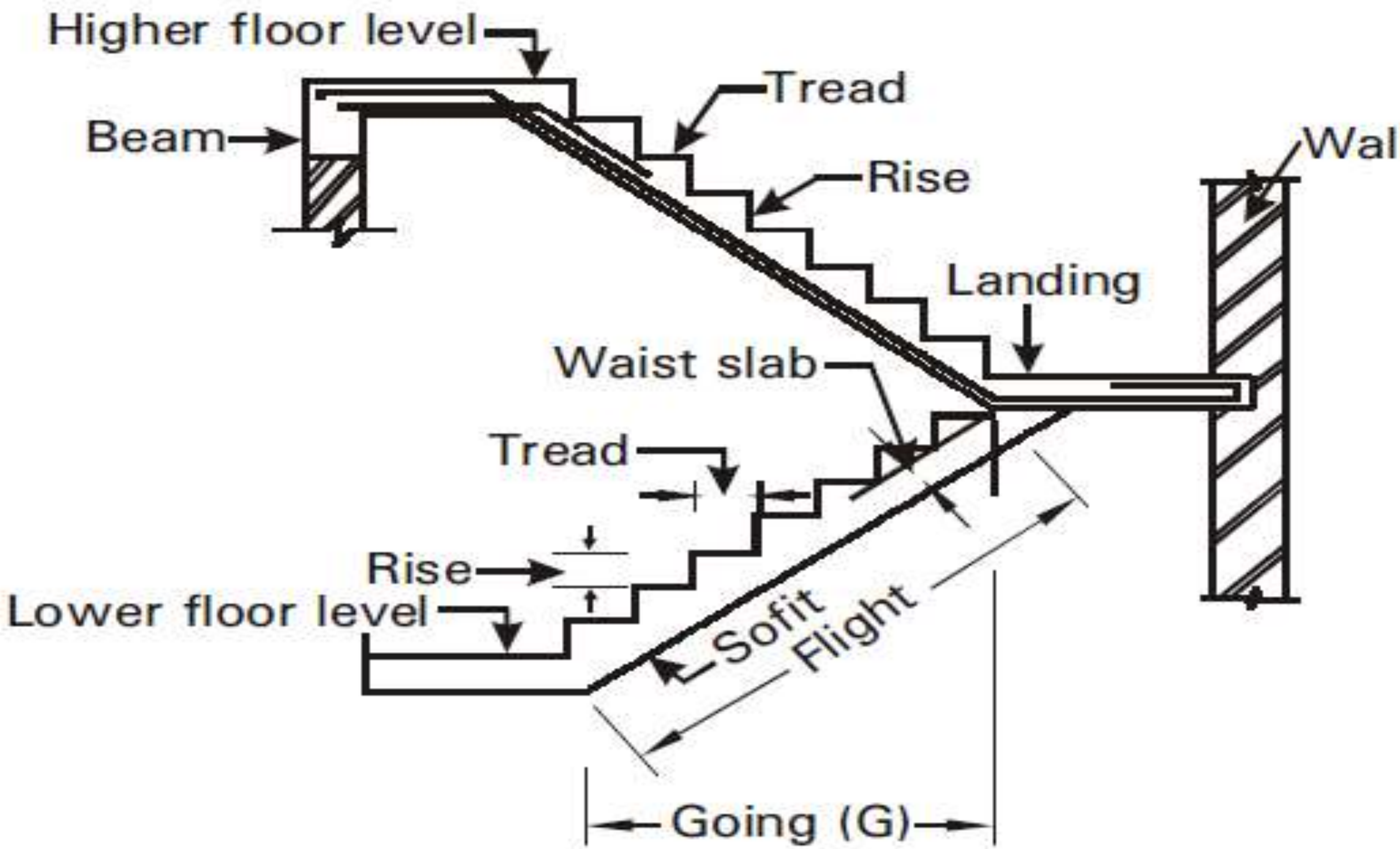


Fig. 5 : Components of staircase

- **Dog-legged stair case:** It is the common type used in all types of building. It comprises of two adjacent flights running parallel with a landing slab at mid height.

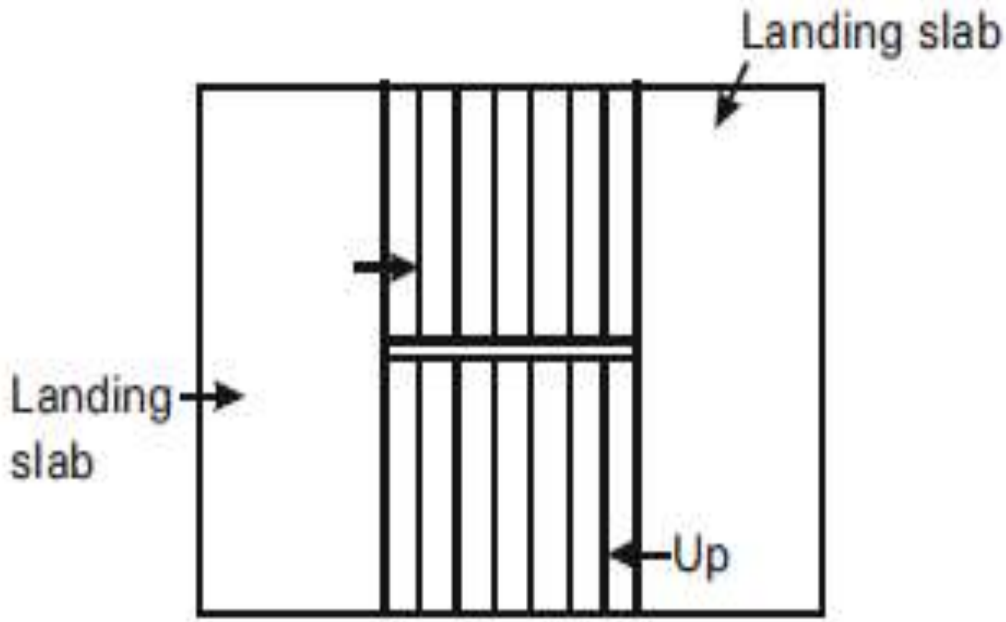


Fig. : Dog-legged stair case (plan)

➢ **Open well stair case:** It is generally adopted in public buildings where large spaces are available. This type of stair case consists of smaller flights and provides better accessibility comfort and good ventilation due to open well at the centre.

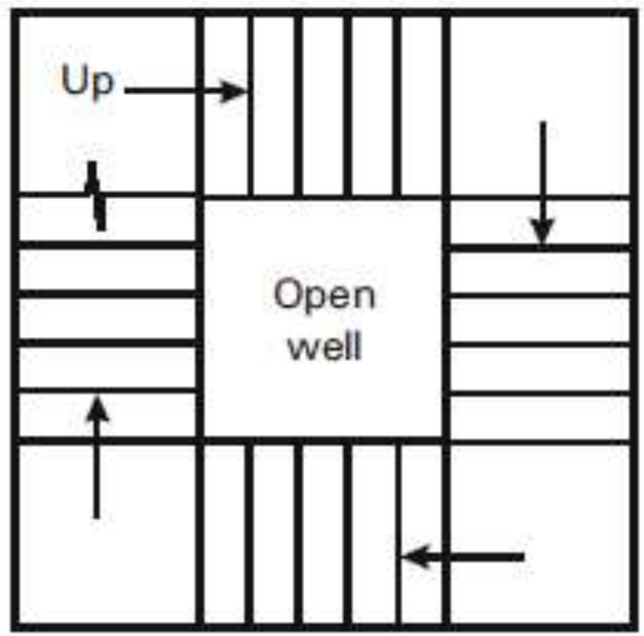


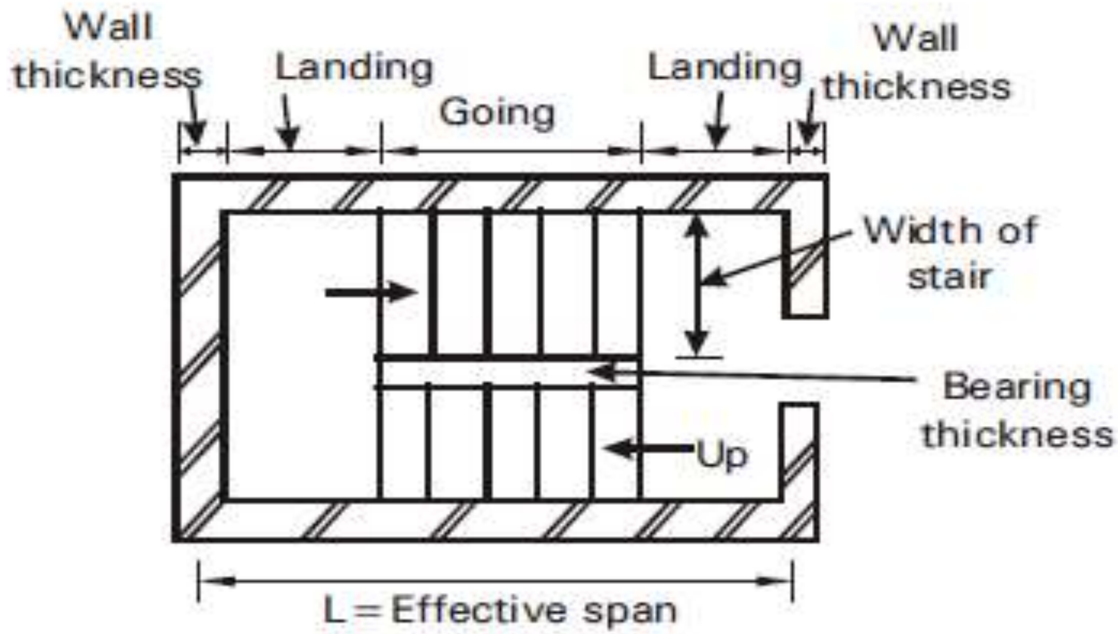
Fig. - : Open well stair case (plan)

DESIGN STEPS

1. Fixing dimensions of stair
   - Vertical distance between floor = H (given)
   - Height of each flight, $h = \frac{H}{2}$ - For Dog-legged staircase

     $h = \frac{H}{3} \text{ or } \frac{H}{4}$ - For Openwell staircase

   [Note: For dog-legged - 2 Flights, For open well - 3 or 4 Flights]
   - Assume Riser (R), suitably
   - No. of risers = $\frac{h}{R}$ [Round it to next value]
   - Actual riser (R) = $\frac{h}{\text{No. of risers}}$
   - No. of tread = No. of risers – 1
   - Bearing thickness, Assume 200 to 300 mm
   - Width of stair = $\frac{\text{Room width} - \text{Bearing thickness}}{2}$



- Width of landing = 1m (minimum) or width of stair
- Width of tread (T), Assume suitably 200 to 300mm
- Going = No. of treads × width of tread (T)
- Effective Horizontal span,

$$L = \frac{\text{Wall thickness}}{2} + \text{Landing} + \text{Going} + \text{Landing} + \frac{\text{Wall thickness}}{2}$$

2. Calculation of waist slab thickness

The thickness of waist slab may be assumed as $\frac{L}{20}$ to $\frac{L}{25}$ for trail section calculations or 40 to 50 mm per m span.

