MODULE - 3

KANIS METHODS

Procedure: 1) FEM
2) Rotation factor (U)
3) Kani's Box & Rotation moment (m')
4) Final moments
5) Diagrams

1) FEM:

Refer Unit-2 → Slope Deflection method.

2) Rotation factor (U):

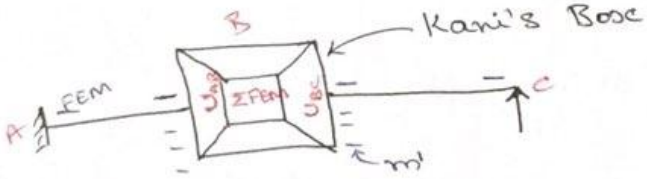
(For intermediate support joints)

Joint	Member	Relative Stiffness (K)	ΣK	$U = -\frac{1}{2} \times \frac{K}{\Sigma K}$

→ Relative Stiffness (K)

(Refer M.D. method notes - Unit-3)

3) Kani's Box & Rotation moment (m'):



(*) $m' = U[\Sigma FEM + \Sigma$ Far end Rotation moment$]$

→ @ Simple, Roller (or) Hinge Support
→ Add equal & opposite FEM
→ Carry ~~50%~~ half of the moment from the added value.
→ Then Calculate ΣFEM

→ For Overhang Portion, no need of Rotation factor & Kani's Box.

4) Final Moments:

$M = (\Sigma FEM + 2 \times$ Near end rotation moment $+ 1 \times$ Far end rotation moment$)$

5) Diagrams:

Refer Unit-2, Slope Deflection notes.

(*) Types of Problems:

I → Continuous Beam.
II → Continuous Beam with deflection.
III → Non-Sway frames.

(*) Important Formulas:

→ Rotation factor, $U = -\frac{1}{2} \times \frac{K}{\Sigma K}$

→ Rotation moment, $m' = U[\Sigma FEM + \Sigma$ Far end Rotation moment$]$

→ Final moment, $M = FEM + 2 \times$ Near end Rotation moment $+ 1 \times$ Far end Rotation moment.

Kani's Method

(i) Rotation Factor = $U = \left(\frac{-1}{2}\right)\frac{K}{\Sigma K}$

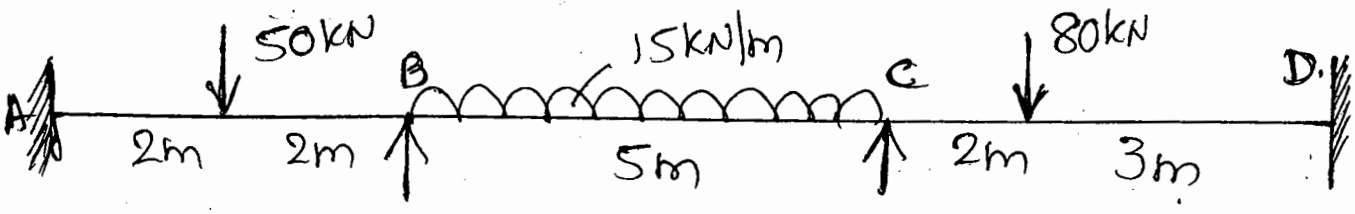
(ii) Rotation Moment :

$$M'_{AB} = U\left[\Sigma M_F + \Sigma \text{ Far end Rotation Moment}\right]$$

(iii) Final Moment

$$M = F.E.M + 2\left(\text{Near End Rotation Moment}\right) + \left(\text{Far end Rotation Moment}\right)$$

Eg:- 1] Analyse the beam shown by Kani's method, Draw BMD.



Sol:- (a) FEM

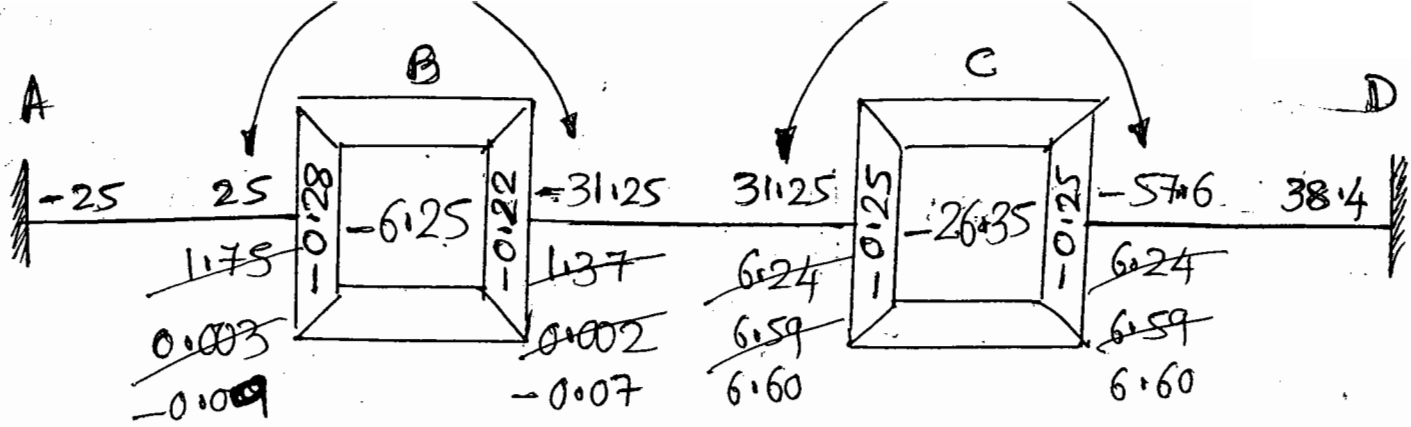
$M_{FAB} = -25\ kN\text{-}m$, $M_{FBA} = +25$

$M_{FBC} = -31.25$, $M_{FCB} = +31.25$

$M_{FCD} = -57.6$, $M_{FDC} = 38.4$.

(b) Rotation Factor (For Intermediate)

		k	Σk	$U = \left(-\frac{1}{2}\right)\frac{k}{\Sigma k}$
B	BA	$I/4 = 0.25I$	0.45I	-0.28
	BC	$I/5 = 0.20I$		-0.22
C	CB	$I/5 = 0.20I$	0.4I	-0.25
	CD	$I/5 = 0.20I$		-0.25



Rotation Moment $m'_{AB} = U\left[\Sigma M_F + \Sigma \text{ Far end Rotation moment}\right]$

Trial (1)

$m'_{BA} = -0.28\,(-6.25 + 0) = 1.75$

$m'_{BC} = -0.22\,(-6.25 + 0) = 1.37$

$m'_{CB} = -0.25\,(-26.35 + 1.37) = 6.24$

$m'_{CD} = -0.25\,(-26.35 + 1.37) = 6.24$

Trial (2)

$m'_{BA} = -0.28\,(-6.25 + 6.24) = 0.003$

$m'_{BC} = -0.22\,(-6.25 + 6.25) = 0.002$

$m'_{CB} = -0.25\,(-26.35 + 0.002) = 6.59$

$m'_{CD} = -0.25\,(-26.35 + 0.002) = 6.59$

Trial (3)

$m'_{BA} = -0.28\,(-6.25 + 6.59) = -0.09$

$m'_{BC} = -0.22\,(-6.25 + 6.59) = -0.07$

$m'_{CB} = -0.25\,(-26.35 - 0.07) = 6.60$

$m'_{CD} = -0.25\,(-26.35 - 0.07) = 6.60$

Final Moment

$$M = FEM + 2(\text{Near End Rotation moment}) + 1(\text{Far end Rotation moment})$$

$M_{AB} = -25 + 2(0) - 0.09 = -25.09$ kN-m ↺

$M_{BA} = +25 + 2(-0.09) + 0 = 24.82$ kN-m ↻

$M_{BC} = -31.25 + 2(-0.107) + 6.60 = -24.79$ kN-m ↺

$M_{CB} = +31.25 + 2(6.60) - 0.07 = 44.38$ " ↻

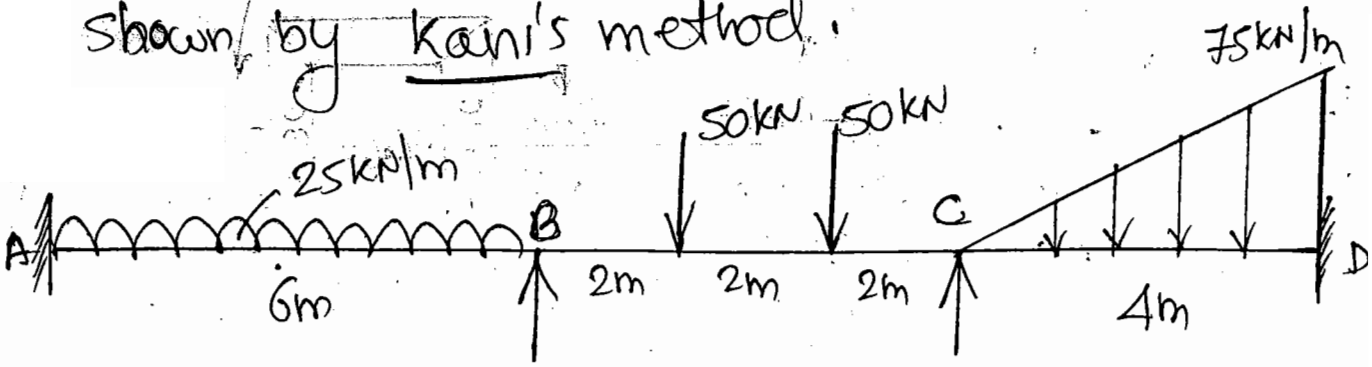
$M_{CD} = -57.6 + 2(6.60) - 0 = -44.40$ " ↺

$M_{DC} = 38.4 + 2(0) + 6.60 = 45$ kN-m ↻

Draw SFD, BMD & EC.

Refer S.D. Notes.

Eg:- Analyse the continuous beam shown by Kani's method.



(a) FEM

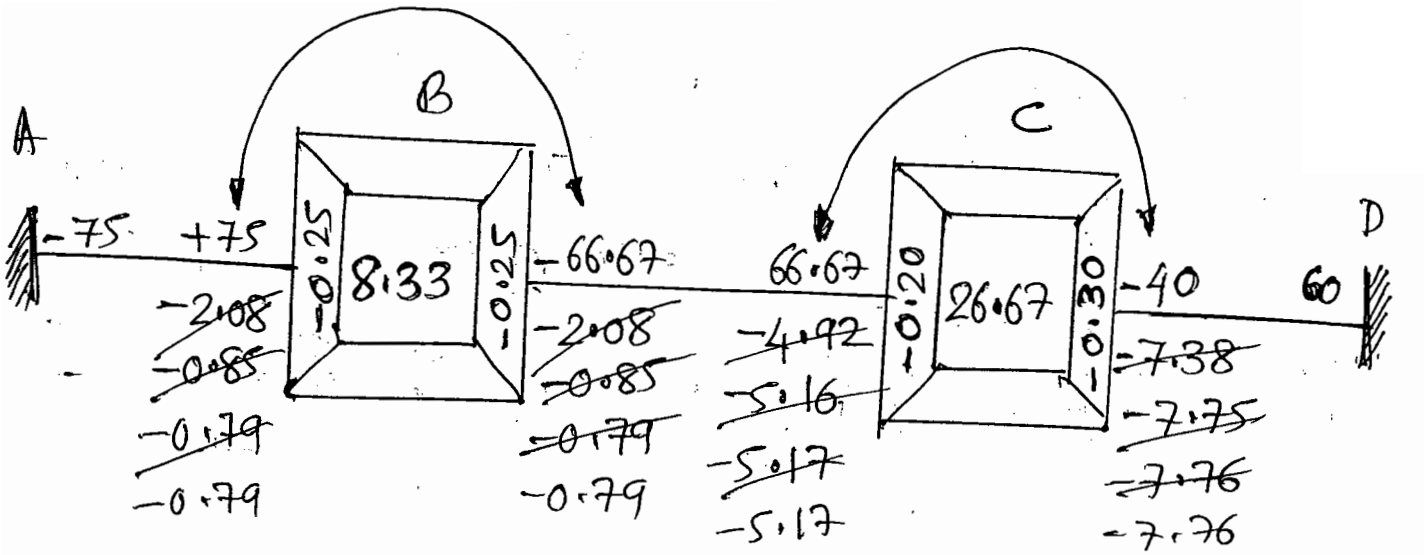
$$M_{FAB} = \frac{-wl^2}{12} = -75, \quad M_{FBA} = +75$$

$$M_{FBC} = \frac{-Wab^2}{l^2} = -66.67 \text{ KN-m}, \quad M_{FCB} = +66.67 \text{ KN-m}$$

$$M_{FCD} = \frac{-Wl^2}{30} = -40, \quad M_{FDC} = \frac{+wl^2}{20} = +60$$

(b) Rotation Factor (For Intermediate)

		K	ΣK	$U = \left(\frac{-1}{2}\right)\frac{K}{\Sigma K}$
B	BA	$I/l = I/6 = 0.167I$	0.334I	-0.25
	BC	$I/l = I/6 = 0.167I$		-0.25
C	CB	$I/l = I/6 = 0.167I$	0.417I	-0.20
	CD	$I/l = \frac{I}{4} = 0.25I$		-0.30



Rotation Moment

$$m = U\left[\Sigma M_F + \Sigma \text{ Far end Rotation Moment}\right]$$ ✓

Trial (1)

$M_{BA} = -0.25\ (8.33 + 0) = -2.08$

$M_{BC} = -0.25\ (8.33 + 0) = -2.08$

$M_{CB} = -0.20\ (26.67 - 2.08) = -4.92$

$M_{CD} = -0.30\ (26.67 - 2.08) = -7.38$

Trial (2)

$M_{BA} = -0.25(8.33 - 4.92) = -0.85$

$M_{BC} = -0.25(8.33 - 4.92) = -0.85$

$M_{CB} = -0.20(26.67 - 0.85) = -5.16$

$M_{CD} = -0.30(26.67 - 0.85) = -7.75$

Trial ③

$M_{BA} = -0.25\,(8.33 - 5.16) = -0.79$

$m_{BC} = -0.25\,(8.33 - 5.16) = -0.79$

$m_{CB} = -0.20\,(26.67 - 0.79) = -5.17$

$m_{CD} = -0.30\,(26.67 - 0.79) = -7.76$

Trial ④

$M_{BA} = -0.79$ kN-m

$M_{BC} = -0.79$

Final Moment :-

$$M = FEM + 2\left[\begin{matrix}\text{Near end}\\ \text{Ro. Moment}\end{matrix}\right] + \left[\begin{matrix}\text{Far End}\\ \text{Ro. moment}\end{matrix}\right]$$

$M_{AB} = -75 + 2(0) - 0.79 = -75.79$ kN-m ↺

$M_{BA} = +75 + 2(-0.79) + 0 = 73.42$ kN-m ↻

$M_{BC} = -66.67 + 2(-0.79) - 5.17 = -73.42$ kN-m ↺

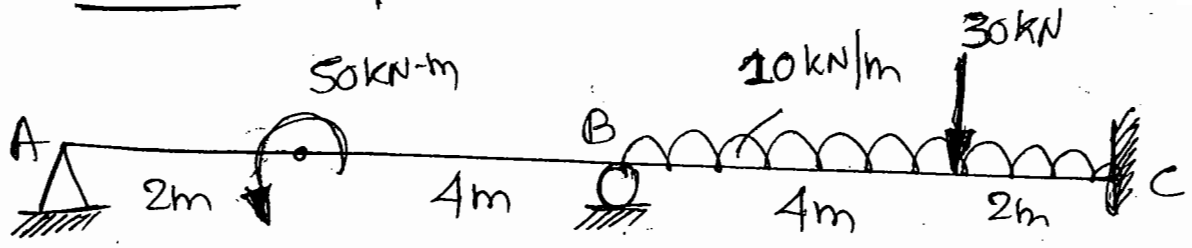
$M_{CB} = +66.67 + 2(-5.17) - 0.79 = 55.54$ kN-m ↻

$M_{CD} = -40 + 2(-7.76) + 0 = -55.52$ kN-m ↺

$M_{DC} = +60 + 2(0) - 7.76 = 52.24$ kN-m ↻

Refer M.D. Notes for SFD & BMD

Eg:-3] Analyse the beam shown by Kani's method



Sol^n (a) FEM

$$M_{FAB} = -\frac{Mb(2a-b)}{l^2} = -\frac{50\times4(2\times2-4)}{6^2} = 0$$

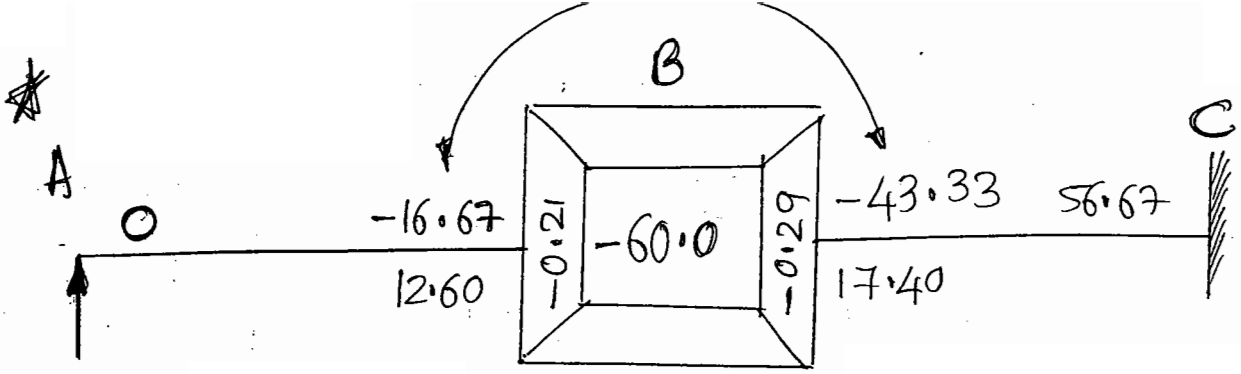
$$M_{FBA} = -\frac{Ma(2b-a)}{l^2} = -\frac{50\times2(2\times4-2)}{6^2} = -16.67$$

$$M_{FBC} = -\frac{wl^2}{12} - \frac{Wab^2}{l^2} = -43.33 \text{ kN-m}$$

$$M_{FCB} = +\frac{wl^2}{12} + \frac{Wa^2b}{l^2} = +56.67$$

(b) Rotation Factor : (For Intermediate)

		k	Σk	$U=\left(-\frac{1}{2}\right)\frac{k}{\Sigma k}$
B	BA	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{I}{6}\right) = 0.125I$	$0.292I$	-0.21
	BC	$\frac{I}{l} = \frac{I}{6} = 0.167I$		-0.29



Rotation Moment

$$M = U\left[\sum M_F + \sum \text{Rotation far end moment}\right]$$

$$M_{BA} = -0.21\ (-60 + 0) = 12.60$$

$$M_{BC} = -0.29\ (-60 + 0) = 17.40$$

Final Moment :

$$M = FEM + 2(\text{Near end Ro. moment}) + (\text{Far end Ro moment})$$

$M_{AB} = 0$ ✱ If last support is simple or Hinge or Roller the above eqn is not applicable.

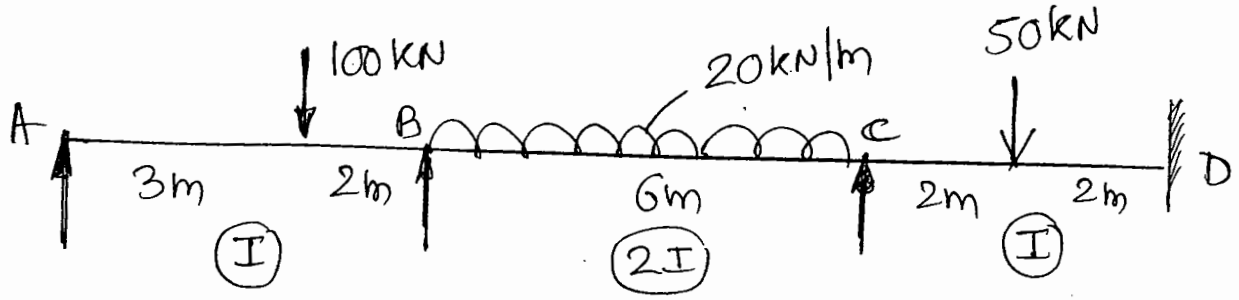
$M_{BA} = -16.67 + 2(12.60) + 0 = 8.53$ ↷

$M_{BC} = -43.33 + 2(17.40) + 0 = -8.53$ kN-m ↶

$M_{CB} = 56.67 + 2(0) + 17.40 = 74.07$ kN-m ↷.