$\therefore d = \frac{L}{20 \text{ to } 25}$ Here L = Effective Span

d = Effective depth

d^1 = Effective cover

3. Load Calculation

(a) Loads on sloping slab

Self weight of slab on slope = $D \times RCC$ density = ____ kN/m²
Ceiling finishes (thickness × density) – Assume = ____ kN/m²
} W_s = ____ kN/m²

(1) Load on horizontal span = $W_S \times \frac{\sqrt{R^2 + T^2}}{T}$ = ____ kN/m²

(2) Dead load on step = $\frac{1}{2} \times R \times RCC$ density = ____ kN/m²

(3) Top finish (if given) or assume = ____ kN/m²

(4) Live load (Given) = ____ kN/m²

$\therefore$ Total load, W = (1) + (2) + (3) + (4) = ____ kN/m²

Factored load per m, $W_u = 1.5 \times W \times 1$ = ____ kN/m²

(b) Loads on Landing slab

Providing thickness of landing

(1) Self weight of landing slab = $D \times RCC$ density = ____ kN/m²

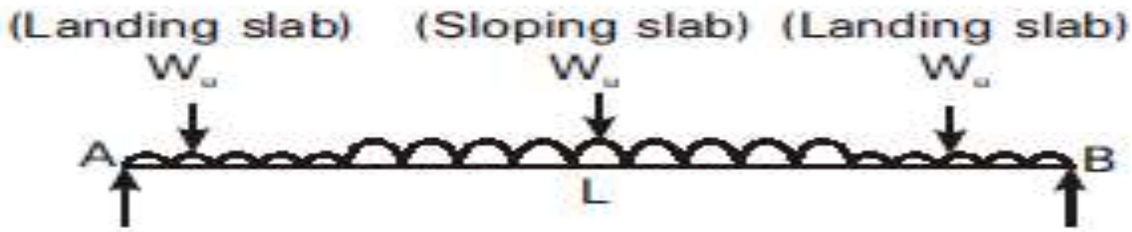
(2) Top finish (if given) or Assume = ____ kN/m²

(3) Live load (given) = ____ kN/m²

Total load, W = ____ kN/m²

Factored load per m, $W_u = 1.5 \times W \times 1$ = ____ kN/m²

4. Bending moment calculation (Design moment)



Calculate M_u using loading conditions

5. Check for depth

Equating $\quad BM\ (M_u) = M_{u_{lim}}$

$$M_{u_{lim}} = 0.148\, f_{ck} bd^2 - Fe\ 250 \text{ (Mild steel)}$$

$$\left.\begin{aligned} M_{u_{lim}} &= 0.138\, f_{ck} bd^2 - Fe\ 415 \\ M_{u_{lim}} &= 0.133\, f_{ck} bd^2 - Fe\ 500 \end{aligned}\right\} \text{ (HYSD bars)}$$

$$\therefore\ d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck} b}} - \text{For Fe 415}$$

Here $b = 1000$ mm or 1 m

if $d_{req} < d$, Design is safe (OK)

if $d_{req} > d$, Design is not safe, Revise the depth

6. Design of main reinforcement

Refer IS : 456-2000, G-1.1(b)

$$M_u = 0.87\, f_y A_{st} d \left[1 - \left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$$

Calculate 'A_{st}' using expression

Assume suitable diameter of bars (eg: $\phi = 8$mm, 10mm, 12mm, 16mm)

Area of one bar, $a_{st} = \frac{\pi}{4}\phi^2$

Spacing, $S = \frac{1000 a_{st}}{A_{st}}$

Maximum spacing, $3d$ or 300mm, whichever is less

7. Distribution steel

A_{st} = 0.12% of gross cross sectional area - For *HYSD* bars

i.e., $A_{st} = \frac{0.12}{100} \times b \times D$

A_{st} = 0.15% of gross cross sectional area - *Fe* 250 (Mild steel)

i.e., $A_{st} = \frac{0.15}{100} \times b \times D$

Assume suitable diameter of bars (eg: $\phi = 8$mm, 10mm, 12mm)

Area of one bar, $a_{st} = \frac{\pi}{4}\phi^2$

Spacing, $S = \frac{1000 a_{st}}{A_{st}}$

Maximum spacing, $5d$ or 450mm, whichever is less

8. Reinforcement details

(1)Plan showing staircase details

(2)Sections

WORKED EXAMPLES ON DOG-LEGGED AND OPEN WELL STAIR CASES

1. **Design a dog-legged staircase for a building in which the vertical distance between floors is 3.6m. The stair hall measures 3m × 6m. Take live load on the stairs is 4 kN/m^2. The flights are supported on 230 mm walls at the ends of outer edges. So, that it spans in the direction of going. Adopt *M*20 concrete and *Fe*415 grade steel. Sketch the reinforcement details.**

   Solution

   Given: Stair hall = 3m × 6m, floor height, $H = 3.6$m

   Here, Length = 6m, width = 3m or 3000mm

   Live load = 4kN/m^2, Wall thickness = 230mm, $f_{ck} = 20\text{N/mm}^2$, and $f_y = 415\text{N/mm}^2$

Step:1 Fixing dimensions of stair

- Vertical distance between the floor $= H = 3.6$m
  For dog-legged stair, consider 2 flights
- Height of each flight, $h = \dfrac{H}{2} = \dfrac{3.6}{2} = 1.8\text{m}$
- Assume, Riser $(R) = 150$mm or 0.15m
- No. of risers $= \dfrac{h}{R} = \dfrac{1.8}{0.15} = 12$

- No. of tread = No. of risers – 1 = 12 – 1 = 11
- Assume, Bearing thickness = 300 mm
- Width of stair $= \dfrac{\text{Room width} - \text{Bearing thickness}}{2}$

$$= \frac{3000 - 300}{2} = 1350\text{mm or } 1.35\text{m}$$

- Width of landing = Width of stair = 1.35m
- Width of tread (T), = 300mm or 0.3m (Assume)
- Going = No. of tread × width of tread (T)

$$= 11 \times 300 = 3300\text{mm or } 3.3\text{m}$$

- Effective Horizontal span, $L = \dfrac{\text{Wall thickness}}{2} + \text{Length of stair} + \dfrac{\text{Wall thickness}}{2}$

$$= \frac{230}{2} + 6000 + \frac{230}{2}$$

$$= 6230\text{mm or } 6.23\text{m}$$

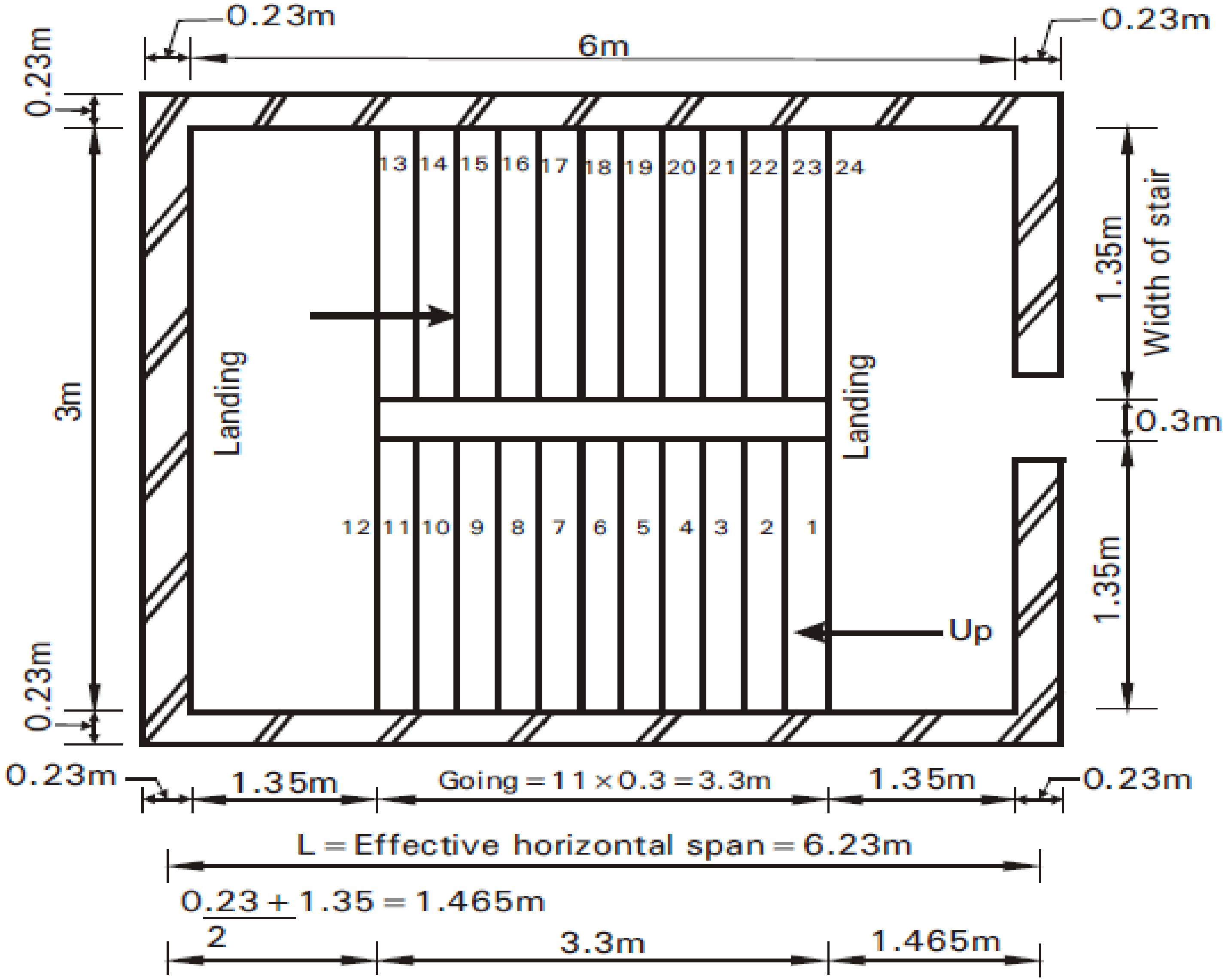
or

Effective horizontal span,

$$L = \frac{\text{Wall thickness}}{2} + \text{Landing} + \text{Going} + \text{Landing} + \frac{\text{Wall thickness}}{2}$$

$$= \frac{230}{2} + 1350 + 3300 + 1350 + \frac{230}{2}$$

$$= 6230\text{mm or } 6.23\text{m}$$



Plan showing staircase details

Step:2 Calculation of waist slab thickness

$$\text{Effective depth, } d = \frac{L}{25} = \frac{6230}{25} = 249.2\text{mm} \approx 250\text{mm}$$

$$\therefore d = 250\text{mm}$$

$$\text{Assume } d^1 = 25\text{mm}$$

$$\therefore \text{Overall depth, } D = d + d^1 = 250 + 25 = 275\text{mm}$$

Step:3 Load Calculation

(a) Loads on sloping slab

$$\text{Self weight of slab on slope} = D \times RCC \times \text{density}$$

$$= 0.275 \times 25$$

$$= 6.87 \text{ kN/m}^2$$

Ceiling finishes (Assume 12mm thick mortar, density of motor = 22kN/m^3)

$$= 0.012 \times 22 = 0.264 \text{ kN/m2}$$

$$\approx 0.3 \text{ kN/m}^2$$

Total load on inclined portion, $(6.87 + 0.3)$

$$W_s = 7.17 \text{ kN/m}^2$$

(1) Load on horizontal span $= W_S \times \dfrac{\sqrt{R^2 + T^2}}{T} = 7.17 \times \dfrac{\sqrt{0.15^2 + 0.3^2}}{0.3}$

$= 8.01 \text{ kN/m}^2$

(2) Dead load on step $= \dfrac{1}{2} \times 0.15 \times 25 = 1.875 \text{ kN/m}^2$

$\approx 1.88 \text{ kN/m}^2$

(3) Top finish (Assume) $= 0.3 \text{ kN/m}^2$

(4) Live load (Given) $= 4.0 \text{ kN/m}^2$

Total load per m $= (1) + (2) + (3) + (4) = 14.19 \times 1 = 14.19 \text{ kN/m}$

$\therefore$ Factored load, $W_u = 1.5 \times 14.19 = 21.29 \text{ kN/m}$

(b) Loads on Landing slab

Assume 275mm thick landing slab is same as waist slab thickness

(1) Self weight of landing slab $= D \times RCC$ density $= 0.275 \times 25 = 6.87 \text{ kN/m}^2$