Draw BMD & SFD.

Eg:- 4] Analyse the beam shown by kani's method



Sol:

(a) FEM

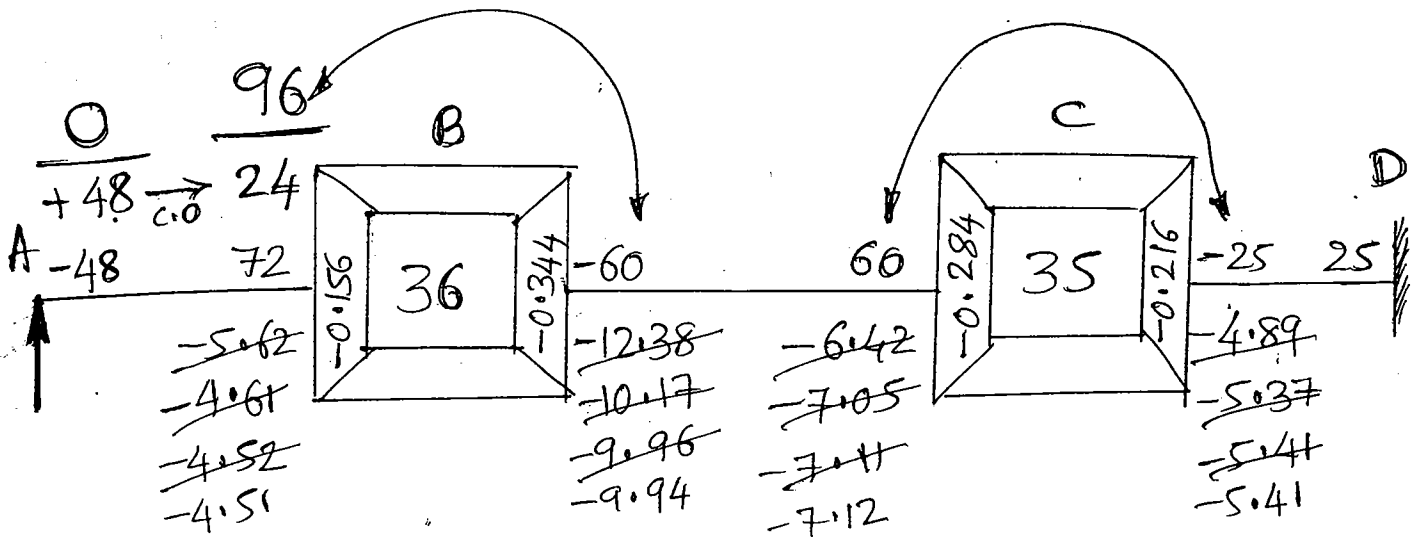
$M_{FAB} = -48$, $M_{FBA} = 72$

$M_{FBC} = -60$, $M_{FCB} = +60$

$M_{FCD} = -25$, $M_{FDC} = 25$.

(b) Rotation Factor :-

		K	ΣK	$U = (-\frac{1}{2})\frac{K}{\Sigma K}$
B	BA	$\frac{3}{4}(\frac{I}{5}) = 0.15I$	0.48I	-0.156
	BC	$\frac{2I}{6} = 0.33I$		-0.344
C	CB	$\frac{2I}{6} = 0.33I$	0.58I	-0.284
	CD	$\frac{I}{4} = 0.25I$		-0.216



Rotation Moment

$$M = U\left[\Sigma M_F + \Sigma \text{Far End Ro. Moment}\right]$$

Trial-(1)

$$m_{BA} = -0.156\ (36 + 0) = -5.62$$

$$m_{BC} = -0.344\ (36 + 0) = -12.38$$

$$m_{CB} = -0.284\ (35 - 12.38) = -6.42$$

$$m_{CD} = -0.216\ (35 - 12.38) = -4.89$$

Final Moment : $M = FEM + 2(\text{Near}) + (\text{Far})$

$M_{AB} = 0$ * The above eqⁿ is not applicable *

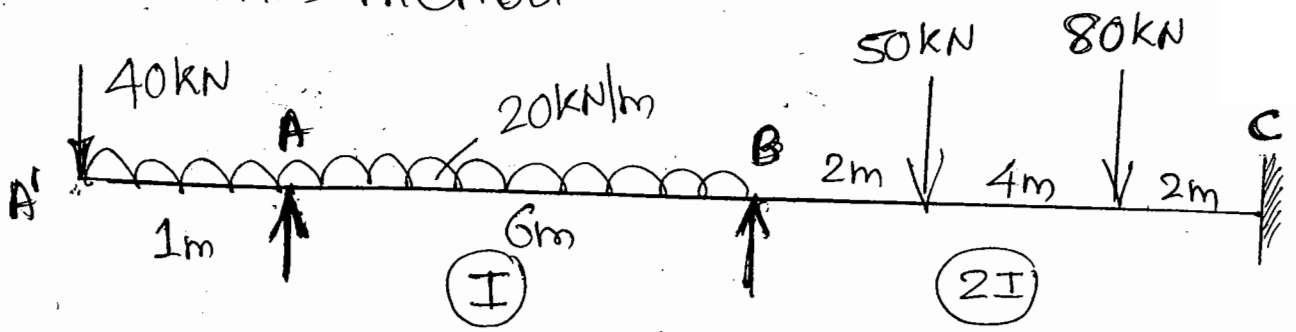
$M_{BA} = 96 + 2(-4.51) + 0 = 86.98$ kN-m

$M_{BC} = -60 + 2(-9.94) - 7.12 = -87.00$ kN-m

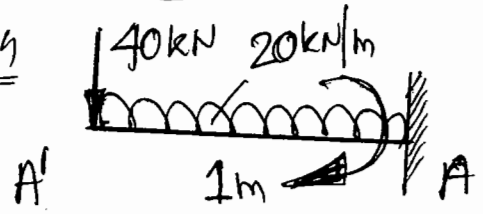
$M_{CB} = +60 + 2(-7.12) - 9.94 = 35.82$ kN-m

$M_{CD} = -35.82$ kN-m, $M_{DC} = 19.59$ kN-m

Eg: 5] Analyse the beam shown by kani's method.



Sol²: (a) FEM



$M_{AA'} = +40\times1 + 20\times1\times1/2 = +50$ kN-m (Final moment)

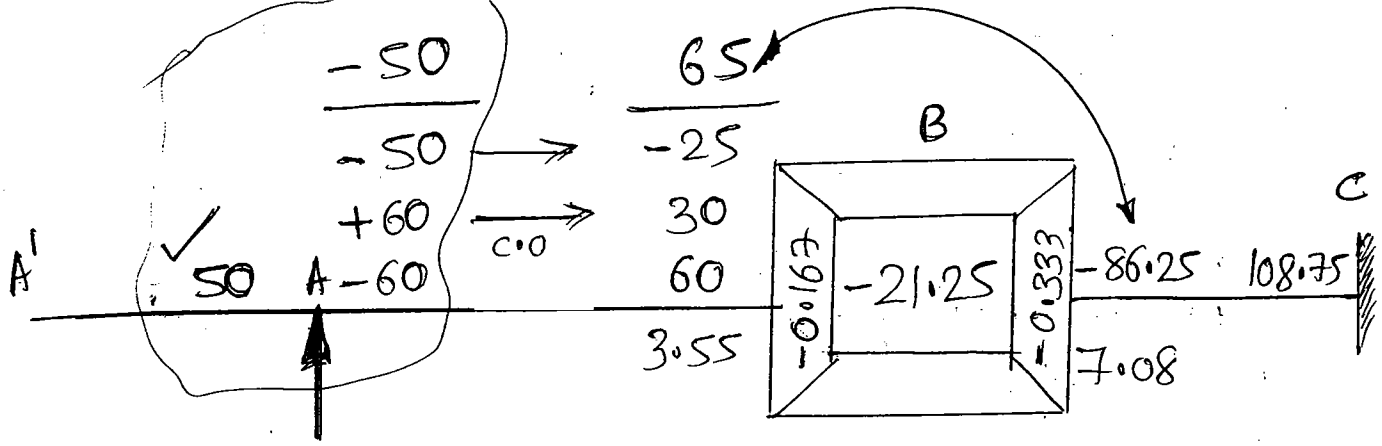
$M_{FAB} = -60$, $M_{FBA} = +60$

$M_{FBC} = -86.25$, $M_{FCB} = 108.75$.

(b) Rotation Factor :

★ The support with overhang portion, ~~section~~ rotation factor and Kani's Box ★ both are not required.

		K	ΣK	$U = \left(-\frac{1}{2}\right)\frac{K}{\Sigma K}$
B	BA	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{I}{6}\right) = 0.125I$	0.375I	-0.167
	BC	$\frac{I}{l} = \frac{2I}{8} = 0.25I$		-0.333



Rotation moment

$$M_{BA} = -0.167(-21.25+0) = 3.55 \text{ kN-m}$$

$$M_{BC} = -0.333(-21.25+0) = 7.08 \text{ kN-m}$$

Final Moment: $\boxed{M = FEM + 2(Near) + (Far)}$ ✓

$M_{AA'} = 50$ kN-m

$M_{AB} = -50$ kN-m

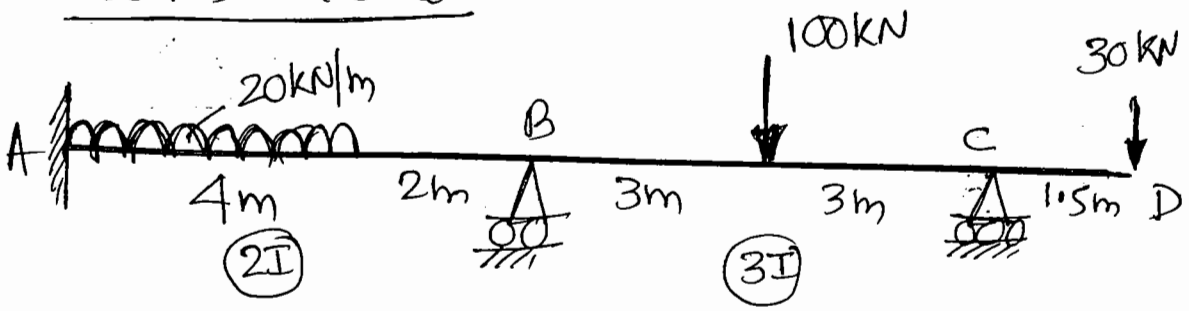
} ★ For these two the eqⁿ is not applicable. They are final moments.

$$M_{BA} = 65 + 2(3.55) + 0 = 72.1 \text{ kN-m} \curvearrowleft$$

$$M_{BC} = -86.25 + 2(7.08) + 0 = -72.10 \text{ kN-m} \curvearrowright$$

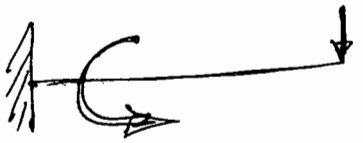
$$M_{CB} = 108.75 + 2(0) + 7.08 = 115.83 \text{ kN-m} \curvearrowleft$$

Eg:-6] Analyse the beam shown by Kani's method



Sol2 (a) $M_{FAB} = -53.33$, $M_{FBA} = +35.56$

$M_{FBC} = -75$, $M_{FCB} = +75$



$M_{CD} = -30 \times 1.5 = -45$ KN-m.

(b) R.F : (only at "B").

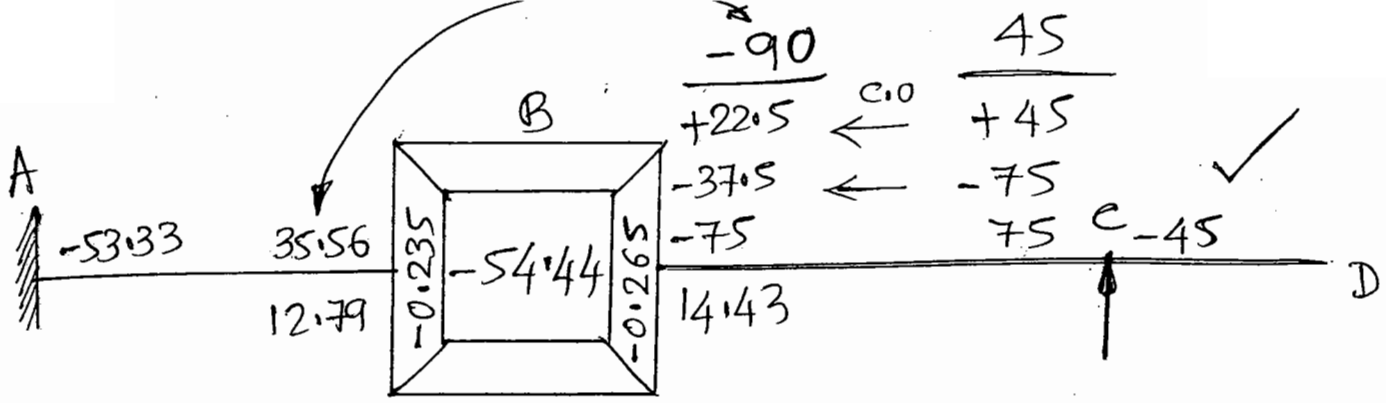
		K	ΣK	U
B	BA	$I/l = 2I/6 = 0.333I$	0.708I	-0.235
	BC	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{3I}{6}\right) = 0.375I$		-0.265

☆ Any overhanging moments are final.

∴ $M_{CD} = -45$ KN-m

∴ From equilibrium point of view

"M_{CB}" should be +45KN-m



Rotation Moment

$$m = U\left[\Sigma FEM + \Sigma \text{Farend Ro. Moment}\right]$$

$$m_{BA} = -0.235(-54.44 + 0) = 12.79$$

$$m_{BC} = -0.265(-54.44 + 0) = 14.43$$

Final Moments :

$$M = FEM + 2(\text{Near}) + (\text{Far})$$

$$M_{AB} = -53.33 + 2(0) + 12.79 = -40.54 \text{ kN-m} \curvearrowright$$

$$M_{BA} = 35.56 + 2(12.79) + 0 = 61.14 \text{ kN-m} \curvearrowleft$$

$$M_{BC} = -90 + 2(14.43) + 0 = -61.14 \text{ kN-m} \curvearrowright$$

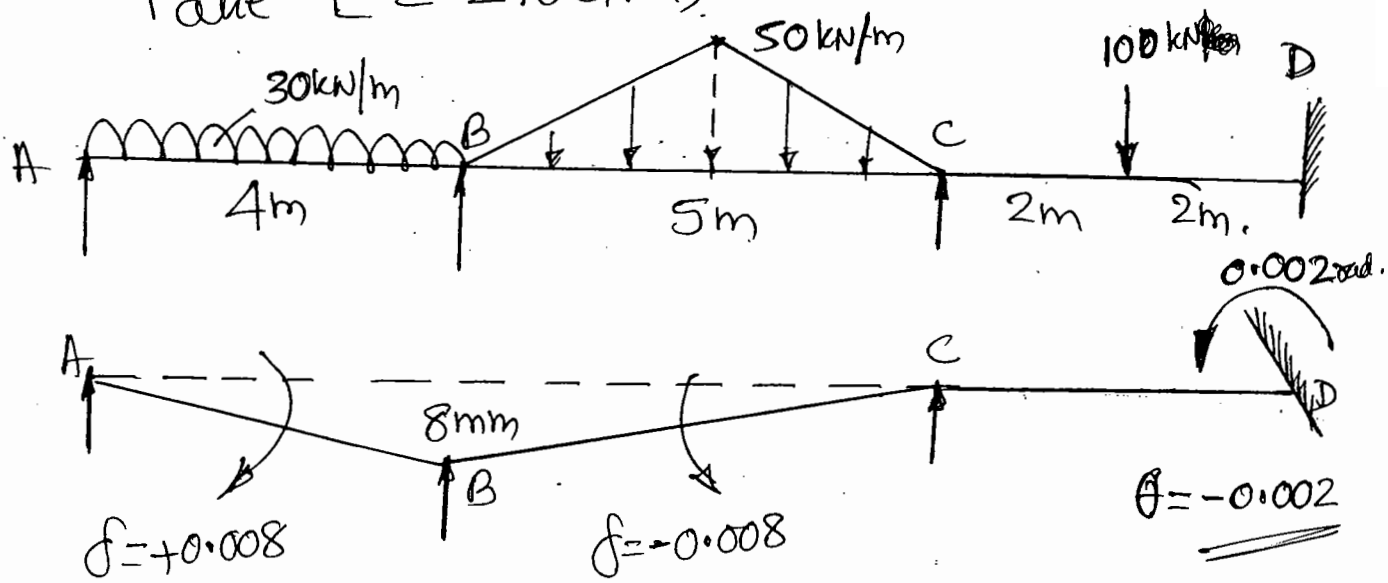
$$M_{CB} = +45 \text{ ★} \curvearrowleft$$

$$M_{CD} = -45 \text{ ★} \curvearrowright$$

Sinking of Support

Eg:- Analyse the beam shown by Kani's method. The support 'D' rotates by 0.002 rad. anti-clockwise. The support 'B' sinks by 8mm.

Take $E = 210 GPa$, $I = 0.1G\,mm^4$.



$$E = 210 \times 10^9 \times 10^{-6} = 210 \times 10^3\ N/mm^2$$

$$I = 0.1 \times 10^9\ mm^4$$

$$EI = \frac{(210 \times 10^3)(0.1)10^9}{(10^3)(10^3)^2}\ N\text{-}mm^2 = 21000\ kN\text{-}m^2$$

(a) FEM :

$$\text{Additional moment} = -\frac{6EI\delta}{l^2} \quad \text{(Sinking)}$$

$$= \frac{4EI\theta}{l} \rightarrow \text{Near end Rotation}$$

$= \frac{2EI\theta}{l}$ → Far end rotation

$$M_{FAB} = -\frac{\omega l^2}{12} - \frac{6EI\delta}{l^2} = -\frac{30\times4^2}{12} - \frac{6(21000)(0.008)}{4^2}$$

$$= -103.0 \text{ kN-m}$$

$$M_{FBA} = +\frac{\omega l^2}{12} - \frac{6EI\delta}{l^2} = -23 \text{ kN-m}.$$

$$M_{FBC} = \frac{-5\omega l^2}{96} - \frac{6EI\delta}{l^2} = -\frac{5(50)5^2}{96} - \frac{6(21000)(-0.008)}{5^2}$$

$$= -24.78$$

$$M_{FCB} = \frac{+5}{96}\omega l^2 - \frac{6EI\delta}{l^2} = 105.42 \text{ kN-m}.$$