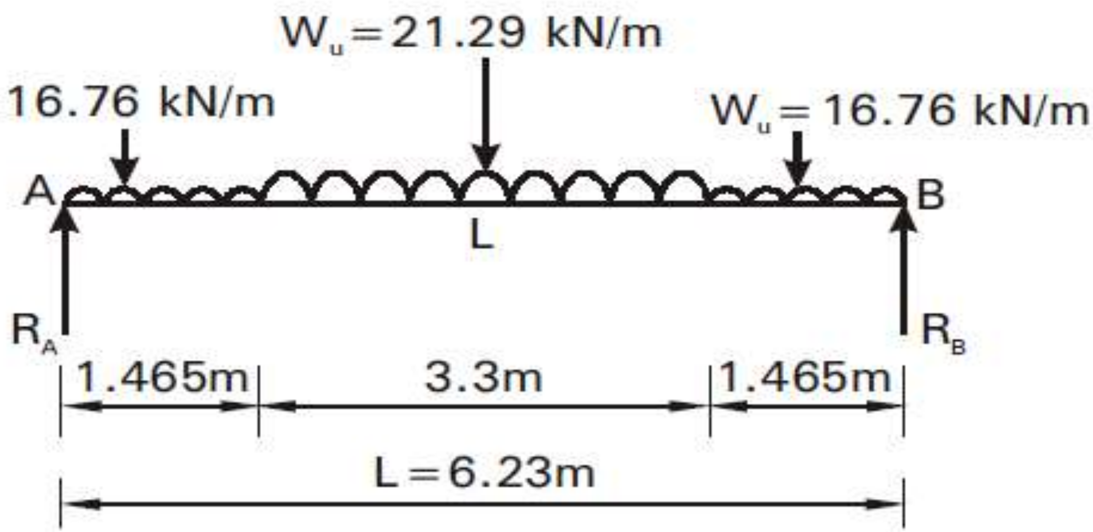
(2) Top finish (Assume) $= 0.3 \text{ kN/m}^2$

(3) Live load (given) $= 4.0 \text{ kN/m}^2$

Total load $= 11.17 \text{ kN/m}^2$

$\therefore$ Factored load per m, $W_u = 1.5 \times 11.17 \times 1 = 16.76 \text{ kN/m}^2$

Step:4 Bending moment calculation (Design moment)



Due to symmetry

$$R_A = R_B = \frac{\text{Total load}}{2}$$

$$= \frac{1}{2}[16.76 \times 1.465 + 21.29 \times 3.3 + 16.76 \times 1.465]$$

$$= 56.68 \text{ kN}$$

or

$$\Sigma M_B = 0$$

$$R_A \times 6.23 - 16.76 \times 1.465 \times \left[\frac{1.465}{2} + 3.3 + 1.465\right] - 21.29 \times$$

$$3.3 \times \left[\frac{3.3}{2} + 1.465\right] - 16.76 \times 1.465 \times \frac{1.465}{2}$$

$$R_A = \frac{134.98 + 218.85 + 17.98}{6.23} = 59.68 \text{ kN}$$

$$\Sigma V = 0$$

$$R_A + R_B - 16.76 \times 1.465 - 21.29 \times 3.3 - 16.76 \times 1.465 = 0$$

$$\therefore R_B = 119.36 - 59.68 = 59.68 \text{ kN}$$

Maximum moment occurs at mid span,

$$M = R_A \times \frac{6.23}{2} - 16.76 \times 1.465 \times \left(\frac{1.465}{2} + \frac{3.3}{2}\right) - 21.29 \times \frac{1}{2} \times 3.3 \times \frac{1}{2} \times \left(\frac{3.3}{2}\right)$$

$$= 59.68 \times 3.115 - 16.76 \times 3.49 - 21.29 \times 1.36$$

$$\therefore M = 98.45 \text{ kN.m}$$

$$\Rightarrow M_u = M = 98.45 \text{ kN.m}$$

Step:5 Check for depth

$$d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{98.45 \times 10^6}{0.138 \times 20 \times 1000}}$$

$= 188.88\text{mm} < d_{\text{provided}}$ (250mm)

Hence, Design is safe

Step:6 Design of main reinforcement

Refer IS : 456-2000, G-1.1(b)

$$M_u = 0.87\ f_y A_{st} d \left[1 - \left(\frac{f_y A_{st}}{f_{ck} b d}\right)\right]$$

$$98.45 \times 10^6 = 0.87 \times 415 \times 250\, A_{st}\left[1 - \left(\frac{415 A_{st}}{20 \times 1000 \times 250}\right)\right]$$

$$7.49\, A_{st}^{\ 2} - 90262.5\, A_{st} + 98.45 \times 10^6 = 0$$

$$\therefore\ A_{st} = 1212.75\ \text{mm}^2$$

Provide 12mm ϕ bars

$$\therefore \text{Spacing, } S = \frac{1000 a_{st}}{A_{st}} = \frac{1000 \times \frac{\pi}{4} \times 12^2}{1212.75} = 93.25 \text{ mm} \qquad | \, 3d = 3 \times 250 = 750\text{mm}$$

∴ Provide 12mm ϕ bars @ 90 mm c/c (< 3 d or 300 mm)

Step:7 Distribution steel

A_{st} = 0.12% of gross cross sectional area

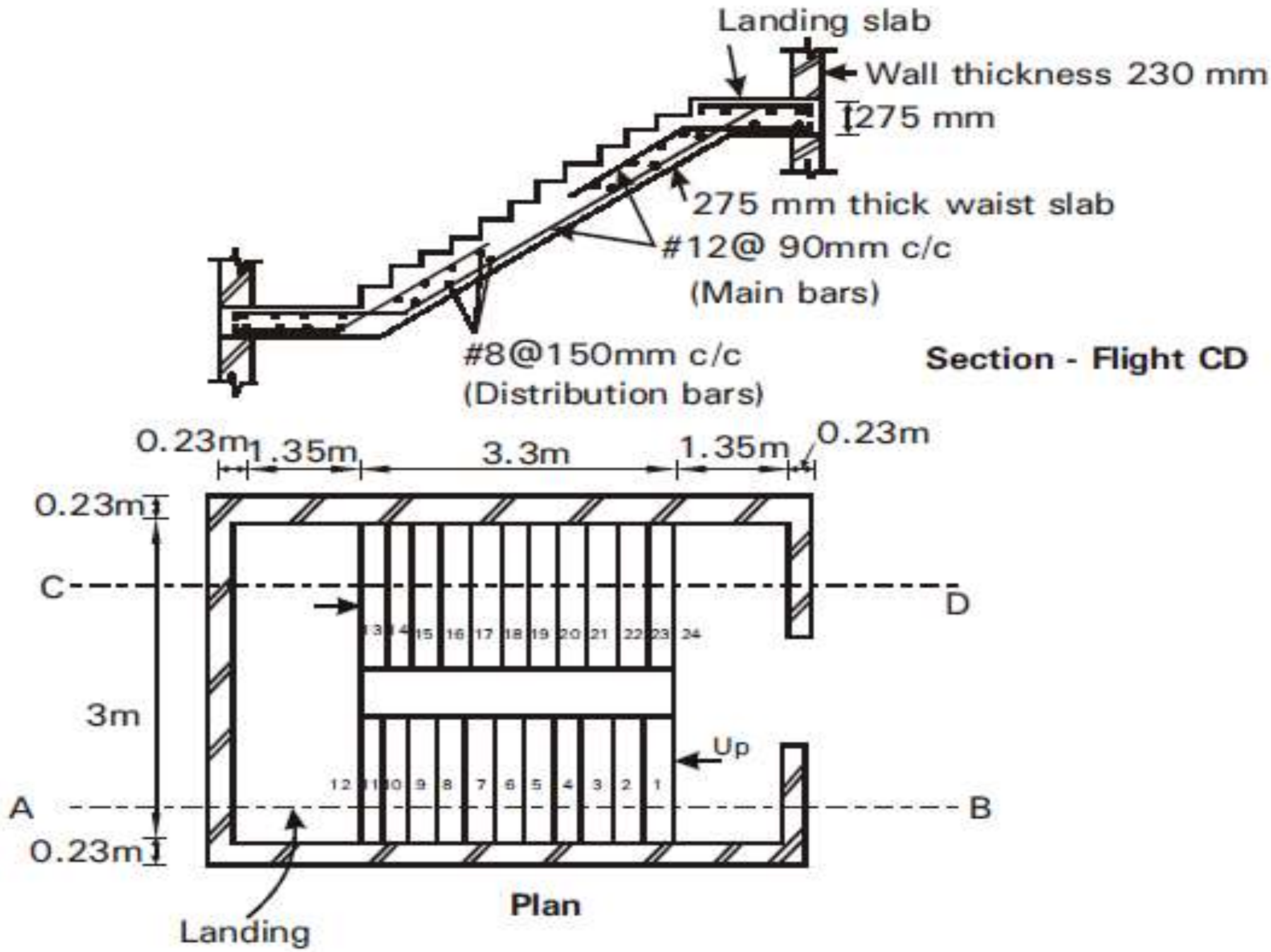
$$A_{st} = \frac{0.12}{100} \times b \times D = \frac{0.12}{100} \times 1000 \times 275 = 330\text{mm}^2$$

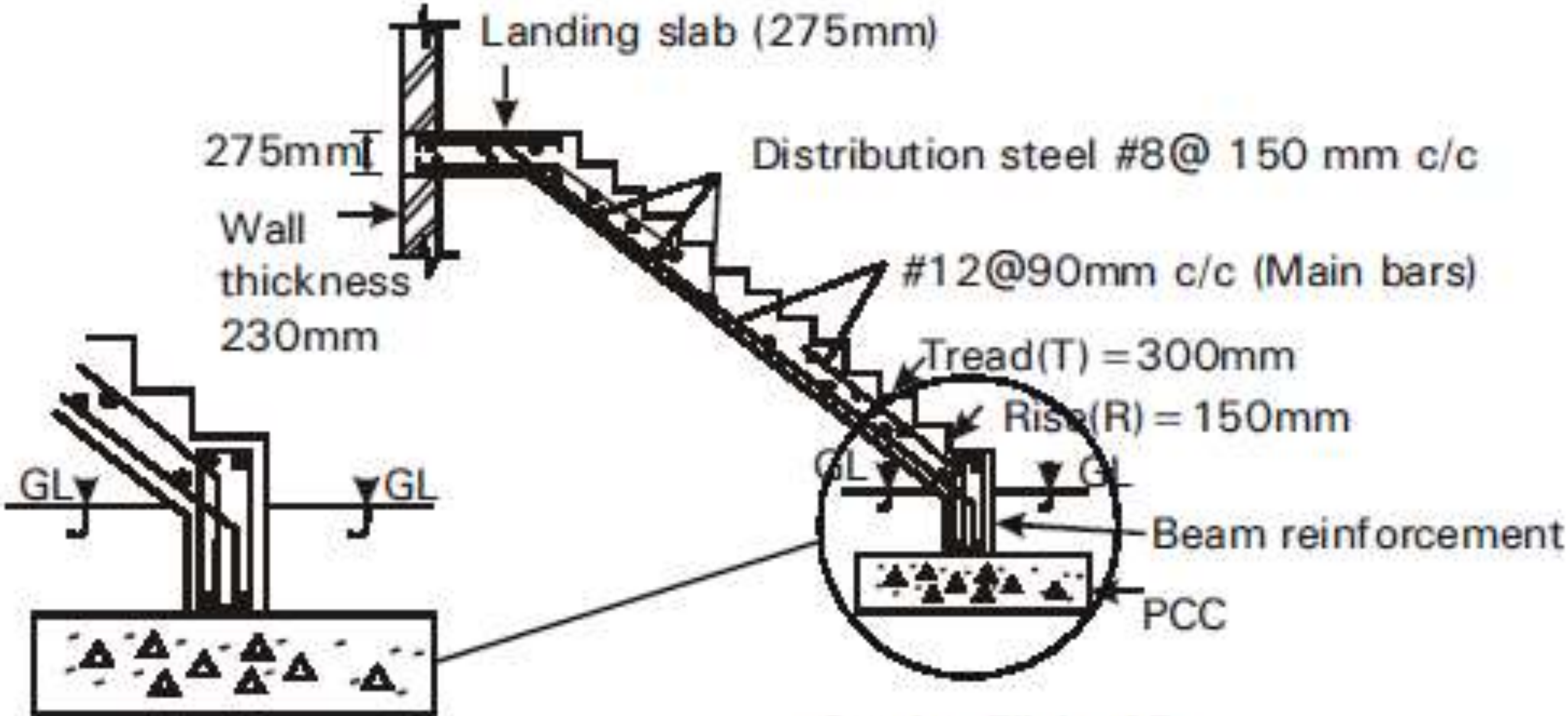
Provide 8 mm dia bars $\qquad | \, 5d = 5 \times 250 = 1250$ mm

$$\text{Spacing, } S = \frac{1000 a_{st}}{A_{st}} = \frac{1000 \times \frac{\pi}{4} \times 8^2}{330} = 152.31 \text{ mm}$$