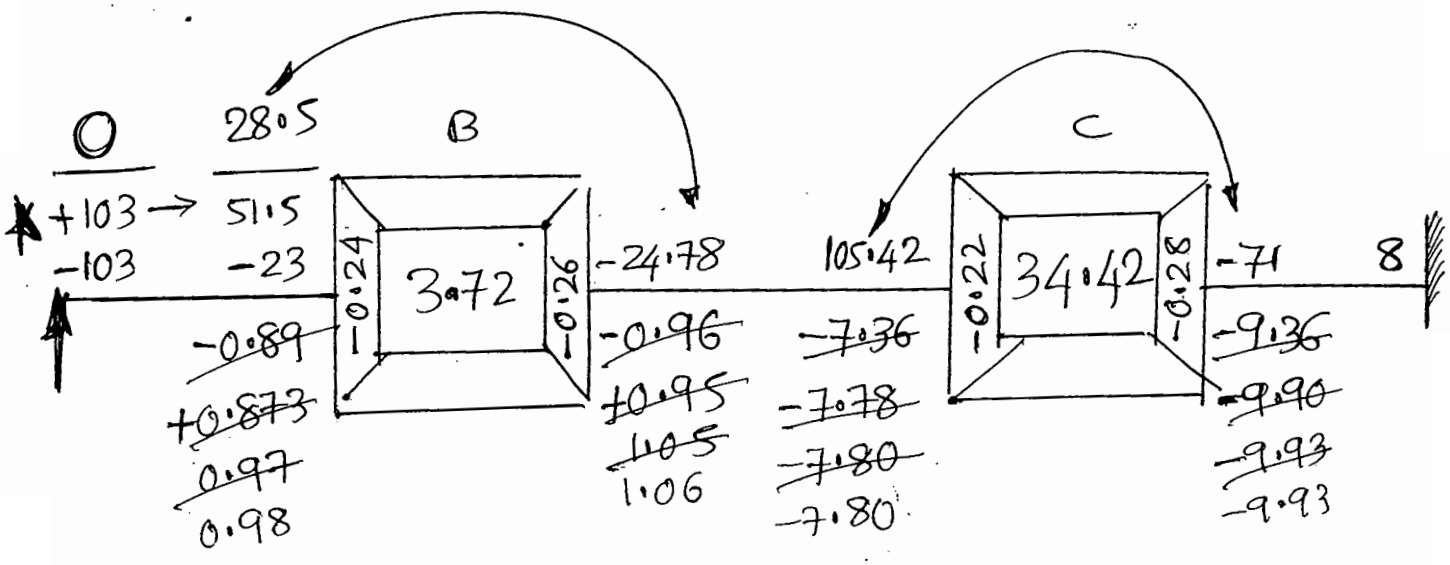
$$M_{FCD} = -\frac{Wl}{8} + \frac{2EI\theta}{l} \quad \text{(Far end)} = -\frac{100\times4}{8} + \frac{2(21000)(-0.002)}{4}$$

$$= -71 \text{ kN-m}$$

$$M^F_{DC} = +\frac{Wl}{8} + \frac{4EI\theta}{l} \quad \text{(Near)} = \frac{100\times4}{8} + \frac{4(21000)(-0.002)}{4}$$

$$= +8$$

		K	ΣK	U
B	BA	$\frac{3}{4}(I/4) = 0.1875I$	0.3875I	-0.24
	BC	$I/5 = 0.2I$		-0.26
C	CB	$I/5 = 0.2I$	0.45I	-0.22
	CD	$I/4 = 0.25J$		-0.28



$$m = U\left[\Sigma FEM + \Sigma \text{ Far end Ro. moment}\right]$$

Trial (1)

$$m_{BA} = -0.24\,(3.72 + 0) = -0.89$$

$$m_{BC} = -0.26\,(3.78 + 0) = -0.96$$

$$m_{CB} = -0.22\,(34.42 - 0.96) = -7.36$$

$$m_{CD} = -0.28\,(34.42 - 0.96) = -9.36$$

Final Moment

$M_{AB} = 0$ ★

$M_{BA} = 28.5 + 2(0.98) + 0 = 30.46$ kN-m ↻

$M_{BC} =$ $= -30.46$ kN-m ↺

$M_{CB} =$ $= 90.88$ kN-m ↻

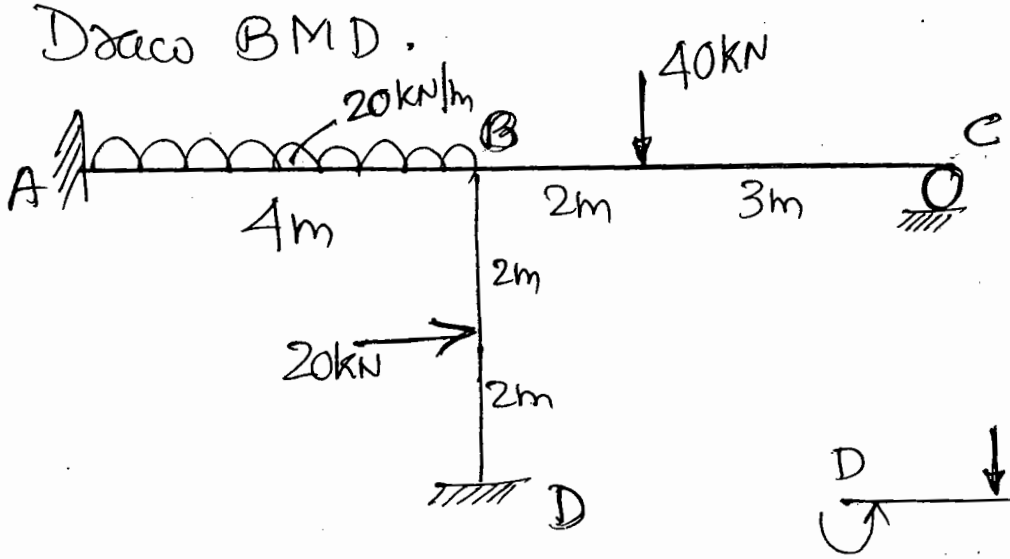
$M_{CD} =$ $= -90.86$ kN-m ↺

$M_{DC} =$ $= -1.93$ kN-m ↺

Non Sway Frames

Eg:- Analyse the frame by kani's method.

Draco BMD.



Sol2

(a) FEM

$M_{FAB} = -26.67$, $M_{FBA} = +26.67$

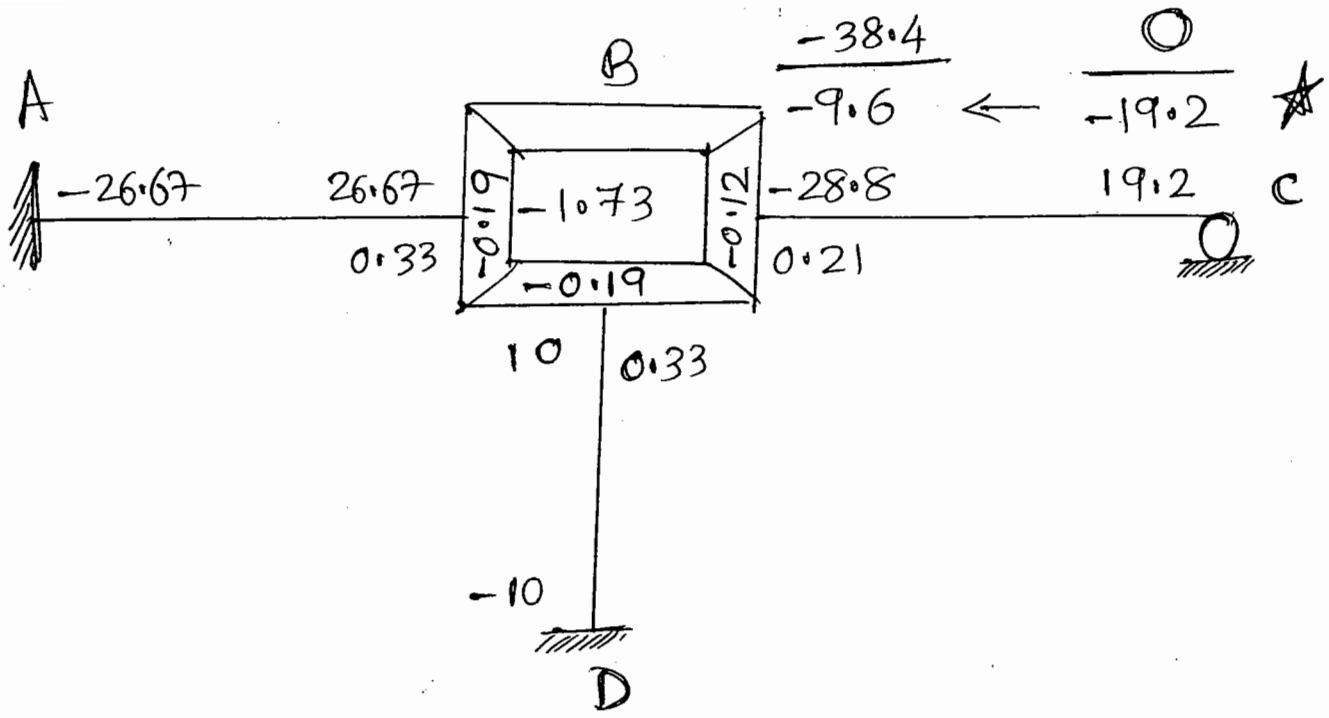
$M_{FBC} = -28.8$, $M_{FCB} = +19.20$

$M_{FDB} = -10$, $M_{FBD} = +10$

(b) R.F

		k	Σk	U
	BA	$I/4 = 0.25I$		-0.19
B	BC	$\frac{3}{4}(I/5) = 0.15I$	0.65I	-0.12
	BD	$I/4 = 0.25I$		-0.19

At B → 26.67 − 38.4 + 10 = (−1.73)
BA BC BD



$m_{BA} = -0.19(-1.73+0) = 0.33$

$m_{BC} = -0.12(-1.73+0) = 0.21$

$m_{BD} = -0.19(-1.73+0) = 0.33$

Final :−

$M_{AB} = -26.67 + 2(0) + 0.33 = -26.34$ KN-m ↺

$M_{BA} = 26.67 + 2(0.33) + 0 = 27.33$ " ↻

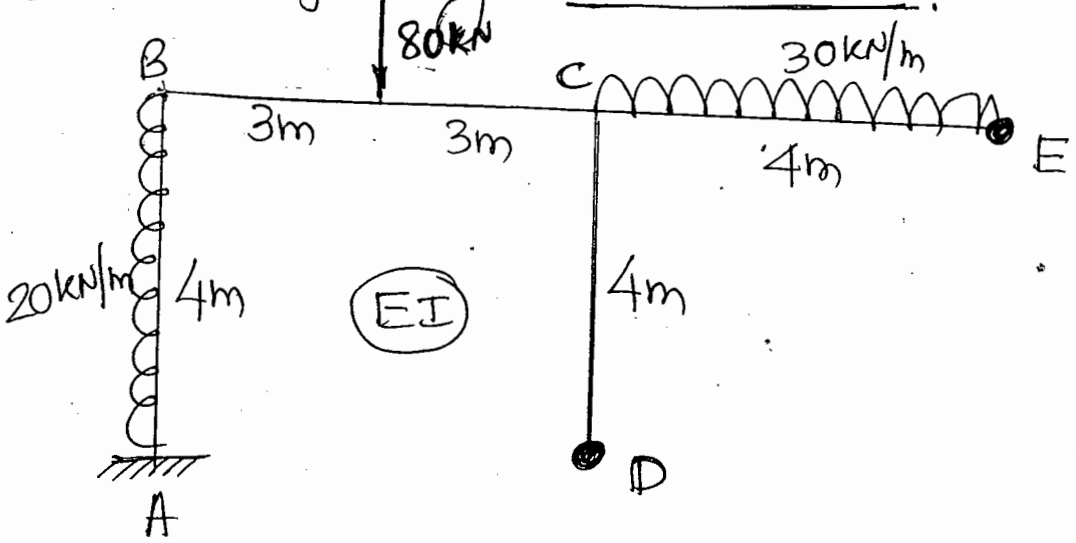
$M_{BC} = -38.4 + 2(0.21) + 0 = -37.98$ " ↺

$M_{CB} = 0$ ★

$M_{BD} = 10 + 2(0.33) + 0 = 10.66$ " ↻

$M_{DB} = -10 + 2(0) + 0.33 = -9.67$ " ↺

Eg:- Analyse by Kani's Method



Sol:

(a) FEM

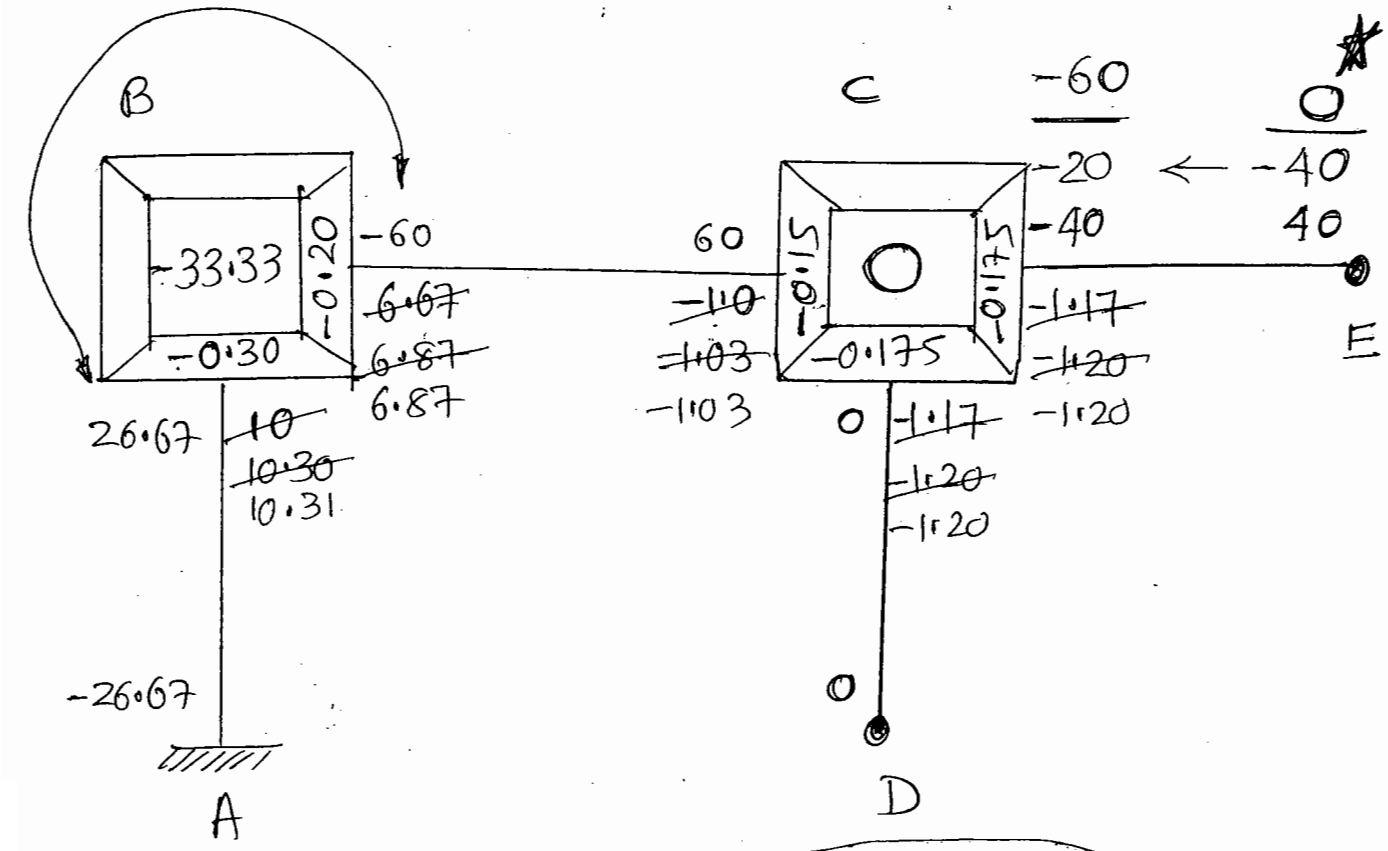
$M_{FAB} = -26.67$, $M_{FBA} = +26.66$ kN

$M_{FBC} = -60$, $M_{FCB} = +60$

$M_{FCE} = -40$, $M_{FEC} = +40$

(b) Rotation Factor (For "B" & "C")

		K	ΣK	$U=\left(\frac{-1}{2}\right)\frac{K}{\Sigma K}$
B	BA	$I/4 = 0.25I$	$0.416I$	-0.3
	BC	$I/6 = 0.167I$		-0.2
C	CB	$I/6 = 0.167I$	$0.542I$	-0.15
	CD	$\frac{3}{4}\left(\frac{I}{4}\right) = 0.1875I$		-0.175
	CE	$\frac{3}{4}\left(\frac{I}{4}\right) = 0.1875I$		-0.175



$$m = U\left[\sum FEM + \sum \text{Far end Rot. moment}\right]$$

Final Moment

$M_{AB} = -26.67 + 2(0) + 10.31 = -16.36$ ↺

$M_{BA} = 26.67 + 2(10.31) + 0 = 47.29$ ↻

$M_{BC} = -60 + 2(6.87) - 1.03 = -47.29$ ↺

$M_{CB} = 60 + 2(-1.03) + 6.87 = 64.81$ ↻

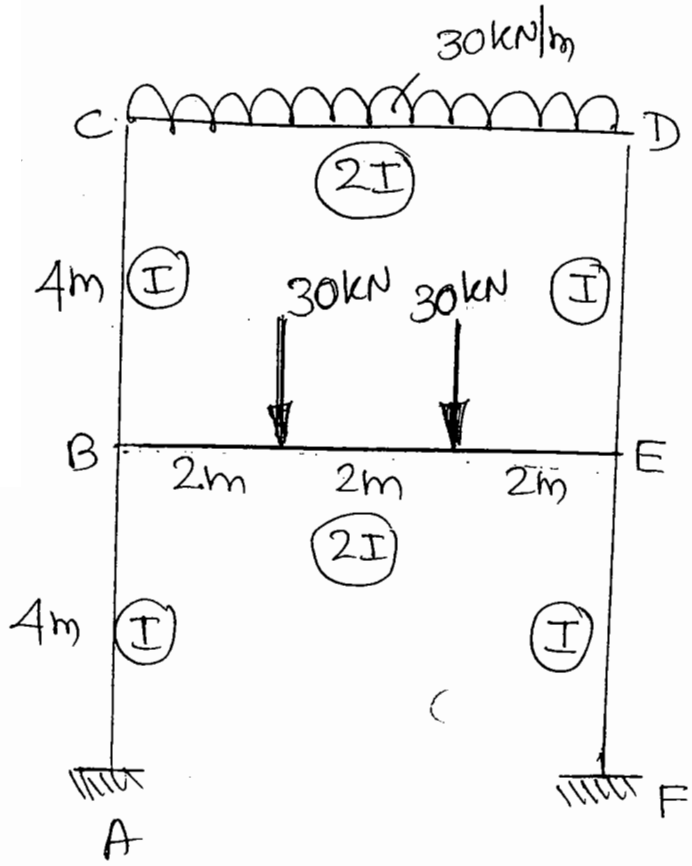
$M_{CD} = 0 + 2(-1.20) + 0 = -2.40$ ↺

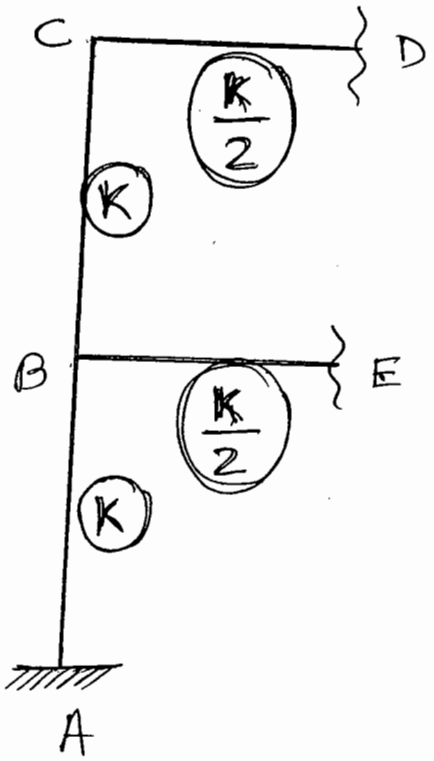
$M_{DC} = 0$

$M_{CE} = -60 + 2(-1.20) + 0 = -62.40$ kN-m ↺

Draw BMD.