∴ Provide 8 mm ϕ bars @ 150 mm c/c (< 5d or 450mm)

Step:8 Reinforcement details





Section-Flight AB

2. Design a dog-legged stairs for an office building in a room of clear size of 2.8m × 5.8m. The vertical distance between the floors is 3.6m, width of each flights is 1.25m. Take a live load of 3 kN/m². The stairs are so provided that the landing slab are supported on 230 mm walls at their outer edges, leading to a structural arrangement of landing slab spanning in the direction of going. Use M20 concrete and *Fe*415 steel.

Solution

Given: Room size = 2.8m × 5.8m, (Here, Length = 5.8m, width = 2.8m)

floor height, $H = 3.6\text{m}$, Live load = 3 kN/m²

Width of each flight = 1.25m, Wall thickness = 230mm, $f_{ck} = 20\text{N/mm}^2$, and $f_y = 415\text{N/mm}^2$

Step:1 Fixing dimensions of stair

- Vertical distance between the floor = $H = 3.6\text{m}$

  For dog-legged stair, consider 2 flights

- Height of each flight, $h = \dfrac{H}{2} = \dfrac{3.6}{2} = 1.8\text{m}$

- Assume Riser $(R) = 150\text{mm}$ or 0.15m

- No. of risers $= \dfrac{h}{R} = \dfrac{1.8}{0.15} = 12$

- No. of tread = No. of risers – 1 = 12 – 1 = 11

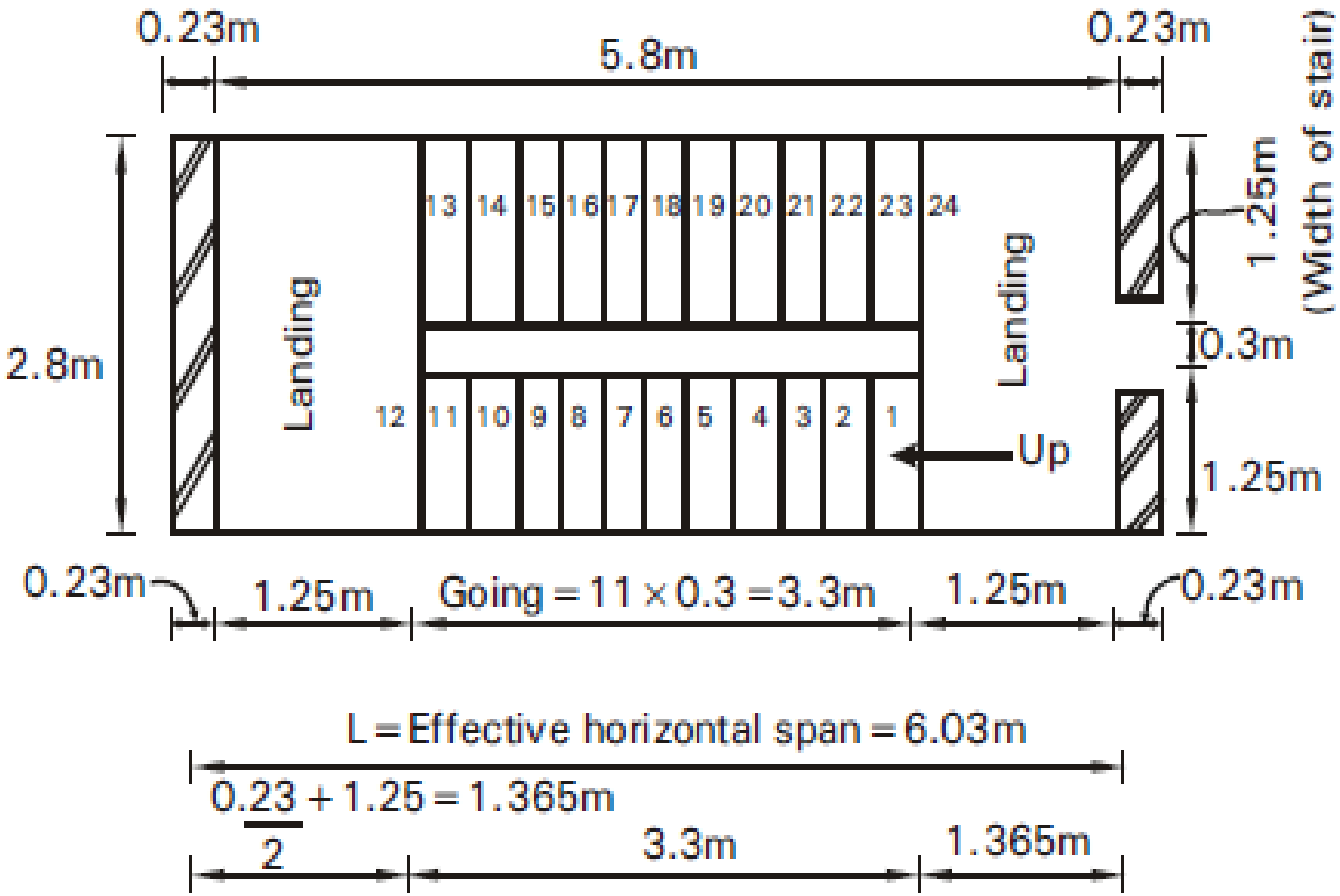
- Assume Bearing thickness = 300 mm
- Width of flight or width of stair = 1.25 m (given)
- Width of landing = Width of stair = 1.25m
- Width of tread (T) = 300mm or 0.3m (Assume)
- Going = No. of tread × width of tread (T)

  $= 11 \times 300 = 3300\text{mm or } 3.3\text{m}$

- Effective Horizontal span, $L = \dfrac{\text{Wall thickness}}{2} + \text{Length of stair} + \dfrac{\text{Wall thickness}}{2}$

$$= \frac{230}{2} + 5800 + \frac{230}{2}$$

$$L = 6030\text{mm or } 6.03\text{m}$$



0.23m
5.8m
0.23m
2.8m
Landing
13 14 15 16 17 18 19 20 21 22 23 24
12 11 10 9 8 7 6 5 4 3 2 1
Landing
Up
1.25m
(Width of stair)
0.3m
1.25m
0.23m
1.25m
Going = 11 × 0.3 = 3.3m
1.25m
0.23m
L = Effective horizontal span = 6.03m
0.23/2 + 1.25 = 1.365m
3.3m
1.365m

Step:2 Calculation of waist slab thickness

$$\text{Effective depth, } d = \frac{L}{25} = \frac{6030}{25} = 241.2\text{mm}$$

$$\text{take, } d = 250\text{mm}$$

Assume $d^1 = 25$mm

$\therefore$ Overall depth, $D = d + d^1 = 250 + 25 = 275$mm

Step:3 Load Calculation

(a) **Loads on sloping slab**

$$\text{Self weight of slab on slope} = D \times RCC \times \text{density}$$
$$= 0.275 \times 25$$
$$= 6.87 \text{ kN/m}^2$$

Ceiling finishes (Assume 12mm thick mortar, density of motor = 22kN/m^3)

$$= 0.012 \times 22 = 0.264 \text{ kN/m}^2$$
$$\approx 0.3 \text{ kN/m}^2$$

Total load on inclined portion,

$$W_s = 7.17 \text{ kN/m}^2$$

(1) Load on horizontal span $= W_S \times \dfrac{\sqrt{R^2 + T^2}}{T} = 7.17 \times \dfrac{\sqrt{0.15^2 + 0.3^2}}{0.3}$

$$= 8.01 \text{ kN/m}^2$$

(2) Dead load on step $= \dfrac{1}{2} \times 0.15 \times 25 = 1.875 \text{ kN/m}^2 \approx 1.88 \text{ kN/m}^2$

(3) Top finish (Assume) $= 0.3\ kN/m^2$

(4) Live load (Given) $= 3.0\ kN/m^2$

Total load per m $= (1) + (2) + (3) + (4) = 13.19 \times 1 = 13.19\ kN/m$

$\therefore$ Factored load, $W_u = 1.5 \times 13.19 = 19.785\ kN/m \approx 20\ kN/m$

(b) Loads on Landing slab

Assume 275mm thick landing slab is same as waist slab thickness

(1) Self weight of landing slab $= D \times RCC$ density $= 0.275 \times 25 = 6.87\ kN/m^2$

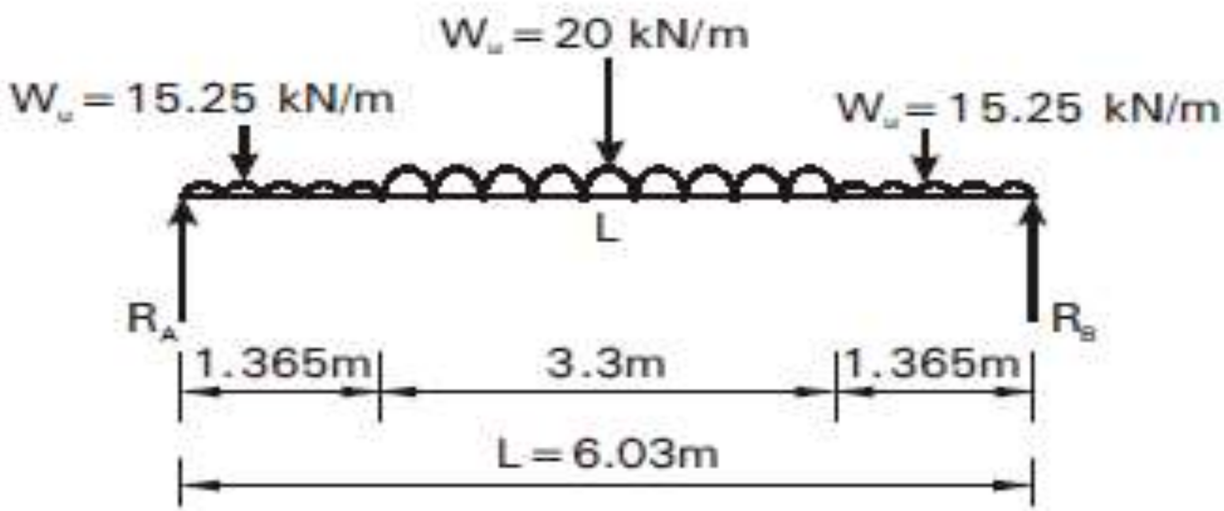
(2) Top finish (Assume) $= 0.3\ kN/m^2$

(3) Live load (given) $= 3.0\ kN/m^2$

Total load $= 10.17\ kN/m^2$

$\therefore$ Factored load per m, $W_u = 1.5 \times 10.17 \times 1 = 15.25\ kN/m$

Step:4 Bending moment calculation (Design moment)



Due to symmetry

$$R_A = R_B = \frac{\text{Total load}}{2}$$

$$= \frac{1}{2}[15.25 \times 1.365 + 20 \times 3.3 + 15.25 \times 1.365]$$

$$= 53.81 \text{ kN}$$

Maximum moment occurs at mid span,

$$M = R_A \times \frac{6.03}{2} - 15.25 \times 1.365 \times \left(\frac{1.365}{2} + \frac{3.3}{2}\right) - 20 \times \frac{1}{2} \times 3.3 \times \frac{1}{2} \times \left(\frac{3.3}{2}\right)$$

$$= 53.81 \times \frac{6.03}{2} - 15.25 \times 3.18 - 20 \times 1.36$$

$$\therefore M = 86.54 \text{ kN.m}$$

$$\Rightarrow M_u = M = 86.54 \text{ kN.m}$$

Step:5 Check for depth

$$d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{86.54 \times 10^6}{0.138 \times 20 \times 1000}}$$

$$= 177.07\text{mm} < d_{provided} \text{ (250mm)}$$

Hence, Design is safe

Step:6 Design of main reinforcement

Refer IS : 456-2000, G-1.1(b)

$$M_u = 0.87\ f_y A_{st} d \left[1-\left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$$

$$86.54 \times 10^6 = 0.87 \times 415 \times 250\ A_{st}\left[1-\left(\frac{415 A_{st}}{20\times1000\times250}\right)\right]$$

$$7.49\ A_{st}^{\ 2} - 90262.5\ A_{st} + 86.54 \times 10^6 = 0$$

$$\therefore\ A_{st} = 1050.3\ \text{mm}^2$$

Provide 12mm ϕ bars

$$\therefore \text{Spacing, } S = \frac{1000 a_{st}}{A_{st}} = \frac{1000\times\frac{\pi}{4}\times12^2}{1050.3} = 107.68\ \text{mm}$$

$$3d = 3\times250 = 750mm$$

$\therefore$ Provide 12mm ϕ bars @ 100 mm c/c (< 3 d or 300 mm)

Step:7 Distribution steel

A_{st} = 0.12% of gross cross sectional area

$$A_{st} = \frac{0.12}{100}\times b\times D = \frac{0.12}{100}\times 1000\times275 = 330\text{mm}^2$$

Provide 8 mm dia bars

$$\text{Spacing, } S = \frac{1000 a_{st}}{A_{st}} = \frac{1000\times\frac{\pi}{4}\times8^2}{330} = 152.31\ \text{mm}$$

$$5d = 5\times250 = 1250mm$$

$\therefore$ Provide 8 mm ϕ bars @ 150 mm c/c (< 5d or 450mm)

3. **Design a stair for an office building for the plan shown in Fig. 4.13. Rise 150mm, Tread 250mm, wall thickness 300mm. Use *M*20 concrete and *Fe*415 steel. Sketch the details of reinforcement for span *AB* and *CD*.**

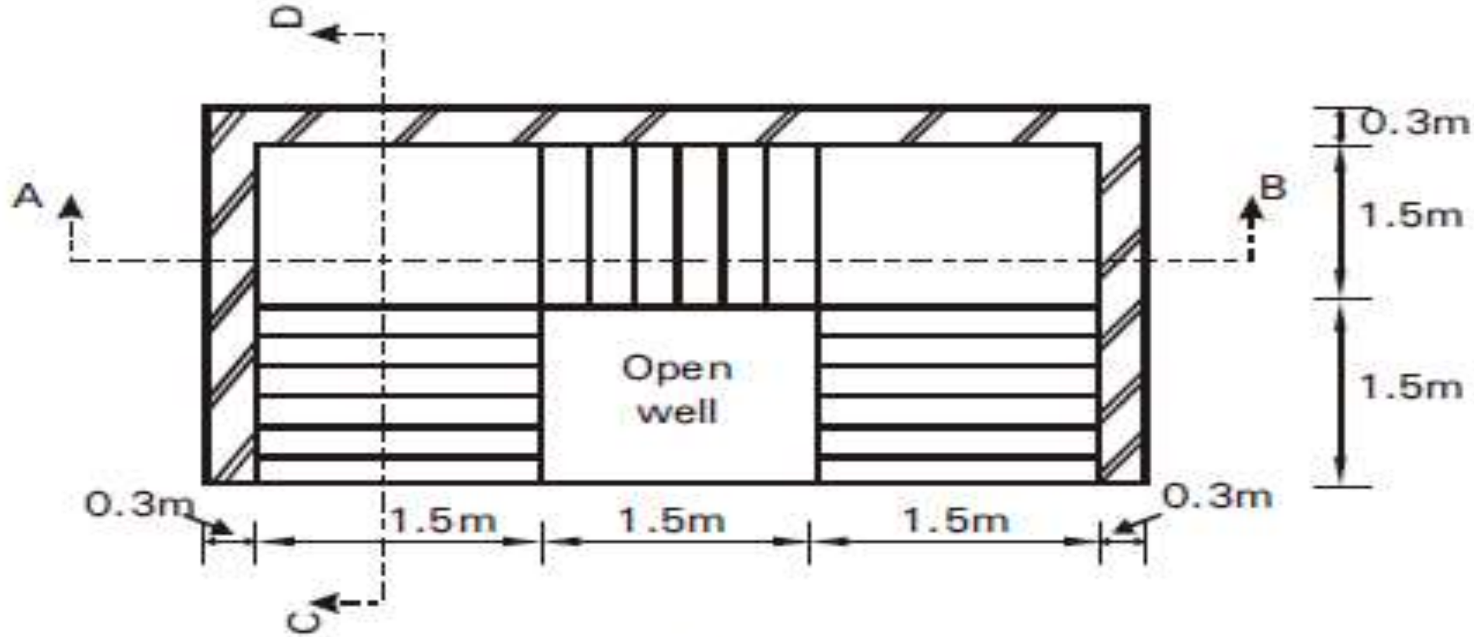


Fig. 4.13

Solution

Given: Rise (R) = 150mm, Tread (T) = 250m

Wall thickness = 300mm, $f_{ck} = 20\text{N/mm}^2$, and $f_y = 415\text{N/mm}^2$

Step:1 Effective Horizontal span (L)

For Flight AB, $L = \dfrac{\text{Wall thickness}}{2} + 1.5 \times 3 + \dfrac{\text{Wall thickness}}{2}$

$$= \frac{0.3}{2} + 1.5 \times 3 + \frac{0.3}{2} = 4.8\text{m or } 4800\text{mm}$$

For Flight CD, $L = 1.5 + 1.5 + \dfrac{0.3}{2} = 3.15 \text{ m or } 3150\text{mm}$

Step:2 Calculation of waist slab thickness

$$\text{Effective depth, } d = \frac{L}{25} = \frac{4800}{25} = 192\text{mm}$$

$$\therefore d = 200\text{mm}$$

Assume $d^1 = 25$mm

$\therefore$ Overall depth, $D = d + d^1 = 200 + 25 = 225$mm

Step:3 Load Calculation

(a) Loads on sloping slab

Self weight of slab on slope $= D \times RCC \times$ density

$= 0.225 \times 25 = 5.625 \text{ kN/m}^2 \approx 5.63 \text{ kN/m}^2$

Ceiling finish (Assume) $= 0.3 \text{ kN/m}^2$

$\therefore$ Total load on inclined portion,

$$W_s = 5.93 \text{ kN/m}^2$$

(1) Load on horizontal span $= W_S \times \dfrac{\sqrt{R^2 + T^2}}{T} = 5.93 \times \dfrac{\sqrt{0.15^2 + 0.25^2}}{0.25}$

$$= 6.91 \text{ kN/m}^2$$

(2) Dead load on step $= \dfrac{1}{2} \times 0.15 \times 25 = 1.875 \text{ kN/m}^2$

$$\approx 1.88 \text{ kN/m}^2$$

(3) Top finish (Assume) $= 0.3 \text{ kN/m}^2$

(4) Live load (Given) $= 4.0 \text{ kN/m}^2$

Total load per m $= (1) + (2) + (3) + (4) = 13.09 \times 1 = 13.09$ kN/m

$\therefore$ Factored load, $W_u = 1.5 \times 13.09 = 19.63 \approx 20$ kN/m

(b) Loads on Landing slab

Assume 225mm thick landing slab is same as waist slab thickness

(1) Self weight of landing slab = $D \times RCC$ density

$$= 0.225 \times 25 = 5.625 \text{ kN/m}^2$$

$$\approx 5.63 \text{ kN/m}^2$$

(2) Top finish (Assume) $= 0.3 \text{ kN/m}^2$

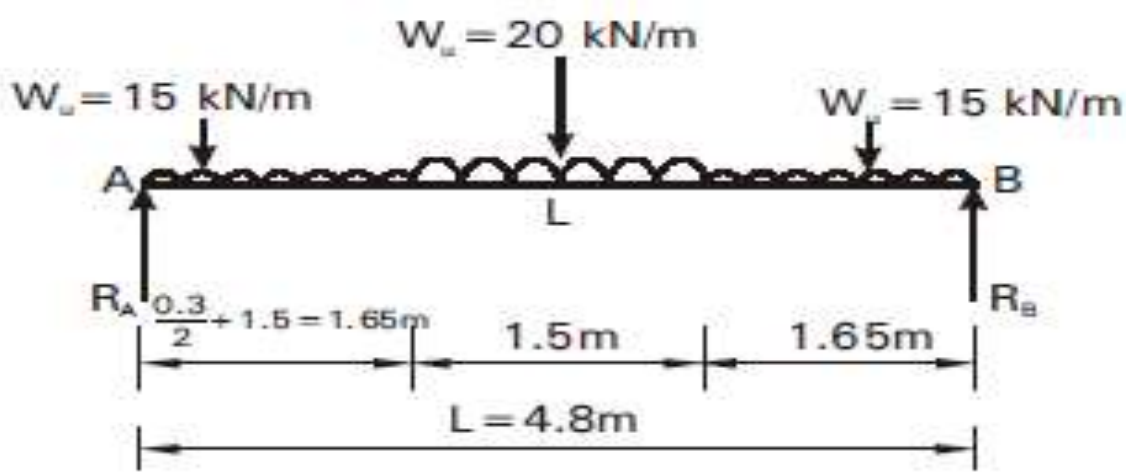
(3) Live load (Assume) $= 4.0 \text{ kN/m}^2$

Total load $= 9.93 \text{ kN/m}^2$

$\therefore$ Factored load per m, $W_u = 1.5 \times 9.93 \times 1 = 14.89 \approx 15 \text{ kN/m}$

Step:4 Bending moment calculation (Design moment)