Eg:- Analyse the frame shown by Kani's method





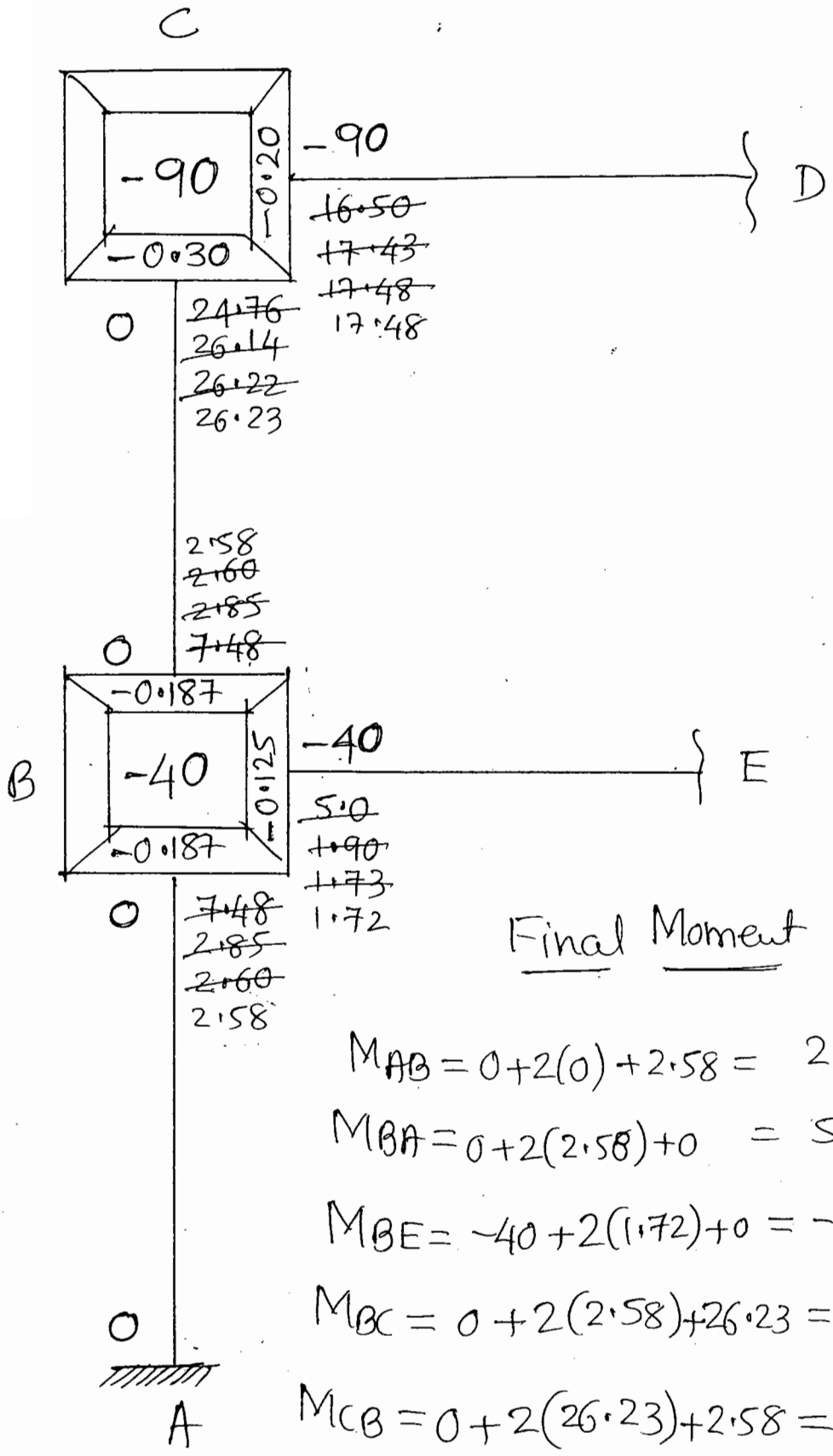
(a) FEM

$$M_{FBE} = \frac{-Wab^2}{l^2} = -\left[\frac{30\times2\times4^2}{6^2} + \frac{30\times4\times2^2}{6^2}\right] = -40$$

$$M_{FCD} = \frac{-wl^2}{12} = -90$$

(b) R.F (Only For "B" & "C")

		K	ΣK	U
B	BA	$K = I/l = I/4 = 0.25I$	0.667I	-0.187
	BC	$K = I/l = I/4 = 0.25I$		-0.187
	BE	$\frac{K}{2} = \frac{1}{2}\left(\frac{I}{l}\right) = \frac{1}{2}\left(\frac{2I}{6}\right) = 0.167I$		-0.125
C	CB	$K = I/l = I/4 = 0.25I$	0.417I	-0.30
	CD	$\frac{K}{2} = \frac{1}{2}\left(\frac{I}{l}\right) = \frac{1}{2}\left(\frac{2I}{6}\right) = 0.167I$		-0.20



Final Moment

$M_{AB} = 0 + 2(0) + 2.58 = 2.58$ kN-m

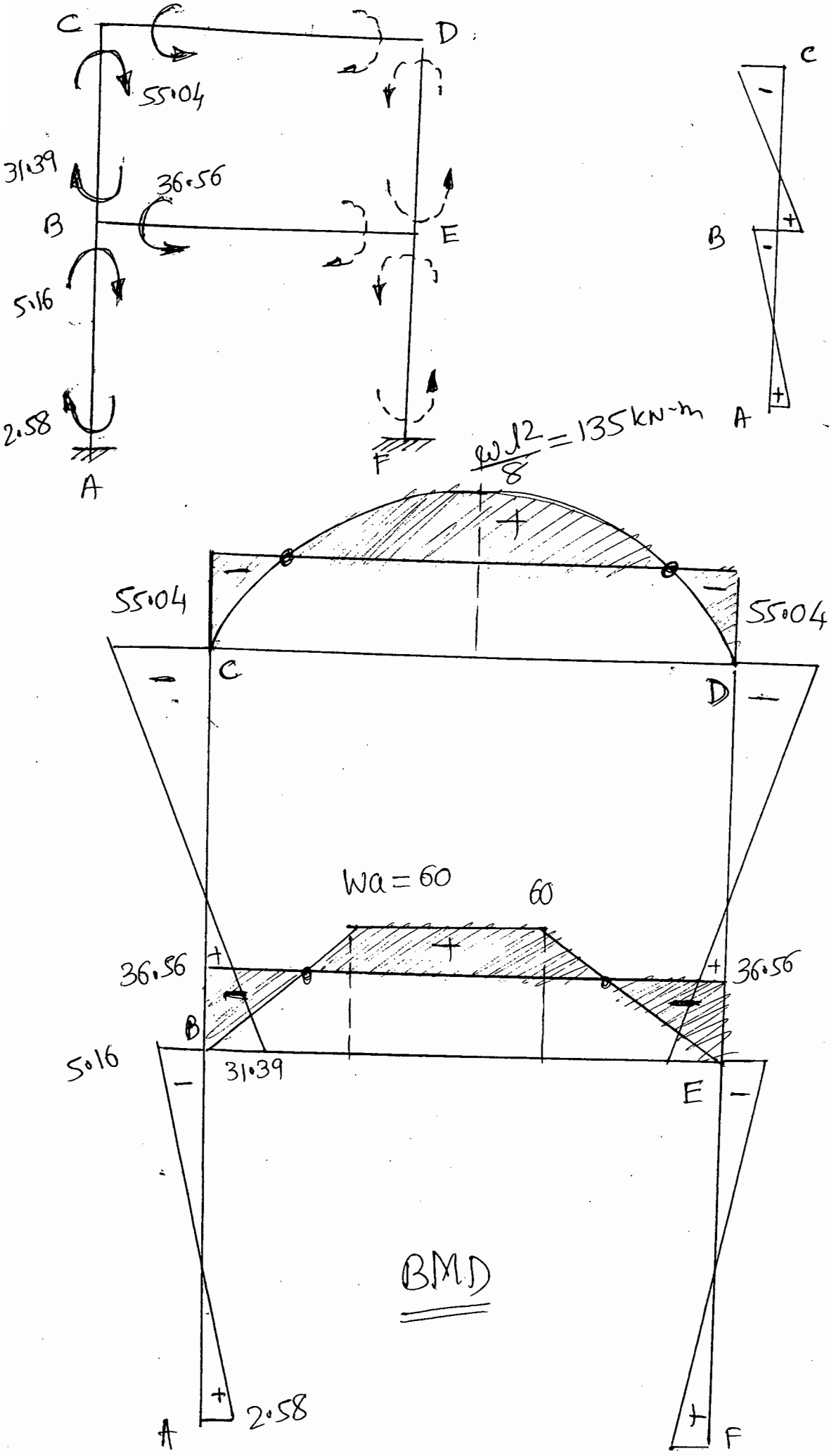
$M_{BA} = 0 + 2(2.58) + 0 = 5.16$ kN-m

$M_{BE} = -40 + 2(1.72) + 0 = -36.56$ "