(1) For Flight AB



Due to symmetry

$$R_A = R_B = \frac{\text{Total load}}{2}$$

$$= \frac{1}{2}[15 \times 1.65 + 20 \times 1.5 + 15 \times 1.65]$$

$$= 39.75 \text{ kN}$$

Maximum moment occurs at mid span,

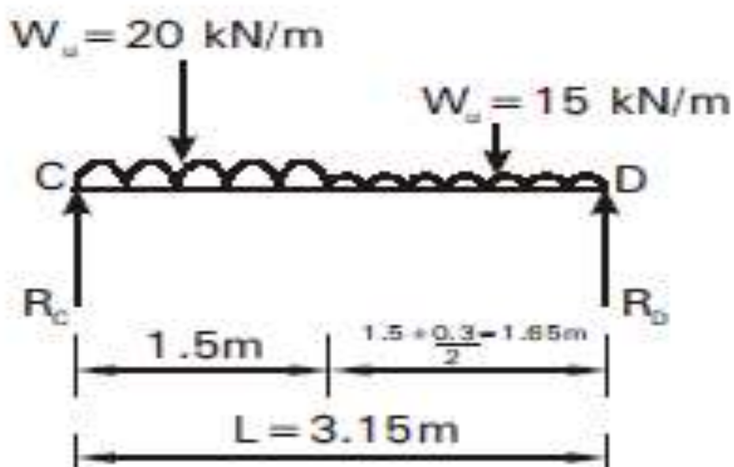
$$M = R_A \times \frac{4.8}{2} - 15 \times 1.65 \times \left(\frac{1.65}{2} + \frac{1.5}{2}\right) - 20 \times \frac{1}{2} \times 1.5 \times \frac{1}{2} \times \left(\frac{1.5}{2}\right)$$

$$= 39.75 \times 2.4 - 15 \times 2.59 - 20 \times 0.28$$

$$\therefore M = 50.95 \text{ kN.m}$$

$$\Rightarrow M_u = M = 50.95 \text{ kN.m}$$

(2) For Flight CD



Taking moment about D

$\Sigma M_D = 0$

$$= R_C \times 3.15 - 20 \times 1.5 \times \left(\frac{1.5}{2} + 1.65\right) - 15 \times 1.65 \times \frac{1.65}{2}$$

$$\therefore R_C = \frac{72 + 20.41}{3.15}$$

$= 29.40$ kN

Shear force is zero at $x = \frac{R_A}{\text{Waist slab load}} = \frac{R_A}{W_u} = \frac{29.40}{20}$

$x = 1.47\text{m from } C$

Maximum moment occurs at 1.47m from C

$$M = R_A \times x - 20 \times x \times \frac{x}{2}$$

$$= 29.40 \times 1.47 - 20 \times 1.47 \times \frac{1.47}{2}$$

$$\therefore M = 21.61 \text{ kN.m}$$

$$\Rightarrow M_u = M = 21.61 \text{ kN.m}$$

Step:5 Check for depth

(1) For Flight AB, $M_u = 50.95$ kN.m or 50.95×10^6 N.mm

$$d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{50.95 \times 10^6}{0.138 \times 20 \times 1000}}$$

$= 135.86\text{mm} < d_{provided}$ (200mm)

Hence, Design is safe

(2) For Flight CD, $M_u = 21.61$ kN.m or 21.61×10^6 N.mm

$$d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{21.61 \times 10^6}{0.138 \times 20 \times 1000}}$$

$= 88.48\text{ mm} < d_{provided}$ (200mm)

Hence, Design is safe

Step:6 Design of main reinforcement

(1) For Flight AB, $M_u = 50.95 \times 10^6$ N.mm

Refer IS : 456-2000, G-1.1(b)

$$M_u = 0.87\ f_y A_{st} d \left[1 - \left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$$

$$50.95 \times 10^6 = 0.87 \times 415 \times 200\ A_{st}\left[1 - \left(\frac{415 A_{st}}{20 \times 1000 \times 200}\right)\right]$$

$$7.49\ A_{st}^2 - 72210\ A_{st} + 50.95 \times 10^6 = 0$$

$$\therefore\ A_{st} = 766.52\ \text{mm}^2$$

Provide 12mm φ bars

$3d = 3 \times 200$
$= 600 mm$

$$\therefore \text{Spacing, } S = \frac{1000 a_{st}}{A_{st}} = \frac{1000 \times \frac{\pi}{4} \times 12^2}{766.52} = 147.54 \text{ mm}$$

∴ Provide 12mm φ bars @ 140 mm c/c (< 3 d or 300 mm)

(2) For Flight CD, $M_u = 21.61 \times 10^6$ N.mm

$$M_u = 0.87\, f_y A_{st} d \left[1 - \left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$$

$$21.61 \times 10^6 = 0.87 \times 415 \times 200\, A_{st}\left[1 - \left(\frac{415 A_{st}}{20 \times 1000 \times 200}\right)\right]$$

$$7.49\, A_{st}^2 - 72210\, A_{st} + 21.61 \times 10^6 = 0$$

$$\therefore A_{st} = 309.18 \text{ mm}^2$$

Provide 12mm φ bars

$$\therefore \text{Spacing, } S = \frac{1000 a_{st}}{A_{st}} = \frac{1000 \times \frac{\pi}{4} \times 12^2}{309.18} = 365.8 \text{ mm}$$

∴ Provide 12mm φ bars @ 300 mm c/c (< 3 d or 300 mm)

Step:7 Distribution steel

$A_{st} = 0.12\%$ of gross cross sectional area

$$A_{st} = \frac{0.12}{100} \times b \times D = \frac{0.12}{100} \times 1000 \times 225 = 270 \text{mm}^2$$

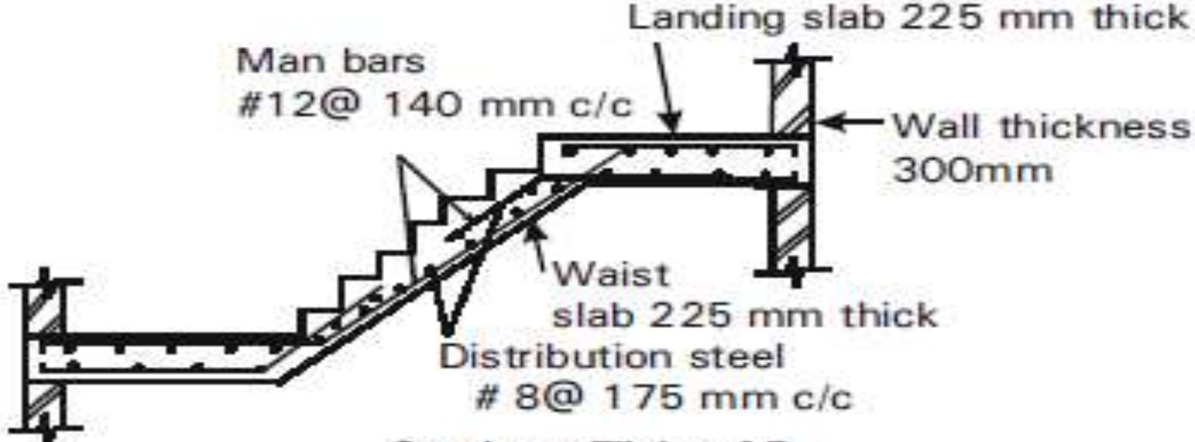
Provide 8 mm ϕ bars

$$5d = 5 \times 200$$
$$= 1000mm$$

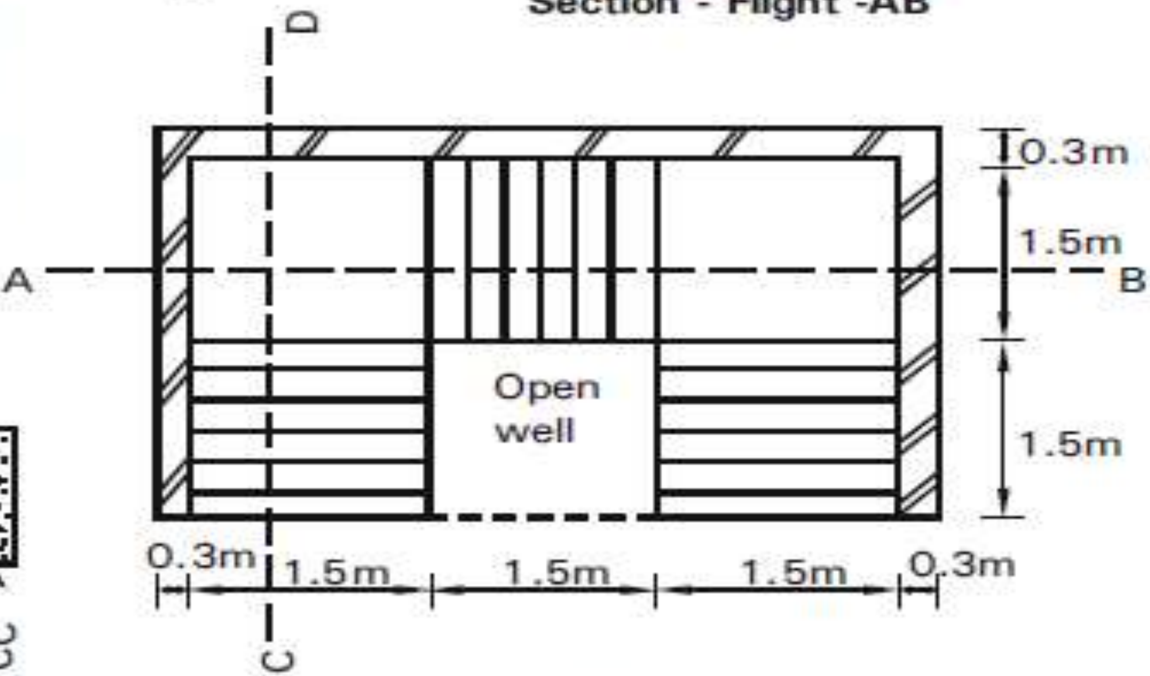
$$\text{Spacing, S} = \frac{1000 a_{st}}{A_{st}} = \frac{1000 \times \frac{\pi}{4} \times 8^2}{270} = 186.16 \text{ mm}$$

∴ Provide 8 mm ϕ bars @ 175 mm c/c (< 5d or 450mm)

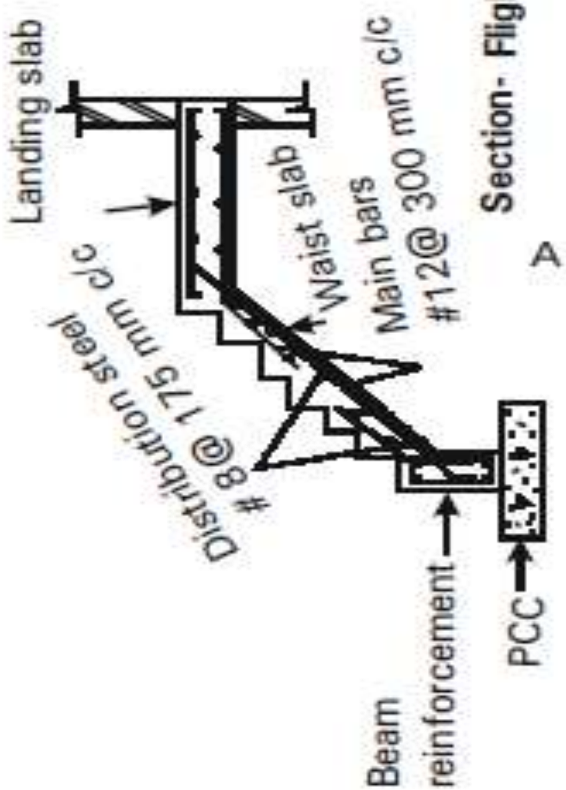
Step:8 Reinforcement details



Section - Flight -AB



Plan

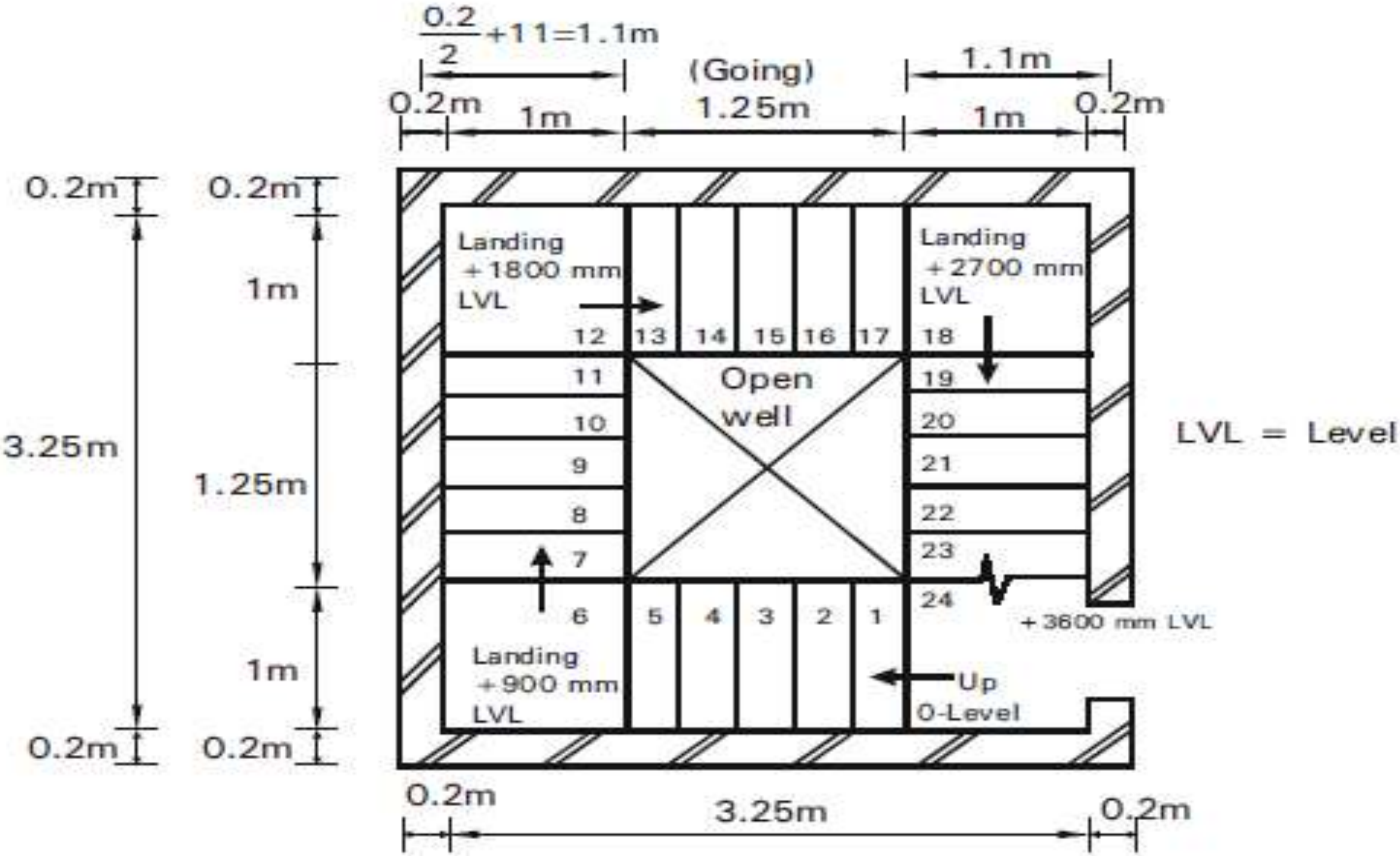


Section- Flight - CD

4. An open well stair case is to be provided for a stair hall of size 3.25m × 3.25m. Size of open well at the centre = 1.25 × 1.25m, floor to floor height = 3.6m. Size of landing at each corner = 1m × 1m. The stair slab is embedded in to the wall by 200mm. The live load on stair = 3kN/m². Sketch the reinforcement details. Use f_{ck} = 20N/mm² and f_y = 415N/mm².

Solution

Step:1 Fixing dimensions of stair



Assume $R = 150\text{mm}$, $T = 250\text{mm}$

- Vertical distance between the floor $= H = 3.6\text{m}$

For open well stair, consider here 4 flights

- Height of each flight, $h = \dfrac{H}{4} = \dfrac{3.6}{4} = 0.9\text{m}$
- No. of risers $= \dfrac{h}{R} = \dfrac{0.9}{0.15} = 6$
- No. of tread = No. of risers $- 1 = 6 - 1 = 5$
- Going = No. of tread × width of tread (T)

$$= 5 \times 250 = 1250\text{mm or } 1.25\text{m}$$

- Effective Horizontal span, $L = \dfrac{\text{Wall thickness}}{2} + \text{Landing} + \text{Going} + \text{Landing} + \dfrac{\text{Wall thickness}}{2}$

$$= \frac{200}{2} + 1000 + 1250 + 1000 + \frac{200}{2}$$

$$= 3450\text{mm or } 3.45\text{m}$$

or

$$L = \frac{\text{Wall thickness}}{2} + 3.25 + \frac{\text{Wall thickness}}{2}$$

$$L = \frac{0.2}{2} + 3.25 + \frac{0.2}{2} = 3.45\text{m}$$

The flights AB, BC, CD and DA are equal in length

Step:2 Calculation of waist slab thickness

$$\text{Effective depth, } d = \frac{L}{25} = \frac{3450}{25} = 138\text{mm}$$

$$\text{Take, } d = 150\text{mm}$$

Assume $d^1 = 25$mm

$\therefore$ Overall depth, $D = d + d^1 = 150 + 25 = 175$mm

Step:3 Load Calculation

(a) **Loads on sloping slab**

$$\text{Self weight of slab on slope} = D \times RCC \times \text{density}$$

$$= 0.175 \times 25 = 4.375 \text{ kN/m}^2$$

$$\approx 4.38 \text{ kN/m}^2$$

Ceiling finish (Assume) $= 0.3 \text{ kN/m}^2$

$\therefore$ Total load on inclined portion,

$$W_s = 4.68 \text{ kN/m}^2$$

(1) Load on horizontal span $= W_s \times \dfrac{\sqrt{R^2 + T^2}}{T} = 4.68 \times \dfrac{\sqrt{0.15^2 + 0.25^2}}{0.25}$

$$= 5.45 \text{ kN/m}^2$$

(2) Dead load on step $= \dfrac{1}{2} \times 0.15 \times 25 = 1.875 \text{ kN/m}^2$

$$\approx 1.88 \text{ kN/m}^2$$

(3) Top finish (Assume) $= 0.3 \text{ kN/m}^2$

(4) Live load (Given) $= 3.0 \text{ kN/m}^2$

Total load per m $= (1) + (2) + (3) + (4) = 10.63 \times 1 = 10.63 \text{ kN/m}$

$\therefore$ Factored load, $W_u = 1.5 \times 10.63 = 15.94 \approx 16 \text{ kN/m}$

(b) Loads on Landing slab

Assume 175mm thick landing slab is same as waist slab thickness

(1) Self weight of landing slab $= D \times RCC$ density $= 0.175 \times 25 = 4.375 \text{ kN/m}^2 \approx 4.38 \text{ kN/m}^2$

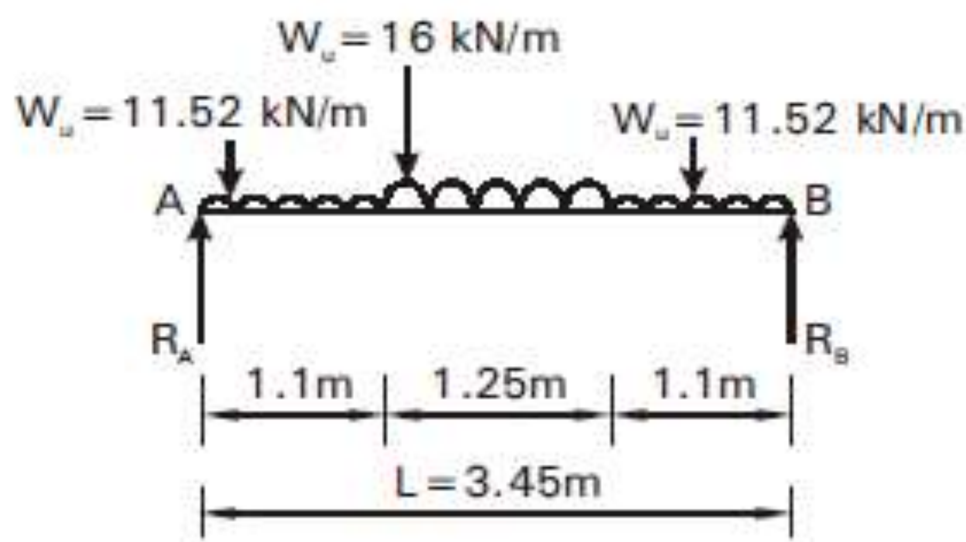
(2) Top finish (Assume) $= 0.3 \text{ kN/m}^2$

(3) Live load (Given) $= 3.0 \text{ kN/m}^2$

Total load $= 7.68 \text{ kN/m}^2$

$\therefore$ Factored load per m, $W_u = 1.5 \times 7.68 \times 1 = 11.52 \text{ kN/m}$

Step:4 Bending moment calculation (Design moment)



Due to symmetry

$$R_A = R_B = \frac{\text{Total load}}{2}$$

$$= \frac{1}{2}[11.52 \times 1.1 + 16 \times 1.25 + 11.52 \times 1.1]$$

$$= 22.67 \text{ kN}$$

Maximum moment occurs at mid span,

$$M = R_A \times \frac{3.45}{2} - 11.52 \times 1.1 \times \left(\frac{1.1}{2} + \frac{1.25}{2}\right) - 16 \times \frac{1}{2} \times 1.25 \times \frac{1}{2} \times \left(\frac{1.25}{2}\right)$$

$$= 22.67 \times \frac{3.45}{2} - 14.88 - 3.125$$

$$\therefore M = 21.10 \text{ kN.m}$$

$$\Rightarrow M_u = M = 21.10 \text{ kN.m}$$

Step:5 Check for depth

$$d_{req} = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{21.10 \times 10^6}{0.138 \times 20 \times 1000}}$$

$$= 87.43\text{mm} < d_{\text{provided}}\ (150\text{mm})$$

Hence, Design is safe

Step:6 Design of main reinforcement

$$M_u = 0.87\, f_y A_{st} d \left[1 - \left(\frac{f_y A_{st}}{f_{ck} bd}\right)\right]$$

$$21.10 \times 10^6 = 0.87 \times 415 \times 150\, A_{st} \left[1 - \left(\frac{415 A_{st}}{20 \times 1000 \times 150}\right)\right]$$

$$7.49\, A_{st}^2 - 54157.5\, A_{st} + 21.10 \times 10^6 = 0$$

$$\therefore\ A_{st} = 413.21\ \text{mm}^2$$

Provide 10mm ϕ bars

$$\therefore \text{Spacing, } S = \frac{1000\, a_{st}}{A_{st}} = \frac{1000 \times \frac{\pi}{4} \times 10^2}{413.21} = 190.07\ \text{mm}$$

$\therefore$ Provide 10mm ϕ bars @ 190 mm c/c ($< 3d$ or 300 mm)

$3d = 3 \times 150$
$= 450mm$

Step:7 Distribution steel

A_{st} = 0.12% of gross cross sectional area

$$A_{st} = \frac{0.12}{100} \times b \times D = \frac{0.12}{100} \times 1000 \times 175 = 210\text{mm}^2$$