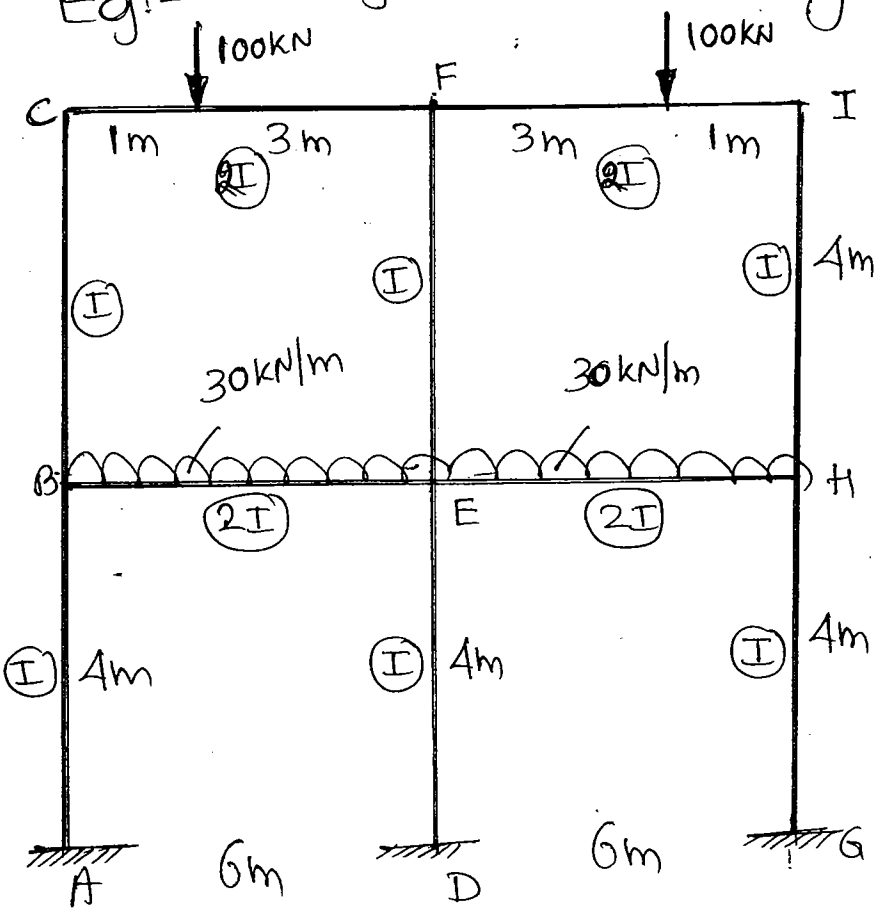
$M_{BC} = 0 + 2(2.58) + 26.23 = 31.39$ "

$M_{CB} = 0 + 2(26.23) + 2.58 = 55.04$ "

$M_{CD} = -90 + 2(17.48) + 0 = -55.04$ "


C
D
55.04
31.39
36.56
B
E
5.16
2.58
A
F
C
B
A
$\frac{wl^2}{8}$ = 135 kN-m
55.04
55.04
C
D
Wa = 60
60
36.56
36.56
B
5.16
31.39
E
2.58
A
F
BMD

Eg:- Analyse the frame by Kani's

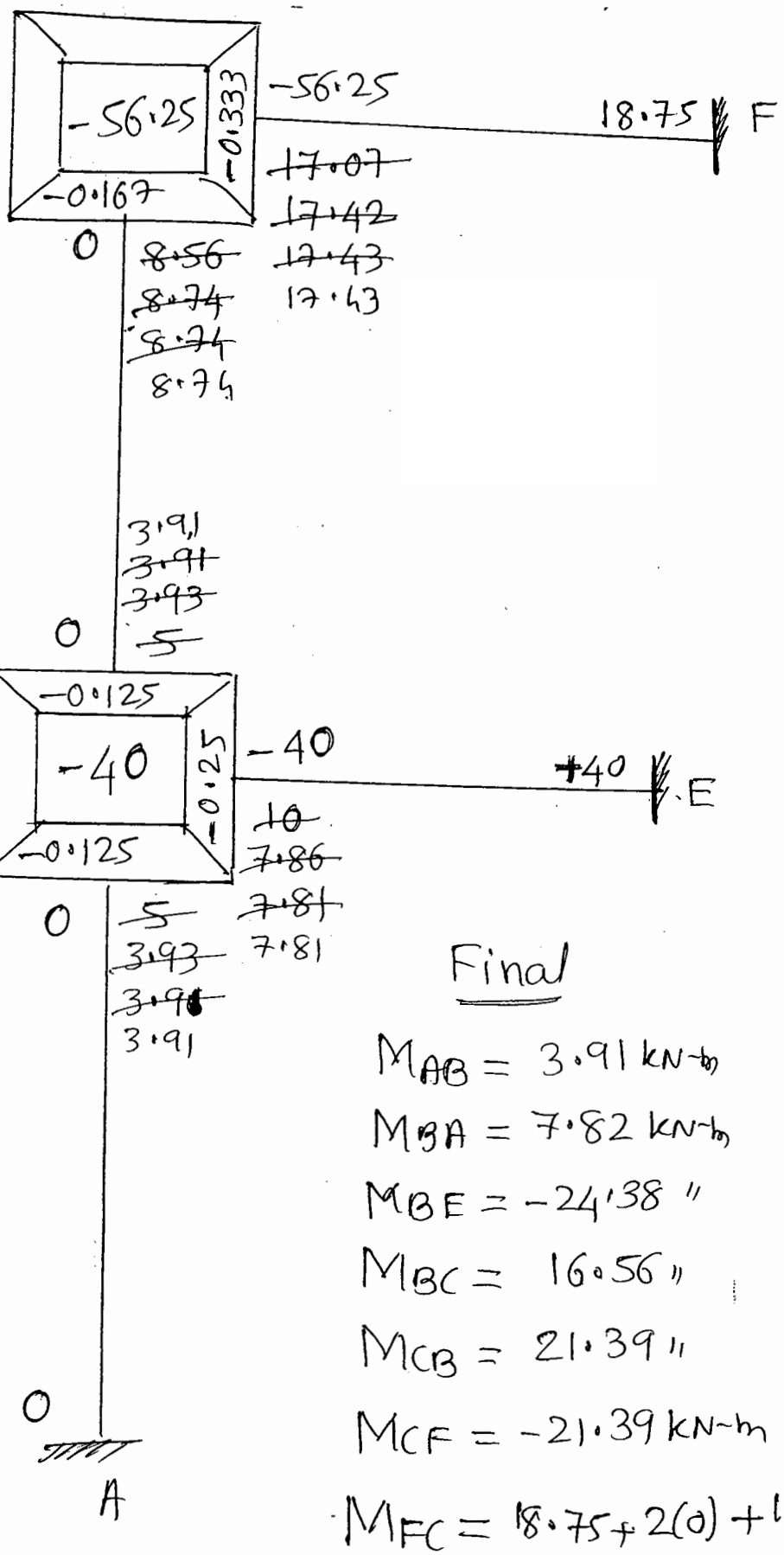


Sol2 (a) FEM

$M_{FBE} = -40$ kN-m, $M_{FCF} = \frac{-Wab^2}{l^2} = -56.25$ kN-m

(b) R.F (only at B & C) $M_{FFC} = \frac{Wa^2b}{l^2} = 18.75$

		K	Σk	U
B	BA	$K = I/4 = 0.25I$	1.0 I	-0.125
	BE	$K = 2I/4 = 0.5I$		-0.25
	BC	$K = I/4 = 0.25I$		-0.125
C	CB	$K = I/4 = 0.25I$	0.75I	-0.167
	CF	$K = 2I/4 = 0.5I$		-0.333



Final

$M_{AB} = 3.91$ kN-m

$M_{BA} = 7.82$ kN-m

$M_{BE} = -24.38$ "

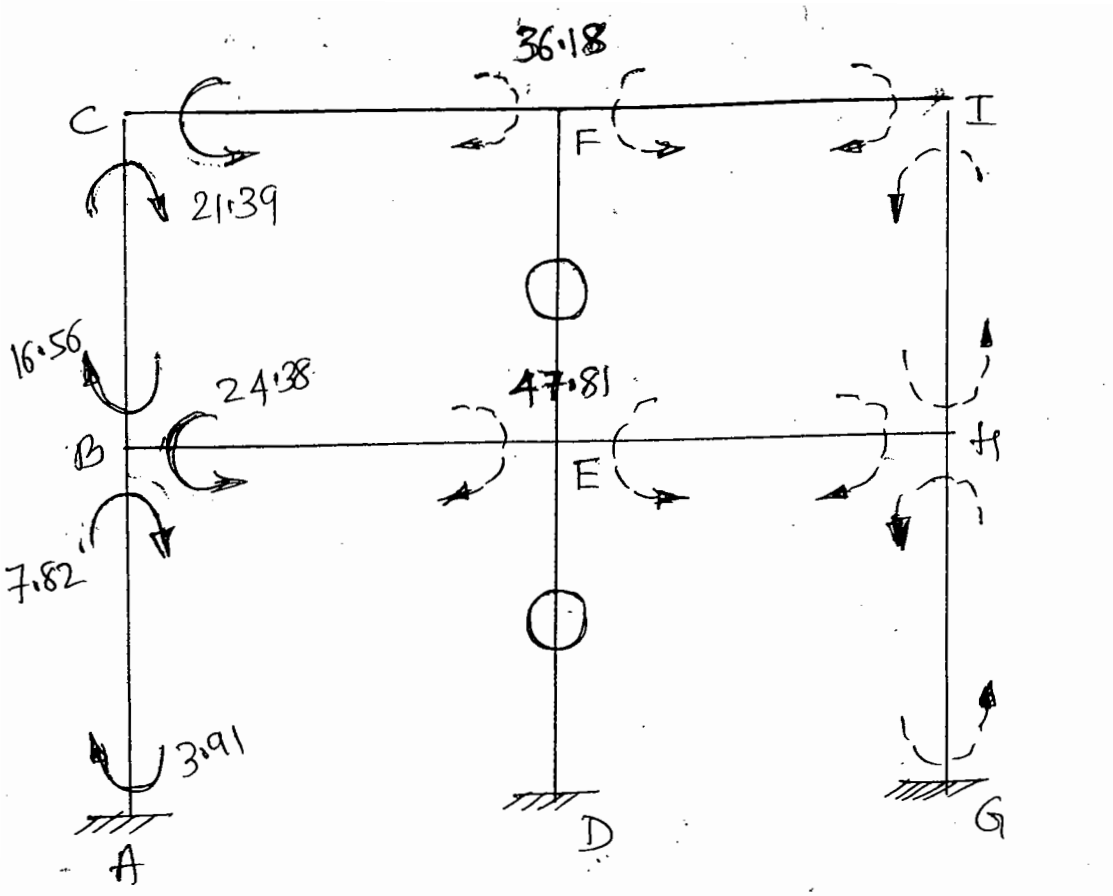
$M_{BC} = 16.56$ "

$M_{CB} = 21.39$ "

$M_{CF} = -21.39$ kN-m

$M_{FC} = 18.75 + 2(0) + 17.43 = 36.18$

$M_{EB} = 40 + 2(0) + 7.81 = 47.81$


36.18
C
F
I
21.39
16.56
24.38
47.81
B
E
H
7.82
3.91
A
D
G