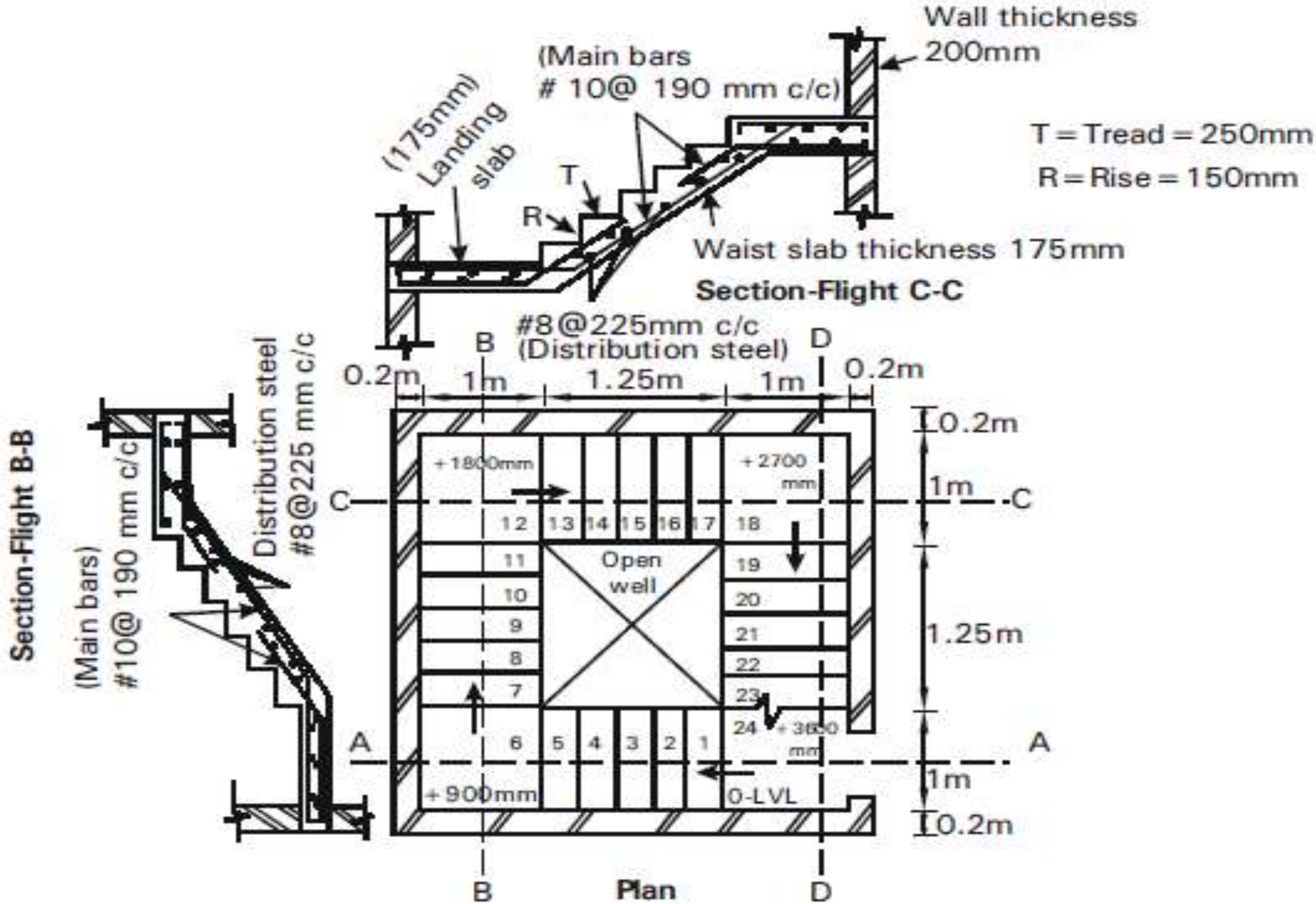
Provide 8 mm ϕ bars

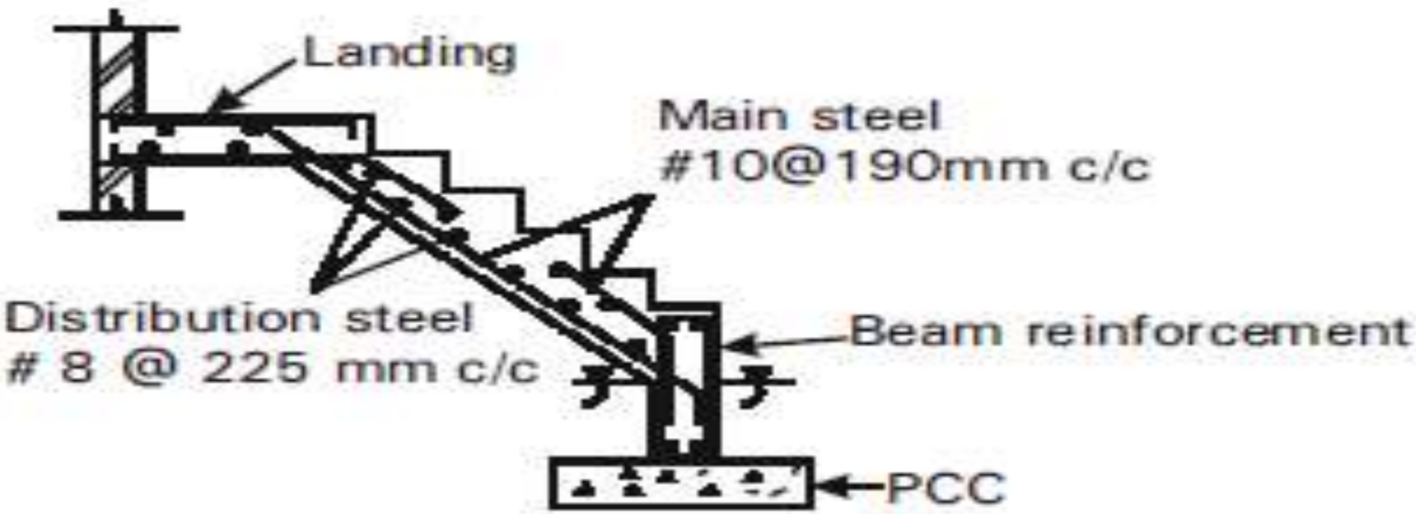
$$\text{Spacing, } S = \frac{1000\, a_{st}}{A_{st}} = \frac{1000 \times \frac{\pi}{4} \times 8^2}{210} = 239.35\ \text{mm}$$

$\therefore$ Provide 8 mm ϕ bars @ 225 mm c/c (< 5d or 450mm)

$5d = 5 \times 150$
$= 750mm$

Step:8 Reinforcement details





Section - Flight A-A

BOND, ANCHORAGE AND DEVELOPMENT LENGTH

Bond

The term bond refers to the adhesion between concrete and steel which resist the slipping of steel bar from the concrete. This bond which is responsible for transfer to stresses from steel to concrete and thereby providing composite action of steel and concrete in R.C.C. The bond develops due to setting of concrete on drying which results in gripping of the steel bars.

The bond resistance in reinforced concrete is obtained by following mechanisms :

Chemical adhesion : It is due to gum like property of the substances, formed after setting of concrete.

Frictional resistance : It is due to friction between steel and concrete.

Gripping action : It is due to gripping of steel by the concrete on drying.

Mechanical Interlock : It is provided by the corrugations or ribs present on the surface of the deformed bars.

The bond is assumed to be perfect in the design of reinforced concrete the bond between steel and concrete can be increased by the following methods :

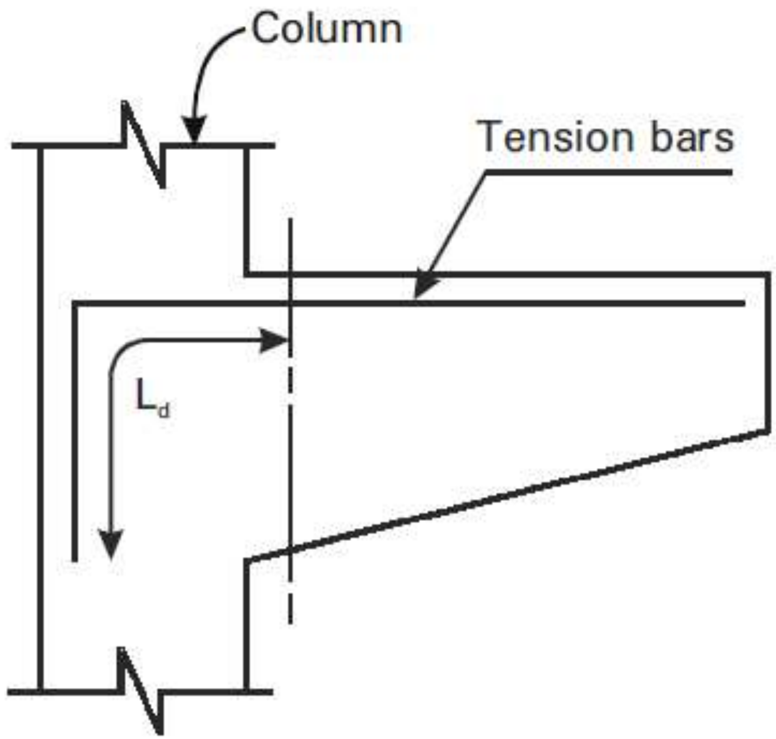
- Using deformed or twisted bar.
- Using rich mix of concrete.
- Adequate compaction and curing of concrete for proper setting.
- Providing hooks at the end of the reinforcing bars.

Anchorage Bond and Development length

The bond developed in the anchorage area at the ends of a beam or at the cut off points of a reinforcing bar within the span of a beam causes slippage of the bar at the ends. This type of bond resisting slippage of reinforcing bar is called anchorage bond.

A basic requirement in RC structure is that the steel and surrounding concrete act together and there should be no slip of the bar relative to its surrounding concrete.

The additional length of bar is required beyond a given critical section for proper anchorage is called development length.



According to IS : 456-2000, Clause 26.2.1, $L_d = \frac{\phi \sigma_s}{4\tau_{bd}}$

Where $\sigma_s = 0.87 f_y$

$\Rightarrow$ Development length $= L_d = \frac{0.87 f_y \phi}{4\tau_{bd}}$

ϕ = Dia of main bar (Tension bar)

τ_{bd} = Permissible bond stress (Refer clause 26.2.1.1)

WORKED EXAMPLES ON DEVELOPMENT LENGTH

1. Column of a multistoried building is reinforced with 25mm diameter Fe 415 bars. Calculate the lap length required. Use M25 grade concrete.

Solution :

Given : $\phi = 25\text{mm}, f_y = 415\ \text{N/mm}^2, L_d = ?, f_{ck} = 25\ \text{N/mm}^2$

Refer IS : 456-2000, Clause 26.2.1

$$L_d = \frac{0.87 f_y \phi}{4\tau_{bd}}$$

$$\tau_{bd} = 1.4 \text{ for M25 concrete (Refer clause 26.2.1.1)}$$

For deformed bars, these values shall be increased by 60% ($f_y = 415\text{N/mm}^2$)

For bars in compression, the values of bond stress for bars in tension shall be increased by 25%

Columns are compression members

$$\therefore \tau_{bd} = 1.4 \times 1.6 \times 1.25 = 2.8\text{N/mm}^2$$

$$\therefore L_d = \frac{0.87 \times 415 \times 25}{4 \times 1.4 \times 1.6 \times 1.25} = \frac{9026.25}{11.2} = 805.91\text{mm}$$

2. **A cantilever beam having width of 250mm and effective depth 300mm, supports a UDL and is reinforced with 3 bars of 20mm diameter on the tension side. Use $f_{ck} = 20N/mm^2$ and $f_y = 415N/mm^2$.**

Find (1) The anchorage length required

(2) The average bond stress, if anchorage length provided is 900mm

Solution :

Given : $b = 250mm, d = 300mm$

$\phi = 20mm, f_{ck} = 20N/mm^2$ and $f_y = 415\ N/mm^2$

1. Calculation of anchorage length or development length

Refer IS : 456-2000, clause 26.2.1

$$L_d = \frac{0.87 f_y \phi}{4\tau_{bd}}$$

$$\tau_{bd} = 1.2 \text{ for M20 concrete (Clause 26.2.1.1)}$$

60% increased due to deformed bars ($f_y = 415N/mm^2$)

$$\therefore \tau_{bd} = 1.2 \times 1.6 = 1.92\ N/mm^2$$

$$\therefore L_d = \frac{0.87 \times 415 \times 20}{4 \times 1.2 \times 1.6} = 940.23mm$$

2. Calculation of Average bond stress, $L_d = 900mm$

$$\tau_{bd} = \frac{0.87 f_y \phi}{4 L_d} = \frac{0.87 \times 415 \times 20}{4 \times 900} = 2N/mm^2$$

THANK YOU